eClassroom

A-LEVEL MATHEMATICS • FURTHER PURE 1

Matrices

Worked Solutions

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Full worked solutions for the free Matrices worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 1 Matrices worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\begin{pmatrix} 6+2 & 3-6 \\ 2-4 & 1+12 \end{pmatrix}$: $\boxed{\begin{pmatrix} 8 & -3 \\ -2 & 13 \end{pmatrix}}$

A2 $(1\times 2)(2\times 2)$ gives 1×2: $\begin{pmatrix} 6-4 & 2-0 \end{pmatrix}$: $\boxed{\begin{pmatrix} 2 & 2 \end{pmatrix}}$

A3 $12-10$: $\boxed{2}$

STRETCH

A4 $\det = 15-14 = 1$: $\boxed{\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}}$

A5 $6k-5(k+1) = k-5 = 0$: $\boxed{k=5}$

A6 $\mathbf{X} = (\mathbf{XA})\mathbf{A}^{-1}$, with $\mathbf{A}^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ on the right: $\boxed{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}$

A7 $\det = k(k+2)+3 = k^2+2k+3 = (k+1)^2+2 \geqslant 2$: **never zero, so non-singular for all real k**

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{\begin{pmatrix} 4 & -2 \\ 1 & -3 \end{pmatrix}}$ **(b)** $\boxed{\begin{pmatrix} 2 & 4 \\ 2 & -1 \end{pmatrix}}$ **(c)** $\boxed{\begin{pmatrix} -1 & 8 \\ -4 & 7 \end{pmatrix}}$ **(d)** $\boxed{\begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}}$

$\mathbf{AB}$ *and* $\mathbf{BA}$ *are different: order matters.*

B2 **(a)** $\boxed{2 \times 2}$ **(b)** $\boxed{3 \times 3}$ **(c)** $\boxed{\textbf{undefined: 3 columns, 2 rows}}$ **(d)** $\boxed{2 \times 3}$

The inner numbers must match.

B3 **(a)** $\boxed{5}$ **(b)** $12 - 15$: $\boxed{-3}$ **(c)** $\boxed{k^2 - 6}$ **(d)** $2ab - 2ab$: $\boxed{0}$

A row that is a multiple of another gives determinant 0

B4 **(a)** $\boxed{\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}}$ **(b)** $\boxed{\begin{pmatrix} 1 & -1 \\ -\frac{3}{2} & 2 \end{pmatrix}}$ **(c)** $\boxed{\begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ -2 & -1 \end{pmatrix}}$ **(d)** $\boxed{\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{5} \end{pmatrix}}$

Divide by the determinant after swapping and changing signs.

B5 **(a)** $k^2 - 4 = 0$: $\boxed{k = \pm 2}$ **(b)** $k^2 - k - 6 = 0$: $\boxed{k = 3 \text{ or } -2}$ **(c)** $8k - 3k = 0$: $\boxed{k = 0}$

Singular means the determinant is zero: find every root.

B6 **(a)** $\boxed{\frac{1}{5}\begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}}$ **(b)** $\mathbf{A}^{-1}\begin{pmatrix} 9 \\ 1 \end{pmatrix}$: $\boxed{x = 2, y = -5}$ **(c)** $\mathbf{A}^{-1}\begin{pmatrix} 3 & 0 \\ 2 & 5 \end{pmatrix}$: $\boxed{\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}}$ **(d)** $5\mathbf{A}^{-1}$: $\boxed{\begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}}$

One inverse solves several problems.

STRETCH

B7 **(a)** $\boxed{5}$ **(b)** $\boxed{5}$ **(c)** $\boxed{25}$ **(d)** $3^2 \times 5$: $\boxed{45}$

$\det(\mathbf{AB}) = \det\mathbf{A}\det\mathbf{B}$*; scaling a* 2×2 *matrix by 3 scales the determinant by 9*

B8 **(a)** $\boxed{k^2 - k - 6}$ **(b)** $(k - 3)(k + 2) = 0$: $\boxed{k = 3 \text{ or } -2}$ **(c)** $k^2 - k - 12 = 0$: $\boxed{k = 4 \text{ or } -3}$

(d) $\frac{1}{6}\begin{pmatrix} 3 & -2 \\ -3 & 4 \end{pmatrix}$: $\boxed{\begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{2}{3} \end{pmatrix}}$

The determinant controls everything about the inverse.

