



eClassroom

GCSE MATHEMATICS • PROBABILITY

Tree Diagrams

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • Calculator allowed

Free Tree Diagrams worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: complete tree diagrams, with and without replacement • multiply along branches and add the ends you need • use trees for dependent events, “exactly one”, “at least one” and expected numbers • *Higher*: conditional probability from a tree, three stages and algebra.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: conditional probability from a tree, three-stage trees and trees with algebra. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	15	18
Higher marks	16	14	17	17
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

The probability of a red light at junction 1 is 0.4, and at junction 2 is 0.3, independently. Find the probability of stopping at both.

On a tree: first branch red 0.4, second branch red 0.3. Multiply along the path.

$$0.4 \times 0.3 = 0.12$$

0.12

Your turn A1

A fair coin is thrown and a fair dice is rolled. Draw a tree for heads or tails, then 6 or not 6. Find $P(\text{heads and not a 6})$.

Worked example A2

$P(\text{Ann scores}) = 0.6$ and $P(\text{Bo scores}) = 0.5$, independently. Find $P(\text{exactly one scores})$.

Two ends: Ann yes and Bo no, or Ann no and Bo yes. Add them.

$$0.6 \times 0.5 + 0.4 \times 0.5 = 0.3 + 0.2$$

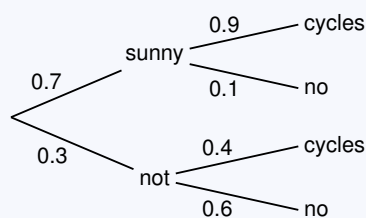
0.5

Your turn A2

$P(\text{Cal scores}) = 0.7$ and $P(\text{Dee scores}) = 0.2$, independently. Find $P(\text{exactly one scores})$.

Worked example A3

$P(\text{sunny}) = 0.7$. If sunny, $P(\text{Tom cycles}) = 0.9$. If not, $P(\text{Tom cycles}) = 0.4$. Find $P(\text{Tom cycles})$.



The second branches depend on the first. Add the two cycling ends.

$$0.63 + 0.12$$

0.75

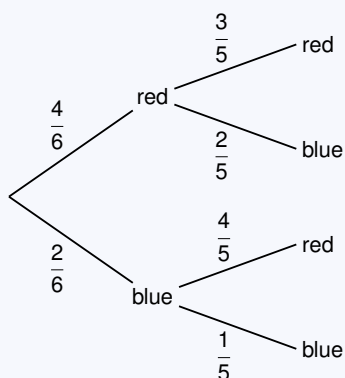
Your turn A3

$P(\text{Jo revises}) = 0.8$. If she revises, $P(\text{pass}) = 0.9$. If not, $P(\text{pass}) = 0.3$. Draw a tree and find $P(\text{Jo passes})$.



HIGHER*Higher only: conditional probability, three stages and algebra***Worked example A4: without replacement**

A bag holds 4 red and 2 blue counters. Two are taken without replacement. Find $P(\text{one of each colour})$.



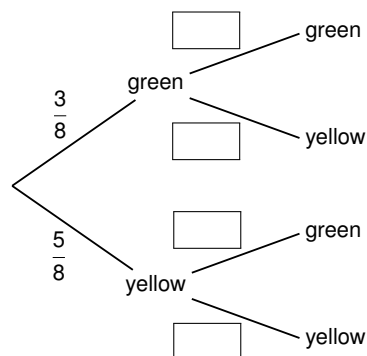
After the first counter only 5 are left, so the second branches change.

$$\frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} = \frac{16}{30}$$

$$\frac{8}{15}$$

Your turn A4

A bag holds 3 green and 5 yellow counters. Two are taken without replacement. Complete the tree and find $P(\text{both the same colour})$.

**Worked example A5: reading the tree backwards**

In worked example A3, Tom cycles. Find the probability that it was sunny.

Only the cycling ends count: $0.63 + 0.12 = 0.75$

$$\frac{0.63}{0.75}$$

$$0.84$$

Your turn A5

In Your turn A3, Jo passes. Find the probability that she revised.

Worked example A6: algebra on a tree

$P(A) = x$. $P(B) = 0.5$ if A happens and 0.2 if not. $P(B) = 0.32$. Find x .

$$0.5x + 0.2(1 - x) = 0.32$$

$$0.3x = 0.12$$

$$x = 0.4$$

Your turn A6

$P(A) = 0.6$. $P(B) = y$ if A happens and 0.3 if not. $P(B) = 0.42$. Find y .



