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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3 (WMA13)

Pure 3 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA13/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1, 5, 9 and 10 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A* **65** • A **58** • B **52** • C **46**

Based on recent WMA13 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Combining algebraic fractions; a solution outside the domain; integrating $\frac{k}{x+2}$	8	Ch 1 Algebraic Methods; Ch 7 Integration	
2	Sign change; rearranging and iterating; a formula that fails	7	Ch 8 Numerical Methods	
3	$\cot^2\theta = \operatorname{cosec}^2\theta - 1$; a quadratic in $\operatorname{cosec}\theta$; all solutions in a range	7	Ch 3 Trigonometric Functions	
4	$R\sin(\theta - \alpha)$; maximum of a model; solving within a time interval	8	Ch 4 Trig Addition Formulae	
5	Sketching an exponential graph; a hidden quadratic in e^x	7	Ch 5 Exponentials and Logarithms	
6	Finding k in an exponential model; equal values; rates of change	8	Ch 5 Exponentials and Logarithms; Ch 6 Differentiation	
7	Product rule; a stationary point from $\tan x$; an exact tangent	8	Ch 6 Differentiation	
8	$\frac{dy}{dx} = 1 \Big/ \frac{dx}{dy}$; a normal; a vertical tangent	7	Ch 6 Differentiation	
9	Standard integrals; integration by recognition; $\tan^2 x = \sec^2 x - 1$	7	Ch 7 Integration	
10	Modulus graph; solving with two branches; transformations	8	Ch 2 Functions and Graphs	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy, or a solution you missed or forgot to reject. Papers 2 and 3 give less help between the parts, so that is the skill to build next.

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Question 1

8 marks

Part	Scheme	Marks
(a)	Uses $x^2 - x - 6 = (x-3)(x+2)$ as the common denominator	**M1**
	Numerator $2x(x+2) - (x+27) = 2x^2 + 3x - 27$	**M1**
	$= \dfrac{(2x+9)(x-3)}{(x-3)(x+2)} = \dfrac{2x+9}{x+2}$ *	**A1***
	A common denominator of $(x-3)(x^2-x-6)$ can still score both M marks, but the final cancelling must be shown. A sign slip on -27 is the usual error.	**(3)**
(b)	$2x+9 = x(x+2)$, so $x^2 = 9$	**M1**
	$\boxed{x = -3}$ only, since $x = 3$ is not in the domain of f	**A1**
	Giving $x = \pm 3$ scores M1 A0. The domain $x \neq 3$ is printed in the question for a reason.	**(2)**
(c)	$\dfrac{2x+9}{x+2} = 2 + \dfrac{5}{x+2}$ (by division or otherwise)	**M1**
	$[2x + 5\ln(x+2)]_4^8$ (form $px + q\ln(x+2)$, limits used)	**M1**
	$(16 + 5\ln 10) - (8 + 5\ln 6) = \boxed{8 + 5\ln\frac{5}{3}}$ oe	**A1**
	Integrating $\frac{2x+9}{x+2}$ directly as $\ln(x+2)$ multiplied by something scores M0. $8 + 5\ln\frac{10}{6}$ is acceptable.	**(3)**
		Total for Question 1: 8 marks

Question 2

7 marks

Part	Scheme	Marks
(a)	Let $g(x) = e^{\frac{1}{2}x} + x^2 - 6$. Finds $g(1.8) = -0.300\ldots$ and $g(1.9) = 0.196\ldots$ Change of sign and g is continuous, so $1.8 < \alpha < 1.9$ *Both values (to at least 1 significant figure), the sign change and a conclusion are needed for A1.*	M1 A1 (2)
(b)	$e^{\frac{1}{2}x} = 6 - x^2 \Rightarrow x^2 = 6 - e^{\frac{1}{2}x} \Rightarrow x = \sqrt{6 - e^{\frac{1}{2}x}}$ (positive root, as $\alpha > 0$) *	B1* (1)
(c)	Uses the formula correctly at least once: $x_2 = 1.8816$ $x_2 = 1.8816,\ x_3 = 1.8542,\ x_4 = 1.8636$ *All three to 4 decimal places for A1. Use the full calculator value of each term, not the rounded one.*	M1 A1 (2)
(d)	$x_2 = 2.0305\ldots$ and $x_3 = 1.2596\ldots$ The values move further away from $\alpha \approx 1.86$ (they diverge), so this formula cannot be used *The explanation must refer to the values moving away from α. "It gives the wrong answer" is not enough.*	M1 A1 (2)
		Total for Question 2: 7 marks

