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INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2

Statistics 2

Chapter 3: Approximations

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WST02/01) • Calculator and Statistical Tables allowed

Multi-step exam questions at the difficulty of the real WST02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Binomial Distributions** Conditions, probabilities and tables, mean and variance, unknown n and p	10	80	separate file	
2	**Poisson Distributions** Probabilities, modelling conditions, scaling the rate, adding Poisson variables	10	80	separate file	
3	**Approximations** Poisson and normal approximations, continuity corrections	10	80	2	
4	**Continuous Random Variables** Probability density and cumulative distribution functions, mean, variance, median, mode	10	81	separate file	
5	**Continuous Uniform Distribution** Rectangular distribution, mean and variance, modelling	10	80	separate file	
6	**Sampling and Sampling Distributions** Populations and samples, statistics, sampling distributions	10	80	separate file	
7	**Hypothesis Testing** Critical regions, significance levels, binomial and Poisson tests	10	80	separate file	

Full mark scheme Free courses More worksheets

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CHAPTER 3 • KEY SKILLS

3 Approximations

Specification S2: Approximations • 10 questions • 80 marks • about 96 minutes

1 Poisson for binomial

n large and p small (close to 0):

$\mathrm{B}(n, p) \approx \mathrm{Po}(np)$

2 When p is close to 1

Count the other outcome first: 98% germinate, so failures

$\sim \mathrm{B}(250, 0.02) \approx \mathrm{Po}(5)$

3 Normal for binomial

n large and p close to 0.5:

$\mathrm{B}(n, p) \approx \mathrm{N}\big(np,\ np(1-p)\big)$

4 Normal for Poisson

λ large: $\mathrm{Po}(\lambda) \approx \mathrm{N}(\lambda, \lambda)$, so the standard deviation is $\sqrt{\lambda}$

5 Continuity correction

$\mathrm{P}(X \geqslant 70) \rightarrow \mathrm{P}(Y > 69.5)$;

$\mathrm{P}(X < 70) \rightarrow \mathrm{P}(Y < 69.5)$;

$\mathrm{P}(X = 40) \rightarrow \mathrm{P}(39.5 < Y < 40.5)$

6 Standardising

$z = \dfrac{x-\mu}{\sigma}$ to 2 decimal places, then the tables. Use the standard deviation, not the variance.

7 Choosing and justifying

Quote the condition with the numbers: "$n = 150$ is large and $p = 0.02$ is small".

8 Working backwards

Use the percentage points: $\mathrm{P}(Z > 2.3263) = 0.01$, then undo the continuity correction and round up to a whole number.

9 Comparing with exact

An approximation is good when it is close to the exact probability; percentage error

$= \dfrac{|\text{approx} - \text{exact}|}{\text{exact}} \times 100$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Poisson for binomial	•			•				•		•
2 When p is close to 1				•						
3 Normal for binomial		•			•		•	•		•
4 Normal for Poisson			•			•			•	
5 Continuity correction		•	•		•	•	•	•	•	•
6 Standardising		•	•		•	•	•	•	•	•
7 Choosing and justifying	•	•		•		•		•		•
8 Working backwards							•		•	
9 Comparing with exact					•		•			•
Interleaved: binomial and Poisson distributions (Chapters 1, 2)	•	•	•	•	•	•	•	•	•	•
Interleaved: the normal distribution and percentage points (S1)		•	•		•	•	•	•	•	•
Interleaved: inequalities (P1)							•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the original distribution, then the approximating distribution with its parameters. Show the continuity correction and the z-value. When p is close to 1, count the other outcome before approximating.

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1. On average, 2% of the eggs from a farm are cracked. A box contains 150 eggs, and X is the number of cracked eggs in the box.

(a) Using a suitable approximation, find $P(X \leqslant 2)$

(3)

(b) Give a reason why your approximation is suitable.

(1)

(c) Using the same approximation, find $P(X = 4)$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. In a large town, 40% of adults support a new bypass. A random sample of 150 adults is taken, and X is the number in the sample who support the bypass.