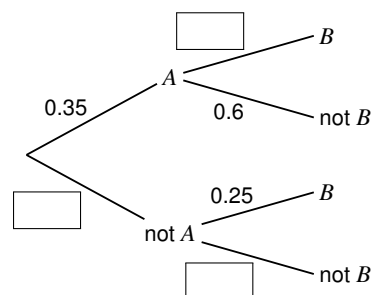
B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION

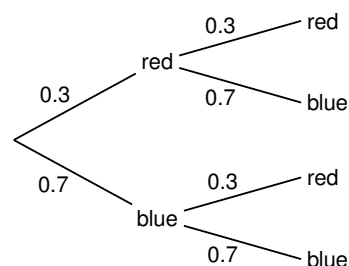
Everyone starts here

- B1** (a) Write down $P(\text{not } A)$.
 (b) Write down $P(B \text{ given } A)$.
 (c) Write down $P(\text{not } B \text{ given not } A)$.
 (d) Find $P(A \text{ and } B)$.



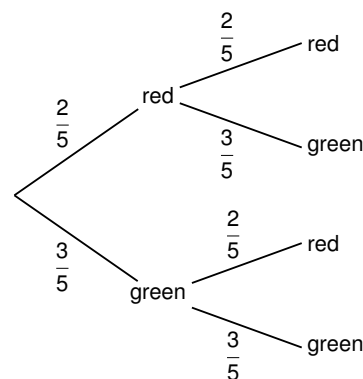
[4 marks]

- B2** A spinner lands on red with probability 0.3 and blue otherwise. It is spun twice. Find
 (a) $P(\text{both red})$ (b) $P(\text{both blue})$
 (c) $P(\text{same colour})$ (d) $P(\text{different colours})$



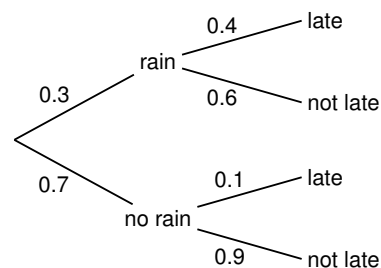
[4 marks]

- B3** A bag holds 2 red and 3 green counters. One is taken, **replaced**, and a second is taken. Find
 (a) $P(\text{red, red})$ (b) $P(\text{green, green})$
 (c) $P(\text{red then green})$ (d) $P(\text{one of each})$



[4 marks]

- B4** $P(\text{rain}) = 0.3$. If it rains, $P(\text{Sam is late}) = 0.4$. If not, $P(\text{Sam is late}) = 0.1$
 (a) $P(\text{rain and late})$ (b) $P(\text{no rain and late})$
 (c) $P(\text{late})$ (d) $P(\text{not late})$



[4 marks]



FOUNDATION

continued: expected numbers and without replacement

B5 Use the tree in B4. Over 200 days, estimate the number of days that

(a) Sam is late

(b) it rains and Sam is late

(c) Sam is not late

[3 marks]

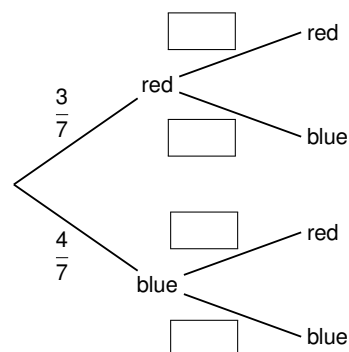
B6 A bag holds 3 red and 4 blue counters. Two are taken **without** replacement.

(a) Write down $P(\text{red second, given red first})$.

(b) Write down $P(\text{blue second, given red first})$.

(c) Write down $P(\text{red second, given blue first})$.

(d) Find $P(\text{both red})$.



[4 marks]

HIGHER

Higher tier only

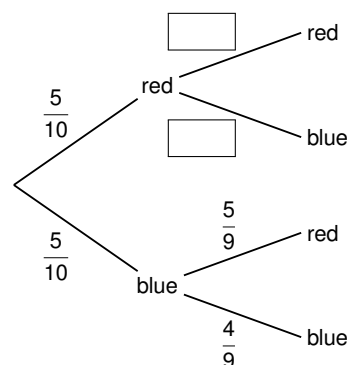
B7 Ten counters, 5 red and 5 blue, are in a bag. Two are picked out, one after the other, and not put back.

(a) Complete the branches after red.

(b) Find $P(\text{both red})$.

(c) Find $P(\text{same colour})$.

(d) Find $P(\text{second is red})$.