B9 **(a)** $\mathbf{A}^{-1}\mathbf{C}$: $\boxed{\begin{pmatrix} 3 & 4 \\ -4 & -5 \end{pmatrix}}$ **(b)** $\mathbf{CA}^{-1}$: $\boxed{\begin{pmatrix} -5 & 4 \\ -4 & 3 \end{pmatrix}}$ **(c)** $\boxed{\textbf{matrix multiplication is not commutative}}$

(d) $\mathbf{A}^{-1}\mathbf{CA}^{-1}$: $\boxed{\begin{pmatrix} -7 & 6 \\ 8 & -7 \end{pmatrix}}$

$\mathbf{A}^{-1}$ *goes on the same side as* $\mathbf{A}$

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B10 **(a)** $\mathbf{I}$ **(b)** $\mathbf{M}^{-1} = \mathbf{M}$ **(c)** $\mathbf{M}^4\mathbf{M}$: $\mathbf{M}$ **(d)** $\mathbf{I}$

A matrix with $\mathbf{M}^2 = \mathbf{I}$ is its own inverse.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $ad - bc = 3(1) - (-2)(4) = 3 + 8$; he lost the sign of -2: **11**

C2 The inverse swaps a and d and changes the signs of b and c, then divides by det = 1: $\begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$

C3 He found $\mathbf{BA}$, not $\mathbf{AB}$: $\begin{pmatrix} 3+2 & 0+2 \\ 0+1 & 0+1 \end{pmatrix}$: $\begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}$

C4 $k^2 - 36 = 0$ has two roots: **singular when $k = 6$ or $k = -6$**

STRETCH

C5 $\mathbf{A}$ is on the right, so $\mathbf{X} = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$: $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

C6 $\mathbf{AB} = \begin{pmatrix} 1-1 & -1+1 \\ 1-1 & -1+1 \end{pmatrix} = \mathbf{0}$, but neither $\mathbf{A}$ nor $\mathbf{B}$ is $\mathbf{0}$: **she is wrong**

C7 $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ has det $= 4 - 4 = 0$. An inverse exists only when $ad - bc \neq 0$

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C8 **(a)** $\mathbf{A}^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$; subtracting $(a+d)\mathbf{A} = \begin{pmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{pmatrix}$ leaves $\begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix}$, so adding $(ad-bc)\mathbf{I}$ gives $\mathbf{0}$, as required

(b) $\mathbf{A}\big((a+d)\mathbf{I}-\mathbf{A}\big) = (ad-bc)\mathbf{I}$, so $\mathbf{A}^{-1} = \dfrac{1}{ad-bc}\big((a+d)\mathbf{I}-\mathbf{A}\big)$, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\det\mathbf{M} = x(x+1)-6 = x^2+x-6 = (x+3)(x-2) > 0$: $\boxed{x < -3 \text{ or } x > 2}$

D2 **(a)** $p(p-1)+6$: $\boxed{p^2-p+6}$

(b) $\left(p-\frac{1}{2}\right)^2 + \frac{23}{4} > 0$, so $\det\mathbf{P} \neq 0$ for all real p, as required

(c) $\boxed{\dfrac{1}{p^2-p+6}\begin{pmatrix} p-1 & -2 \\ 3 & p \end{pmatrix}}$

D3 $\mathbf{M}^{-1} = \dfrac{1}{a-6}\begin{pmatrix} a & -2 \\ -3 & 1 \end{pmatrix}$; the off-diagonal entries give $\dfrac{-2}{a-6} = -2$, so $a = 7$; then $7 = -1+q$: $\boxed{a = 7,\ q = 8}$

D4 **(a)** $4(2k-1)-3k = 5k-4 = 0$: $\boxed{k = \frac{4}{5}}$

(b) $\mathbf{A} = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$, det = 6: $\boxed{\frac{1}{6}\begin{pmatrix} 4 & -3 \\ -2 & 3 \end{pmatrix}}$

