



eClassroom

A-LEVEL MATHEMATICS • PURE 2

Exponentials and Logarithms

Worked Solutions

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Exponentials and Logarithms worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 2 Exponentials and Logarithms worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Exponentials & Logarithms revision engine

More worksheets



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Page 1 of 7

A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $3^4 = 81$, $8^{\frac{1}{3}} = 2$, $2^{-3} = \frac{1}{8}$: $\boxed{4}$, $\boxed{\frac{1}{3}}$, $\boxed{-3}$

A2 $\log_2 216 - \log_2 27 = \log_2 \frac{216}{27} = \log_2 8 = \boxed{3}$

A3 $2x \log 5 = \log 40$, so $x = \frac{\log 40}{2 \log 5} \approx \boxed{1.15}$

STRETCH

A4 $\frac{3x+4}{x-1} = 2^3 = 8$, so $3x+4 = 8x-8$, $5x = 12$: $\boxed{x = \frac{12}{5}}$ (both logs positive)

A5 $u = 3^x$: $u^2 - 4u + 3 = 0$, $(u-1)(u-3) = 0$, so $3^x = 1$ or 3 : $\boxed{x = 0}$, $\boxed{x = 1}$

A6 $\log_9 x = \frac{1}{2} \log_3 x$, so $\frac{3}{2} \log_3 x = 9$, $\log_3 x = 6$: $\boxed{x = 729}$

A7 $ab = 360$ and $ab^3 = 810$: dividing, $b^2 = 2.25$, so $\boxed{b = 1.5}$, and $\boxed{a = 240}$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\boxed{4}$ (b) $\boxed{-2}$ (c) $\boxed{\frac{1}{2}}$ (d) $\boxed{0}$

A negative log means a fraction; a fractional log means a root; $\log_a 1 = 0$ always.

B2 (a) $\boxed{\log 20}$ (b) $\boxed{\log 8}$ (c) $\log 8 + \log 9 = \boxed{\log 72}$ (d) $\log 9 - \log 9 = \boxed{0}$

Apply the power law first, then add or subtract.

B3 (a) $\frac{\log 10}{\log 2} \approx \boxed{3.32}$ (b) $\boxed{-3.32}$ (c) $x + 1 = 3.32 \dots: \boxed{2.32}$ (d) $3x = 3.32 \dots: \boxed{1.11}$

Each change to the power shifts or scales the same value, $\log_2 10$

B4 (a) $\boxed{81}$ (b) $2x - 1 = 9: \boxed{5}$ (c) $\log_3 4x = 2, 4x = 9: \boxed{\frac{9}{4}}$ (d) $x^2 = 25$, and $x > 0: \boxed{5}$

In (d), $x = -5$ is rejected: $\log_3(-5)$ is undefined.

B5 (a) $\boxed{(0, 1)}, \boxed{y = 0}$ (b) $\boxed{(0, 3)}, \boxed{y = 2}$ (c) $\boxed{(0, 1)}, \boxed{y = 0}$, decreasing

Adding 2 moves the curve and its asymptote up; 3^{-x} is the reflection in the y-axis.

B6 (a) $\boxed{p+q}$ (b) $\log_a(2 \times 3^2): \boxed{p+2q}$ (c) $\boxed{3p-q}$ (d) $\log_a a + \frac{1}{2} \log_a 2: \boxed{1 + \frac{1}{2}p}$

Write the number as a product of powers of 2, 3 and a first.

STRETCH

B7 (a) $(u-1)(u-4) = 0: \boxed{x=0, x=2}$ (b) $u^2 - 2u - 8 = 0, u = 4$ ($u = -2$ rejected, $2^x > 0$): $\boxed{x=2}$
 (c) $3u^2 - 10u + 3 = 0, u = 3$ or $\frac{1}{3}: \boxed{x=1, x=-1}$ (d) $(u-5)(u+2) = 0, u = 5: \boxed{x = \log_2 5}$

a^x is always positive, so a negative value of u gives no solution.

B8 (a) $x(x-2) = 8, x^2 - 2x - 8 = 0: \boxed{x=4}$ ($x = -2$ rejected) (b) $x(x+1) = 6: \boxed{x=2}$ ($x = -3$ rejected)
 (c) $\frac{x^2}{x+6} = 3, x^2 - 3x - 18 = 0: \boxed{x=6}$ ($x = -3$ rejected) (d) $x+1 = 2x: \boxed{x=1}$

Reject any value that makes a logarithm of a negative number or of zero.



B9 (a) $\frac{\log_2 8}{\log_2 4} = \frac{3}{2}$ (b) $\frac{\log_2 32}{\log_2 8} = \frac{5}{3}$

(c) $\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} = \frac{2 \log 2}{\log 2} = 2$ **as required** (d) $\log_8 64 = 2$: $x = 4$

Change of base turns logs to different bases into one base.