[4 marks]

B8 Use the tree in B4. Find

(a) $P(\text{rain, given that Sam is late})$ (b) $P(\text{no rain, given that Sam is late})$

(c) $P(\text{rain, given that Sam is not late})$ (d) $P(\text{Sam is late, given that it rains})$

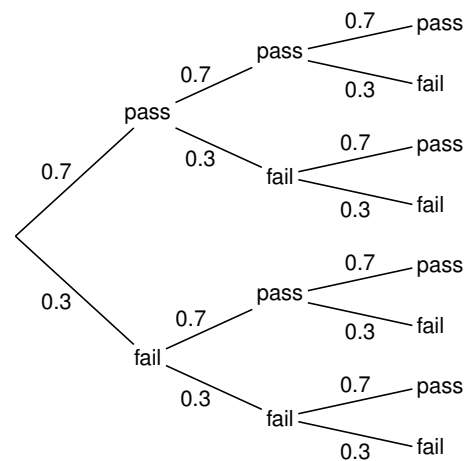
[4 marks]



HIGHER*continued: three stages and algebra*

B9 The probability of passing each of three tests is 0.7, independently. Find

- (a) $P(\text{passes all three})$
- (b) $P(\text{passes none})$
- (c) $P(\text{passes exactly one})$
- (d) $P(\text{passes at least one})$

**[4 marks]**

- B10** (a) Two independent stages each have probability x . $P(\text{both}) = 0.16$. Find x .
- (b) $P(A) = 0.5$. $P(B) = p$ if A happens and 0.3 if not. $P(B) = 0.4$. Find p .
- (c) $P(A) = x$. $P(B) = 0.6$ if A happens and 0.1 if not. $P(B) = 0.3$. Find x .
- (d) A bag of n counters has 2 red. Two are taken without replacement. $P(\text{both red}) = \frac{1}{15}$. Find n .

[4 marks]

C

Reasoning

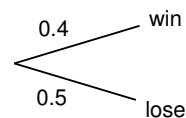
Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** Ben draws this tree for one game. The only outcomes are win and lose.

Explain why Ben's tree must be wrong.



[2 marks]

- C2** $P(\text{red}) = 0.3$ on each of two spins. Tom says, " $P(\text{both red}) = 0.3 + 0.3 = 0.6$ "

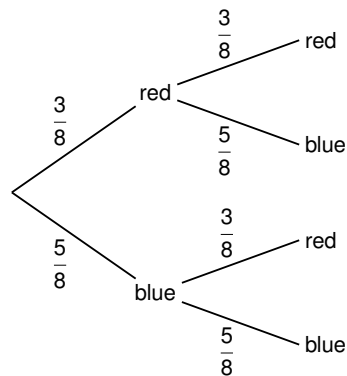
Explain Tom's mistake and find the correct probability.

[2 marks]

- C3** A bag holds 3 red and 5 blue counters. Two are taken **without** replacement. Ali draws this tree.

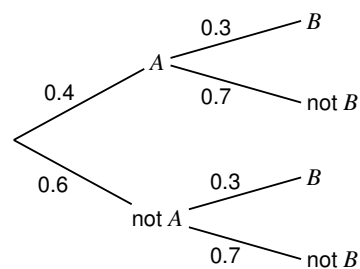
(a) Explain Ali's mistake.

(b) Write down the correct probabilities for the four second branches.



[3 marks]

- C4** Show that the probabilities at the four ends of this tree add up to 1



[3 marks]



HIGHER*Higher tier only*

- C5** A bag holds 3 red and 2 blue counters. Two are taken without replacement.

Jo says, "If the second counter is red, the probability that the first was red is still $\frac{3}{5}$ "

Use a tree diagram to show that Jo is wrong.

[3 marks]

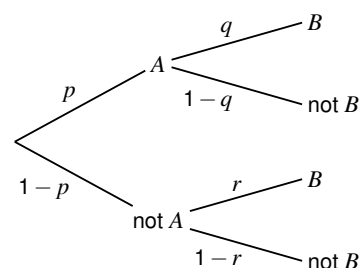
- C6** A bag holds 4 red and 6 blue counters. Two are taken without replacement.

Show that the probability that the **second** counter is red is $\frac{4}{10}$, the same as for the first.

[3 marks]

- C7** The tree shows two events, A and B .

Prove that $P(B) = r + p(q - r)$

**[3 marks]**

- C8** A bag holds r red and b blue counters. Two are taken without replacement.