(a) Using a suitable approximation, find the probability that at least 70 adults in the sample support the bypass.

(4)

(b) Explain why a normal approximation is suitable here.

(1)

(c) Explain why a continuity correction is needed.

(1)

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Question 2 continued

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3. Cars arrive at a car park at a mean rate of 40 per hour. The number of cars arriving in an hour is modelled by a Poisson distribution.

(a) Using a suitable approximation, find the probability that more than 50 cars arrive in an hour.

(4)

(b) Using the same approximation, find the probability that exactly 40 cars arrive in an hour.

(2)

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Question 3 continued

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4. On average 98% of the seeds of a certain plant germinate. A packet contains 250 seeds.

(a) Explain why a normal approximation to the number of seeds that germinate is not suitable.

(1)

(b) Using a suitable approximation, find the probability that more than 245 seeds in the packet germinate.

(4)

(c) Find the probability that at least 240 seeds in the packet germinate.

(1)

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Question 4 continued

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(Total for Question 4 is 6 marks)

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5. The random variable $X \sim \text{B}(30, 0.5)$

(a) Find the exact value of $\text{P}(X \leqslant 12)$

(1)

(b) Use a normal approximation to estimate $\text{P}(X \leqslant 12)$

(3)

(c) Find the percentage error in your estimate, and comment on the accuracy of the approximation.

(2)

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Question 5 continued

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6. Customers arrive at a ticket office at a mean rate of 1.4 per minute, and the number arriving in any interval follows a Poisson distribution.

(a) Find the probability that at least 3 customers arrive in a one-minute period.

(2)

(b) Find the probability that, in 5 separate one-minute periods, exactly one has no customers arriving.

(3)

(c) Using a suitable approximation, find the probability that at least 50 customers arrive in a 30-minute period.

(4)

(d) Explain why the approximation in part (c) is suitable.

(1)

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Question 6 continued

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Question 6 continued

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7. A test has 50 multiple-choice questions, each with 4 possible answers, only one of which is correct. A candidate guesses the answer to every question at random. X is the number of correct answers.

(a) Find the mean and the variance of X.

(2)

(b) Using a normal approximation, find P($X \geqslant 20$)

(4)

The pass mark is k. The examiners want the probability that a candidate who guesses every answer passes to be less than 1%.

(c) Using a normal approximation, find the smallest possible value of k.

(4)

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. On average, 1.5% of the bolts made by a machine are faulty. The bolts are packed in boxes of 400, and a box is rejected if it contains more than 8 faulty bolts.

(a) Using a suitable approximation, show that the probability that a box is rejected is 0.1528

(3)

A sample of 60 boxes is inspected, and R is the number of boxes rejected.

(b) Using a normal approximation, find the probability that at least 12 boxes are rejected.

(5)

(c) Comment on the suitability of the normal approximation in part (b).

(2)

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Question 8 continued

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Question 8 continued

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9. A small company receives emails at a mean rate of 25 per working day. The number of emails received in any period is modelled by a Poisson distribution.

(a) Using a suitable approximation, find the probability that no more than 20 emails are received on a working day.

(3)

The company's system can deal with k emails in a day. It wants the probability of receiving more than k emails in a day to be less than 0.05

(b) Using a normal approximation, find the smallest possible value of k.

(4)

(c) Find the probability that more than 140 emails are received in a 5-day working week.

(3)

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Question 9 continued

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Question 9 continued

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10. The random variable $X \sim \text{B}(n, p)$ has mean 6 and variance 5.88

(a) Find the value of n and the value of p.

(3)

(b) Use a Poisson approximation to estimate $\text{P}(X \leqslant 4)$

(2)

(c) Use a normal approximation to estimate $\text{P}(X \leqslant 4)$

(3)

The exact value of $\text{P}(X \leqslant 4)$ is 0.2824

(d) State, with a reason, which approximation is more suitable here.

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 80 MARKS

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