Question 3

7 marks

Part	Scheme	Marks
(a)	Uses $\cot^2\theta = \operatorname{cosec}^2\theta - 1$ $2\operatorname{cosec}^2\theta - 2 + 5\operatorname{cosec}\theta = 10 \Rightarrow 2\operatorname{cosec}^2\theta + 5\operatorname{cosec}\theta - 12 = 0$ *	M1 A1* (2)
(b)	$(2\operatorname{cosec}\theta - 3)(\operatorname{cosec}\theta + 4) = 0$ $\operatorname{cosec}\theta = \frac{3}{2}$ or $\operatorname{cosec}\theta = -4$ Uses $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$: $\sin\theta = \frac{2}{3}$ or $\sin\theta = -\frac{1}{4}$ Two correct angles $\theta = -165.5°,\ -14.5°,\ 41.8°,\ 138.2°$ and no others in the range *The negative angles are the ones most often missed: for $\sin\theta = -\frac{1}{4}$ the solutions are $-14.5°$ and $-180° + 14.5° = -165.5°$. Using $\sin\theta = \frac{3}{2}$ is impossible and scores M0.*	M1 A1 M1 A1 A1 (5)
		Total for Question 3: 7 marks

Question 4

8 marks

Part	Scheme	Marks
(a)	$R = 5$	B1
	$\tan\alpha = \frac{4}{3}$ (from $R\cos\alpha = 3$, $R\sin\alpha = 4$)	M1
	$\alpha = 0.9273$	A1
	$\tan\alpha = \frac{3}{4}$ *gives* $\alpha = 0.6435$*, the most common error. An answer in degrees* (53.1°) *scores M1 A0.*	(3)
(b)	Maximum when $\sin\left(\frac{t}{2} - \alpha\right) = 1$, so $\frac{t}{2} - \alpha = \frac{\pi}{2}$, $t = \pi + 2\alpha$	M1
	Maximum depth 13 m when $t = 5.00$	A1
		(2)
(c)	$8 + 5\sin\left(\frac{t}{2} - \alpha\right) = 10 \Rightarrow \sin\left(\frac{t}{2} - \alpha\right) = 0.4$, so $\frac{t}{2} - \alpha = 0.4115$	M1
	Second value $\frac{t}{2} - \alpha = \pi - 0.4115 = 2.7301$, then solves for t	dM1
	$t = 2.68$ and $t = 7.31$	A1
	Forgetting to multiply by 2 gives 1.34 and 3.66. With $0 \leqslant t \leqslant 12$, $\frac{t}{2} - \alpha$ *runs from* -0.93 *to* 5.07*, so there are exactly two solutions.*	(3)
		Total for Question 4: 8 marks

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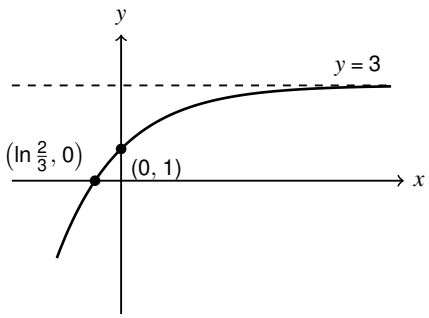
Question 5

7 marks

Part	Scheme	Marks
(a)	 An increasing curve that flattens out below its asymptote as x increases Asymptote $\boxed{y = 3}$ Meets the axes at (0, 1) and $\left(\ln\frac{2}{3}, 0\right)$ oe, such as $\left(-\ln\frac{3}{2}, 0\right)$	B1 B1 B1 (3)
(b)	Multiplies through by e^x: $3e^x - 2 = e^{2x}$ $e^{2x} - 3e^x + 2 = 0$ $(e^x - 1)(e^x - 2) = 0$, so $e^x = 1$ or $e^x = 2$ $\boxed{x = 0 \text{ and } x = \ln 2}$ *Both answers are needed for A1. $x = 0$ is the one most often lost, by not recognising that $e^x = 1$ gives $x = 0$.*	M1 A1 M1 A1 (4)
		Total for Question 5: 7 marks

Question 6

8 marks

Part	Scheme	Marks
(a)	$50 = 20 + 60e^{-15k} \Rightarrow e^{-15k} = \frac{1}{2}$	M1
	$-15k = \ln\frac{1}{2} = -\ln 2$, so $k = \dfrac{\ln 2}{15}$ *	A1*
		(2)
(b)	Sets $20 + 70e^{-0.05t} = 20 + 60e^{-kt}$, so $70e^{-0.05t} = 60e^{-kt}$	M1
	Takes logs correctly: $\ln 70 - 0.05t = \ln 60 - kt$, so $t(0.05 - k) = \ln\frac{7}{6}$	dM1
	$\boxed{t = 40.7}$	A1
	$\ln\left(70e^{-0.05t}\right) = \ln 70 \times (-0.05t)$ *scores dM0.*	
		(3)
(c)	Differentiates at least one model: $\dfrac{d\theta_A}{dt} = -3.5e^{-0.05t}$ or $\dfrac{d\theta_B}{dt} = -60ke^{-kt}$	M1
	Falling at 0.458 °C per minute (A) and 0.423 °C per minute (B)	A1
	$\boxed{\text{Cup } A}$ is cooling faster	A1
	Alternative: at this moment both have the same $\theta - 20$, and the rates are $0.05(\theta - 20)$ and $k(\theta - 20)$. As $0.05 > k = 0.0462$, A cools faster. This scores M1 A1 A1 if the values are also found.	
		(3)
		Total for Question 6: 8 marks