(a) Draw a tree diagram and prove that the probability that the second counter is red is $\frac{r}{r+b}$

(b) A bag holds 7 red and 13 blue counters. Three are taken without replacement. Explain why the probability that the third counter is red is $\frac{7}{20}$

[5 marks]

D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** A game costs 50p. A player rolls a fair dice and then throws a fair coin. The player wins £5 for a 6 and then a head.

The game is played 120 times. Work out the expected profit for the person running the game. **[4 marks]**

- D2** In each match a team wins with probability 0.5, draws with probability 0.2 and loses otherwise, independently. A win scores 3 points and a draw scores 1 point.

The team plays two matches. Find the probability that it scores at least 4 points. **[4 marks]**

- D3** Kim has a bag of 3 orange and 5 lemon sweets. She takes one sweet and eats it, then takes another.

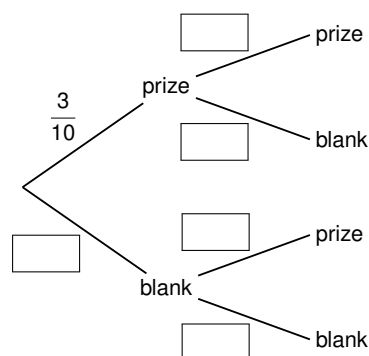
(a) Draw a tree diagram to show all the probabilities.

(b) Find the probability that she takes one of each flavour. **[4 marks]**

- D4** A lucky dip has 3 prizes and 7 blanks. Kim takes a ticket, puts it back, and takes another.

(a) Complete the tree diagram.

(b) Find the probability that she gets at least one prize.



[3 marks]

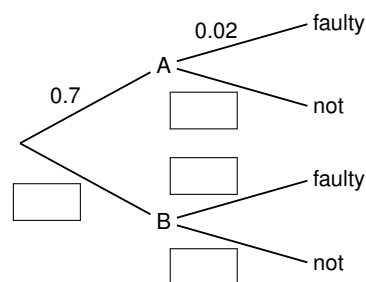


HIGHER*Higher tier only*

- D5** Machine A makes 70% of a factory's items and 2% of its items are faulty. Machine B makes the rest and 5% of its items are faulty.

(a) Complete the tree diagram.

(b) An item is faulty. Find the probability that it was made by machine A.

**[4 marks]**

- D6** A box of 10 balls has 5 red, 3 blue and 2 green. Three balls are picked out, one at a time, and not put back. Find the probability that the three balls are all different colours.

[4 marks]

- D7** Lee has up to three attempts to hit a target. He stops when he hits it. $P(\text{hit}) = 0.4$ on each attempt, independently.

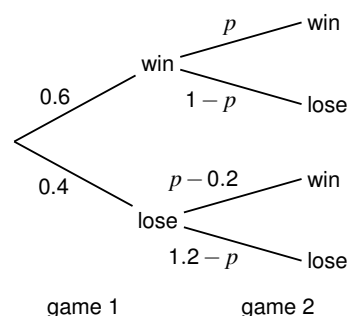
(a) Find the probability that he hits the target. (b) Find the probability that he needs exactly three attempts.

[4 marks]

- D8** Ria plays two games. The tree shows the probabilities.
The probability that she wins the second game is 0.5

(a) Find p .

(b) Ria won the second game. Find the probability that she won the first game.

**[5 marks]**

E

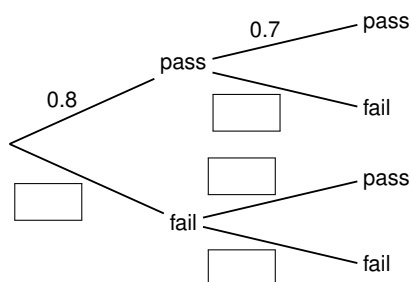
Exam practice

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION

Foundation and Higher crossover

- E1** The probability that Sam passes his theory test is 0.8. The probability that he passes his practical test is 0.7. These are independent. The tree shows the theory test, then the practical test.



- (a) Complete the tree diagram.

(2)

- (b) Find the probability that Sam passes both tests.

.....

(1)

- (c) Find the probability that Sam passes exactly one test.

.....

(1)

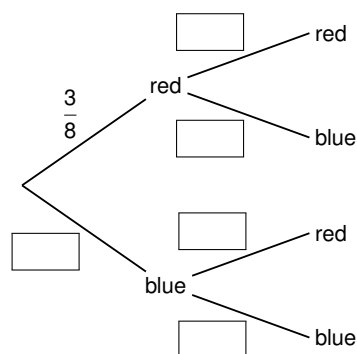
(Total for Question E1 is 4 marks)

- E2** A bag holds 3 red and 5 blue counters. Ella takes a counter, records its colour and puts it back. She then takes a second counter.