STRETCH

D5 **(a)** $\boxed{\begin{pmatrix} 2k & 1 \\ 2k-2 & 0 \end{pmatrix}}$

(b) $\det(\mathbf{AB}) = 0-(2k-2) = 0$: $\boxed{k = 1}$; then $\mathbf{AB}$ is singular and has no inverse

D6 $\mathbf{C} = \mathbf{B}^{-1}(\mathbf{BC}) = \dfrac{1}{a-6}\begin{pmatrix} 1 & -3 \\ -2 & a \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$: $\boxed{\dfrac{1}{a-6}\begin{pmatrix} -8 & -2 \\ 3a-2 & 2a-8 \end{pmatrix}}$

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D7 **(a)** $\mathbf{M}\begin{pmatrix}1\\k\end{pmatrix} = \begin{pmatrix}3k\\3+k^2\end{pmatrix} = \begin{pmatrix}12\\19\end{pmatrix}$: $\boxed{k=4}$

(b) $\det = 16-6 = 10$: $\boxed{\frac{1}{10}\begin{pmatrix}4 & -2\\-3 & 4\end{pmatrix}}$

D8 **(a)** $\boxed{\mathbf{R} = \begin{pmatrix}4 & -3\\7 & -5\end{pmatrix}}$

(b) $\det\mathbf{R} = 1$, $\mathbf{R}^{-1} = \begin{pmatrix}-5 & 3\\-7 & 4\end{pmatrix}$; $(3\mathbf{R})^{-1} = \frac{1}{3}\mathbf{R}^{-1}$: $\boxed{\frac{1}{3}\begin{pmatrix}-5 & 3\\-7 & 4\end{pmatrix}}$

(c) $\frac{1}{3}\begin{pmatrix}-30+27\\-42+36\end{pmatrix}$: $\boxed{P(-1,-2)}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $(k+1)(k-1)-8$: $\boxed{k^2-9}$ *B1*

(b) $k^2-9=0$ *M1*

$\boxed{k=\pm 3}$ *A1*

(c) $\boxed{\frac{1}{k^2-9}\begin{pmatrix}k-1 & -4\\-2 & k+1\end{pmatrix}}$ *M1 A1*

(d) $\boxed{\frac{1}{7}\begin{pmatrix}3 & -4\\-2 & 5\end{pmatrix}}$ *B1*

E2 **(a)** $\det \mathbf{M} = 4 - 2a$ *B1*

$$\boxed{\mathbf{M}^{-1} = \frac{1}{4-2a}\begin{pmatrix} 4 & -a \\ -2 & 1 \end{pmatrix}}$$ *M1 A1*

(b) $\frac{1}{2}\mathbf{M} + q\mathbf{I} = \begin{pmatrix} \frac{1}{2}+q & \frac{a}{2} \\ 1 & 2+q \end{pmatrix}$ *M1*

bottom left: $\frac{-2}{4-2a} = 1$, so $a = 3$ *M1 A1*

top left: $\frac{4}{-2} = \frac{1}{2} + q$: $\boxed{a = 3,\ q = -\frac{5}{2}}$ *A1*

STRETCH

E3 **(a)** $\mathbf{A}^2 = \begin{pmatrix} 4-5 & -2+2 \\ 10-10 & -5+4 \end{pmatrix}$ *M1*

$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathbf{I}$, as required *A1**

(b) $\mathbf{A}(-\mathbf{A}) = \mathbf{I}$: $\boxed{\mathbf{A}^{-1} = -\mathbf{A}}$ *B1*

(c) $\mathbf{A}^4 = \mathbf{I}$ and $2027 = 4(506) + 3$, so $\mathbf{A}^{2027} = \mathbf{A}^3 = -\mathbf{A}$ *M1*

$$\boxed{\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}}$$ *A1*

(d) $\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = -\begin{pmatrix} 6-4 \\ 15-8 \end{pmatrix}$ *M1*

$\boxed{x = -2,\ y = -7}$ *A1*

(e) $\mathbf{X} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\mathbf{A}^{-1}$, with $\mathbf{A}^{-1}$ on the right *M1*

$= -\begin{pmatrix} 2 & -1 \\ 11 & -5 \end{pmatrix}$: $\boxed{\begin{pmatrix} -2 & 1 \\ -11 & 5 \end{pmatrix}}$ *A1*