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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 7: Series

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	separate file	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	separate file	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	separate file	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	separate file	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	separate file	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	separate file	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	2	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

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CHAPTER 7 • KEY SKILLS

7 Series

Specification FP1: Series • 10 questions • 80 marks • about 96 minutes

1 Standard results

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$,

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$,

$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

2 Constants

$\sum_{r=1}^{n} 5 = 5n$, not 5. Constant multiples come outside: $\sum 3r^2 = 3\sum r^2$

3 Splitting sums

The sum of a sum is the sum of the separate sums. Expand any brackets in r first.

4 Factorising

Take out common factors such as $\frac{1}{6}n(n+1)$ straight away; it makes the remaining bracket simple.

5 Between limits

$\sum_{r=a}^{b} = \mathrm{f}(b) - \mathrm{f}(a-1)$: subtract the sum up to $a-1$, not up to a.

6 From $n+1$ to $2n$

$\mathrm{f}(2n) - \mathrm{f}(n)$. Substitute $2n$ into your formula rather than starting again.

7 Solving for n

Set the formula equal to the value and solve; n must be a positive integer.

8 Given answers

Show the substitution, the common factor and the expanded bracket before the final form.

9 Patterns

Results such as $\sum r^3 = (\sum r)^2$ and products of consecutive integers link the formulae together.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Standard results	•	•	•	•	•	•	•	•	•	•
2 Constants	•				•		•			
3 Splitting sums	•	•		•	•	•	•		•	•
4 Factorising	•	•		•	•	•	•		•	•
5 Between limits			•			•			•	
6 From $n+1$ to $2n$					•		•			
7 Solving for n	•					•		•		•
8 Given answers	•	•		•	•	•	•	•	•	•
9 Patterns								•		•
Interleaved: algebraic manipulation and factorising (P1)	•	•		•	•	•	•		•	•
Interleaved: quadratic and polynomial equations (P1, P2)	•					•		•	•	•
Interleaved: arithmetic series (P2)	•							•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Expand, split, substitute the standard results, then factor out the common fraction and bracket before simplifying. For sums between limits, write $\mathrm{f}(b) - \mathrm{f}(a-1)$ before evaluating.

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1.

(a) Show that $\sum_{r=1}^{n}(4r-1) = n(2n+1)$

(3)

(b) Hence find the value of $\sum_{r=1}^{20}(4r-1)$

(1)

(c) Find the value of n for which $\sum_{r=1}^{n}(4r-1) = 465$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2.

(a) Use the standard results for $\sum r$ and $\sum r^2$ to show that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$

(4)

(b) Hence evaluate $\sum_{r=1}^{10} r(r+1)$

(2)

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Question 2 continued

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3. Using the standard results, find the exact value of

(a) $\sum_{r=10}^{30} r^2$

(3)

(b) $\sum_{r=10}^{30} r^3$

(3)

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Question 3 continued

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4.

(a) Show that $\sum_{r=1}^{n}\left(r^3 - r\right) = \frac{1}{4}(n-1)n(n+1)(n+2)$

(4)

(b) Hence find the value of $\sum_{r=1}^{5}\left(r^3 - r\right)$, and check your answer by direct calculation.

(2)

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Question 4 continued

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5.

(a) Show that $\sum_{r=1}^{n} \left(3r^2 - r + 2\right) = n\left(n^2 + n + 2\right)$

(4)

(b) Hence show that $\sum_{r=n+1}^{2n} \left(3r^2 - r + 2\right) = n\left(7n^2 + 3n + 2\right)$

(2)

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Question 5 continued

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6.

(a) Show that $\sum_{r=1}^{n} r^2(2r-1) = \frac{1}{6}n(n+1)\left(3n^2+n-1\right)$

(5)

(b) Hence find the value of $\sum_{r=11}^{20} r^2(2r-1)$

(3)

(c) Find the smallest value of n for which $\sum_{r=1}^{n} r^2(2r-1) > 10000$

(2)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7.

(a) Show that $\sum_{r=1}^{n}(r+1)(r+4) = \frac{1}{3}n(n+4)(n+5)$

(4)

(b) Hence show that $\sum_{r=n+1}^{2n}(r+1)(r+4) = \frac{1}{3}n(n+1)(7n+20)$

(4)

(c) Hence find the value of $\sum_{r=11}^{20}(r+1)(r+4)$

(2)

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Question 7 continued

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Question 7 continued

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8.

(a) Use the standard results to show that $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$

(2)

(b) Find the value of n for which $\sum_{r=1}^{n} r = 210$

(3)

(c) Hence write down the value of $\sum_{r=1}^{n} r^3$ for this value of n.

(1)

(d) By considering $\sum_{r=1}^{40} r^3$, find the value of $\sum_{r=1}^{20} (2r-1)^3$

(4)

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9.

(a) Show that $\sum_{r=1}^{n}\left(r^2+kr\right)=\frac{1}{6}n(n+1)(2n+1+3k)$, where k is a constant.

(4)

(b) Given that $\sum_{r=1}^{10}\left(r^2+kr\right)=660$, find the value of k.

(3)

(c) For this value of k, find the value of $\sum_{r=11}^{20}\left(r^2+kr\right)$

(3)

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Question 9 continued

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Question 9 continued

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10. **(a)** Show that $\sum_{r=1}^{n} r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$

(5)

(b) Find the value of n for which $\sum_{r=1}^{n} r(r+1)(r+2) = 756$, explaining why there is only one value.

(2)

(c) Using this result and the result $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$, suggest a formula for $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$, and check it for $n = 1$ and $n = 2$

(3)

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Question 10 continued

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TOTAL FOR CHAPTER 7 IS 80 MARKS

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