- (a) Complete the tree diagram.

(1)

- (b) Find the probability that the two counters are different colours.



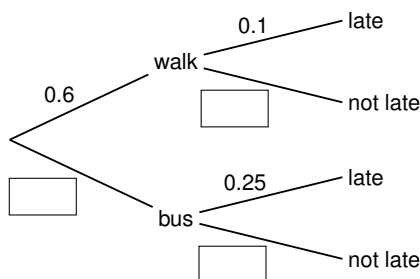
.....

(2)

(Total for Question E2 is 3 marks)



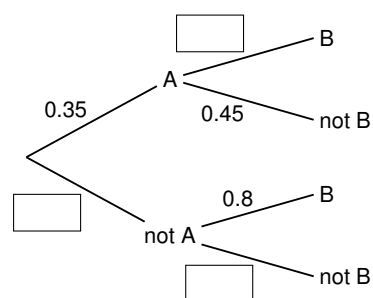
- E3** The probability that Jo walks to school is 0.6. Otherwise she takes the bus.
If she walks, the probability that she is late is 0.1. If she takes the bus, the probability that she is late is 0.25



Find the probability that Jo is late.

.....
(Total for Question E3 is 3 marks)

- E4** Here is part of a tree diagram. Write in the missing probabilities.



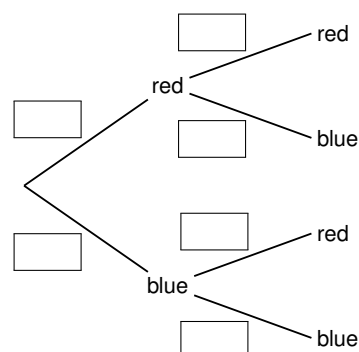
(Total for Question E4 is 2 marks)

- E5** A bag holds 2 red and 3 blue counters. Amir takes a counter at random and does not put it back. He then takes a second counter.

(a) Complete the tree diagram.

(2)

(b) Find the probability that both counters are blue.



.....
(1)

(Total for Question E5 is 3 marks)



- E6** $P(\text{red}) = \frac{1}{4}$ on each spin of a spinner. It is spun twice.

Find the probability that it lands on red at least once.

.....
(Total for Question E6 is 3 marks)

HIGHER

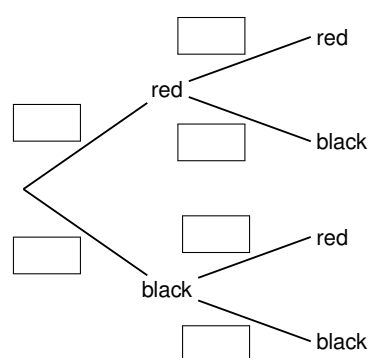
Higher tier only

- E7** A box holds 6 red pens and 4 black pens. Aisha takes two pens at random, one after the other, without putting the first back.

(a) Complete the tree diagram.

(2)

(b) Find the probability that both pens are the same colour.



.....
(2)

(Total for Question E7 is 4 marks)

- E8** Use the information in E3. Jo is late for school. Find the probability that she walked.

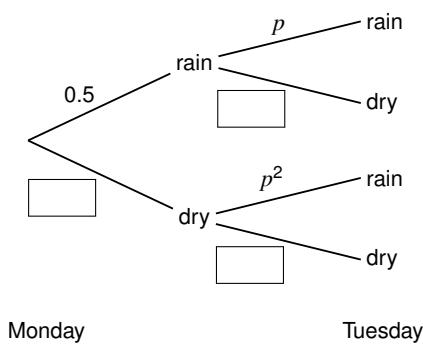
.....
(Total for Question E8 is 4 marks)



- E9** A jar holds 4 red sweets and 5 yellow sweets. Leo eats three sweets, chosen at random one at a time.
Find the probability that exactly two of the sweets he eats are red.

.....
(Total for Question E9 is 4 marks)

- E10** The probability of rain on Monday is 0.5
If it rains on Monday, the probability of rain on Tuesday is p . If it is dry on Monday, the probability of rain on Tuesday is p^2



The probability of rain on Tuesday is 0.28

- (a) Show that $p^2 + p - 0.56 = 0$

(2)

- (b) Find p .

.....
(1)

- (c) It rains on Tuesday. Find the probability that it rained on Monday.

.....
(2)

(Total for Question E10 is 5 marks)

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