B10 (a) **200** (b) $200 \times 1.05^{10} \approx 326$ (c) $1.05^t = 2$, $t = \frac{\log 2}{\log 1.05} \approx 14.2$ **years**

(d) **the population grows by 5% each year**

a is the starting value and b the growth factor per unit of time.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 There is no law for the log of a sum: $\log(x+3)$ cannot be split. The addition law works the other way:
 $\log x + \log 3 = \log 3x$

C2 The power law moves the 3 to a power, not a multiplier inside: $3 \log_2 x = \log_2 x^3$

C3 $x = -6$ gives $\log_2(-2)$ and $\log_2(-8)$, which are undefined, so **reject $x = -6$** ; $x = 4$ gives $\log_2 8 + \log_2 2 = 4$

C4 What matters is whether each log is of a positive number: $1 - (-2) = 3 > 0$ and $-2 + 3 = 1 > 0$
 $\log_3 3 - \log_3 1 = 1 - 0 = 1$, so **$x = -2$ is a valid solution**

STRETCH

C5 The intercept is $2^0 + 3 = 4$; $y = 3$ is the asymptote, which the curve never reaches. **$(0, 4)$** , asymptote **$y = 3$**

C6 $\log_3 9 = 2$ because $3^2 = 9$, so 2 is $\log_3 9$, not $\log_3 6$

$\log_3(x+1) = \log_3 9x$, so $x+1 = 9x$: **$x = \frac{1}{8}$**

C7 After 4 years the value has been multiplied by 0.85 four times, not three.
 $18000 \times 0.85^4 \approx$ **£9396**



C8 (a) Let $\log_a b = k$, so $a^k = b$. Take $\log_b$ of both sides: $k \log_b a = \log_b b = 1$, so $k = \frac{1}{\log_b a}$

as required

(b) $\log_2 x = \frac{4}{\log_2 x}$, so $(\log_2 x)^2 = 4$, $\log_2 x = \pm 2$: $x = 4$ or $x = \frac{1}{4}$

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\log_3(x+5)(x-1) = 3$, so $(x+5)(x-1) = 27$, $x^2 + 4x - 32 = 0$, $(x+8)(x-4) = 0$

$x = -8$ makes $x+5$ and $x-1$ negative: $x = 4$ only

D2 $\log_2(10-x) = \log_2 8 + \log_2(x-1)^2 = \log_2 8(x-1)^2$

$10-x = 8(x-1)^2$, so $8x^2 - 15x - 2 = 0$, $(8x+1)(x-2) = 0$

$x = -\frac{1}{8}$ makes $x-1 < 0$, so it is rejected: $x = 2$

D3 (a) A decreasing curve through $(0, 4)$, approaching $y = 5$ as $x \rightarrow -\infty$

(b) $2^x = 5$: $x = \log_2 5$

D4 $4^x = (2^x)^2$: with $u = 2^x$, $u^2 = 5u - 6$, so $u^2 - 5u + 6 = 0$, $u = 2$ or 3

$x = 1$ and $x = \log_2 3$

STRETCH

D5 (a) $\log_3 \frac{x^2}{x-2} = 2$, so $x^2 = 9(x-2)$: $x^2 - 9x + 18 = 0$ as required

(b) $(x-3)(x-6) = 0$; both give $x-2 > 0$: $x = 3$, $x = 6$

D6 (a) $2^{-\frac{12}{k}} = \frac{1}{4} = 2^{-2}$, so $k = 6$: the mass halves every 6 days (the half-life).

(b) $80 \times 2^{-\frac{t}{6}} < 1$, so $2^{\frac{t}{6}} > 80$, $t > 6 \log_2 80 \approx 37.9$: 38 days



D7 (a) $\log_2 4 + 2\log_2 x - (\log_2 x - \log_2 2) = 2 + 2\log_2 x - \log_2 x + 1 = 3 + \log_2 x$

as required

(b) $\log_2 x = 4$: $x = 16$ (c) $\log_2 x \leq 2$ and $x > 0$: $0 < x \leq 4$

D8 (a) $3^x = \frac{2}{3^x} + 1$; multiply by $u = 3^x$: $u^2 = 2 + u$, so $u^2 - u - 2 = 0$

as required

(b) $(u - 2)(u + 1) = 0$; $u = -1$ is rejected because $3^x > 0$: $x = \log_3 2$

(c) $y = 3^x = 2$

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (i) $\log_2(x+1)(x-5) = 4$, so $(x+1)(x-5) = 16$

M1: combines and removes logs

$x^2 - 4x - 21 = 0$, $(x-7)(x+3) = 0$

dM1

$x = 7$; $x = -3$ rejected since $x - 5 < 0$

A1

(ii) $\log_a 8 + \log_a a^2 = 3\log_a 2 + 2$

M1

$3p + 2$

A1

E2 (a) A decreasing curve, above $y = 2$, through $(0, 3)$, asymptote $y = 2$

B1 shape, B1, B1

(b) $3^{-x} = 9 = 3^2$: $x = -2$

B1

(c) $3^{-x} = 5$, so $-x = \frac{\log 5}{\log 3}$

M1

$x = -1.46$

A1



STRETCH

E3 (a) $\log_2 \frac{x^3 + 6}{x} = \log_2(4x - 1)$

M1: subtraction law

$$x^3 + 6 = x(4x - 1) = 4x^2 - x$$

M1: removes logs

$$x^3 - 4x^2 + x + 6 = 0 \quad \text{as required}$$

A1*

(b) $(x + 1)(x^2 - 5x + 6)$

M1

$$(x + 1)(x - 2)(x - 3)$$

A1

(c) $x = -1, 2$ or 3

M1

$x = -1$ is rejected: $\log_2 x$ needs $x > 0$

B1

$$x = 2, \quad x = 3 \quad (\text{both give } 4x - 1 > 0)$$

A1

