eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2 (WME02)

Mechanics 2 Practice Paper 1

Worked Solutions

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WME02/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	Variable acceleration in a straight line	7	2
2	An impulse in vector form	8	3
3	A car working at constant power	9	4
4	A composite lamina	9	5
5	A projectile from a cliff	9	6
6	A direct collision and a wall	10	7
7	A ladder against a smooth wall	10	8
8	Work and energy on a rough slope	13	9

Question paper & mark scheme | Exam questions by topic | More practice papers

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

1. A particle P moves along a straight line. At time t seconds, $t \geqslant 0$, the acceleration of P is $(6t - 14)\,\text{m s}^{-2}$, in the positive x direction. At time $t = 0$, P passes through the point O with velocity $8\,\text{m s}^{-1}$, in the positive x direction.

(a) Find the velocity of P at time t seconds.

(2)

(b) Find the values of t at which P is instantaneously at rest.

(2)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

(a) $v = \int (6t - 14)\,dt = 3t^2 - 14t + c$; $t = 0$, $v = 8 \Rightarrow c = 8$ M1

$\boxed{v = 3t^2 - 14t + 8}$ A1

(b) At rest: $3t^2 - 14t + 8 = 0 \Rightarrow (3t - 2)(t - 4) = 0$ M1

$\boxed{t = 2/3 \text{ and } t = 4}$ A1

(c) $s = \int v\,dt = t^3 - 7t^2 + 8t$ ($s = 0$ at $t = 0$, so no constant) M1

Turns back at $t = 2/3$: $s(2/3) = 68/27$; $s(4) = 64 - 112 + 32 = -16$

Out to $68/27$, then back past 0 to -16: distance $= 68/27 + (68/27 + 16)$ M1

$= 568/27 = 21.03\ldots$ $\boxed{21.0\text{ m}}$ (not 16 m: that is the displacement) A1

(Total for Question 1 is 7 marks)

DO NOT WRITE IN THIS AREA

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. *In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular unit vectors in a horizontal plane.*

A ball of mass 0.4 kg is moving with velocity $(5\mathbf{i}+2\mathbf{j})\,\text{m s}^{-1}$ when it is struck by a bat. The impulse exerted on the ball by the bat is $(-3\mathbf{i}+2\mathbf{j})\,\text{N s}$

(a) Find the speed of the ball immediately after it is struck.

(5)

(b) Find the kinetic energy gained by the ball as a result of being struck.

(3)

(a) $I = m(v - u)$: $-3i + 2j = 0.4(v - (5i + 2j))$ M1 A1

$v - (5i + 2j) = (-3i + 2j) \div 0.4 = -7.5i + 5j \Rightarrow v = -2.5i + 7j$ A1

Speed $= \sqrt{2.5^2 + 7^2} = \sqrt{55.25}$ M1

$= 7.433\ldots$ $7.43\ \text{m s}^{-1}$ (a speed, not the vector) A1

(b) KE before $= \frac{1}{2} \times 0.4 \times (5^2 + 2^2) = 0.2 \times 29 = 5.8$ J M1

KE after $= \frac{1}{2} \times 0.4 \times 55.25 = 11.05$ J A1

Gain $= 11.05 - 5.8$ 5.25 J A1

(Total for Question 2 is 8 marks)

DO NOT WRITE IN THIS AREA

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3. A car of mass 1000 kg moves up a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude 500 N. The engine of the car is working at a constant rate of 24 kW.

(a) Show that the maximum speed of the car up the road is $20\,\text{m s}^{-1}$

(4)

(b) Find the acceleration of the car at the instant when its speed is $12\,\text{m s}^{-1}$

(3)

Later the car moves along a straight horizontal road at a constant speed of $30\,\text{m s}^{-1}$. The resistance to motion is unchanged.

(c) Find, in kW, the rate at which the engine of the car is now working.

(2)

(a) Driving force $D = P/v = 24000/v$ M1

Top speed: $a = 0$, so $D = 500 + 1000g \sin\alpha = 500 + 9800/14$ M1

$24000/v = 500 + 700 = 1200$ A1

$v = 24000 \div 1200 = 20\ m\ s^{-1}$ ✓ A1*

(b) At $12\ m\ s^{-1}$: $D = 24000 \div 12 = 2000$ N. $F = ma$ up the road:

$2000 - 500 - 700 = 1000a$ M1 A1

$1000a = 800$ $\boxed{a = 0.8\ m\ s^{-2}}$ A1

(c) Level road, constant speed: D = resistance = 500 N; $P = Dv = 500 \times 30$ M1

$= 15000$ W $\boxed{15\ kW}$ A1

(Total for Question 3 is 9 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4.