Question 7

8 marks

Part	Scheme	Marks
(a)	Uses the product rule: form $Ae^{2x}\sin x + Be^{2x}\cos x$	M1
	$\boxed{\dfrac{dy}{dx} = 2e^{2x}\sin x + e^{2x}\cos x}$	A1
		(2)
(b)	Sets $e^{2x}(2\sin x + \cos x) = 0$; $e^{2x} \neq 0$, so $2\sin x + \cos x = 0$	M1
	Divides by $\cos x$: $\tan x = -\frac{1}{2}$ *	A1*
	$x = \pi - \arctan\frac{1}{2} = \boxed{2.68}$	A1
	$\arctan\left(-\frac{1}{2}\right) = -0.464$ *is outside $0 \leqslant x \leqslant \pi$; adding π is needed.*	
		(3)
(c)	At $x = \frac{\pi}{2}$: $y = e^{\pi}$ and gradient $= e^{\pi}(2 \times 1 + 0) = 2e^{\pi}$	M1
	$y - e^{\pi} = 2e^{\pi}\left(x - \frac{\pi}{2}\right)$	M1
	$\boxed{y = 2e^{\pi}x + e^{\pi}(1 - \pi)}$ oe	A1
		(3)
		Total for Question 7: 8 marks

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Question 8

7 marks

Part	Scheme	Marks
(a)	Product rule on ye^y $\boxed{\frac{dx}{dy} = e^y + ye^y}$	M1 A1 (2)
(b)	At $y = 1$: $x = e$ and $\frac{dx}{dy} = 2e$, so $\frac{dy}{dx} = \frac{1}{2e}$ Normal gradient $-2e$: $y - 1 = -2e(x - e)$ $\boxed{y = -2ex + 2e^2 + 1}$ *The normal gradient is $-2e$, which is $-\frac{dx}{dy}$. Using $-\frac{1}{2e}$ is the most common error and scores the second M0.*	M1 M1 A1 (3)
(c)	Tangent parallel to the y-axis when $\frac{dx}{dy} = 0$: $e^y(1 + y) = 0$, so $y = -1$ $\boxed{\left(-\frac{1}{e}, -1\right)}$ *Setting $\frac{dx}{dy} = 0$ finds where $\frac{dy}{dx}$ is undefined. Setting $\frac{dy}{dx} = 0$ instead looks for a horizontal tangent, and there is none.*	M1 A1 (2)
		Total for Question 8: 7 marks

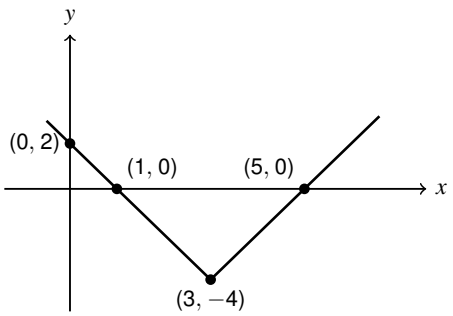
Question 9

7 marks

Part	Scheme	Marks
(a)	$\frac{3}{2}\ln(2x+1)$	B1
	$-e^{1-x}+c$ (both terms and the constant for the second mark)	B1
	$3\ln(2x+1)$ and $+e^{1-x}$ are the common errors.	(2)
(b)	Recognises $\int 8\sin x\cos^3 x\,dx = k\cos^4 x$	M1
	$-2\cos^4 x$	A1
	$\left[-2\cos^4 x\right]_0^{\frac{\pi}{2}} = 0-(-2) = \boxed{2}$	A1
	Checking by differentiation: $\frac{d}{dx}\left(-2\cos^4 x\right) = 8\cos^3 x\sin x$. A calculator value with no integration scores no marks.	(3)
(c)	Uses $\tan^2 x = \sec^2 x - 1$ and integrates to $\tan x - x$	M1
	$[\tan x - x]_0^{\frac{\pi}{4}} = \left(1-\frac{\pi}{4}\right)-0 = 1-\frac{\pi}{4}$ *	A1*
		(2)
		Total for Question 9: 7 marks

Question 10

8 marks

Part	Scheme	Marks
(a)	 A V shape with its vertex below the x-axis Vertex $(3, -4)$ $(1, 0)$, $(5, 0)$ and $(0, 2)$	B1 B1 B1 (3)
(b)	Solves one correct equation: $2(x-3)-4=\frac{1}{2}x$ or $-2(x-3)-4=\frac{1}{2}x$ $\boxed{x=\frac{20}{3}}$ $\boxed{x=\frac{4}{5}}$ *Each answer can be checked on the sketch: $\frac{20}{3}$ is on the right branch ($x>3$) and $\frac{4}{5}$ on the left ($x<3$).* *Rearranging to $\lvert x-3\rvert=\frac{1}{4}x+2$ and squaring also works, if both answers are then checked.*	M1 A1 A1 (3)
(c)	x coordinate $\boxed{6}$ (stretch, scale factor 2, parallel to the x-axis) y coordinate $\boxed{-12}$ (stretch, scale factor 3, parallel to the y-axis) $\left(\frac{3}{2}, -12\right)$, *halving instead of doubling the x coordinate, is the common error.*	B1 B1 (2)
		Total for Question 10: 8 marks

TOTAL FOR PAPER: 75 MARKS

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