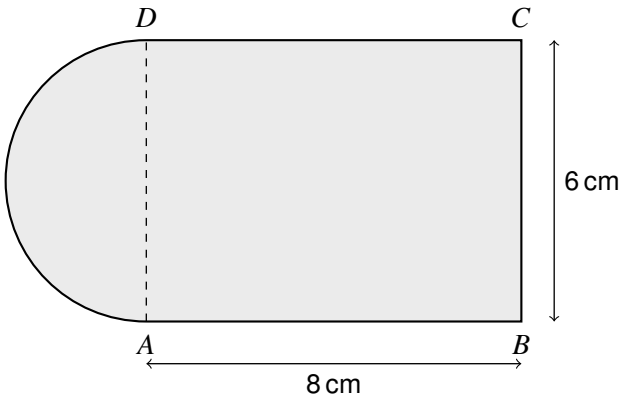

Figure 1

A uniform lamina is formed by joining a rectangle $ABCD$ to a semicircle with diameter AD, as shown in Figure 1. The rectangle has $AB = 8$ cm and $AD = 6$ cm, and the semicircle lies outside the rectangle.

(a) Find the distance of the centre of mass of the lamina from AD

(5)

The lamina is freely suspended from B and hangs in equilibrium.

(b) Find the angle between AB and the vertical.

(4)

(a) Rectangle: area 8 × 6 = 48, centre 4 cm from AD.

Semicircle: r = 3, area ½π(3²) = 4.5π; centre 4r/(3π) = 4/π from AD, on the other side. B1 B1

Moments about AD (x positive towards BC): 48 × 4 + 4.5π × (−4/π) = (48 + 4.5π)$\bar{x}$ M1

192 − 18 = (48 + 4.5π)$\bar{x}$ ⇒ $\bar{x}$ = 174 / 62.137... A1

$\bar{x}$ = 2.800... 2.80 cm from AD A1

(b) By symmetry G is 3 cm from AB. From B: 8 − 2.80 = 5.20 cm along BA, 3 cm across. B1

Hanging from B, BG is vertical. tan θ = 3 / 5.20, θ between BA and the vertical M1 A1

θ = 29.98... 30.0° A1

(Total for Question 4 is 9 marks)

DO NOT WRITE IN THIS AREA

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5.

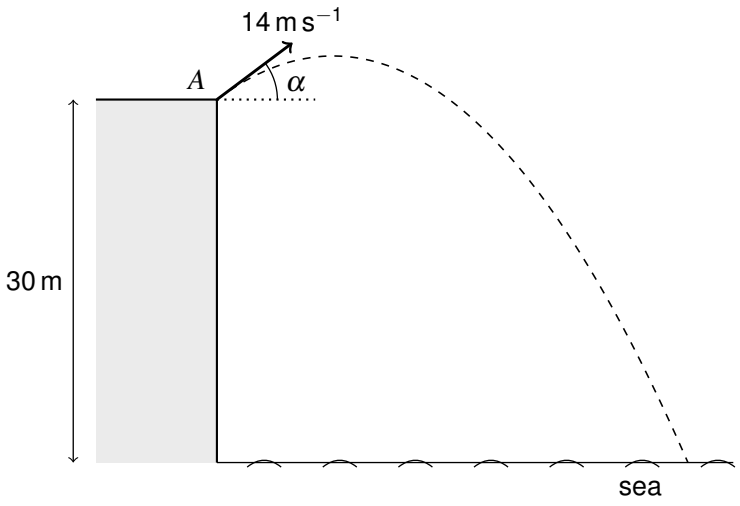

Figure 2

A ball is projected from a point A at the top of a vertical cliff. The point A is 30 m vertically above the sea. The ball is projected with speed 14 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 2. The ball moves freely under gravity and hits the sea.

(a) Find the time taken for the ball to reach the sea.

(4)

(b) Find the horizontal distance from the foot of the cliff to the point where the ball hits the sea.

(2)

(c) Find the speed of the ball as it hits the sea.

(3)

(a) sin α = 0.6, cos α = 0.8: horizontal 14 × 0.8 = 11.2; vertical 14 × 0.6 = 8.4 B1

Up positive, sea is 30 m below A: $-30 = 8.4t - 4.9t^2$ M1

$4.9t^2 - 8.4t - 30 = 0 \Rightarrow t = (8.4 + \sqrt{70.56 + 588}) \div 9.8$ A1

$t = 3.475\ldots$ (the other root is negative) 3.48 s A1

(b) Horizontal speed is constant: 11.2 × 3.475… M1

= 38.92… 38.9 m A1

(c) Vertical: 8.4 − 9.8 × 3.475… = −25.66; horizontal still 11.2 M1

$v^2 = 11.2^2 + 25.66^2 = 125.44 + 658.56 = 784$ (check: $14^2 + 2 \times 9.8 \times 30 = 784$ ✓) A1

$v = \sqrt{784}$ 28 m s^{-1} A1

(Total for Question 5 is 9 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

6. Two particles, A and B, of masses $3m$ and m respectively, are moving in opposite directions along the same straight line on a smooth horizontal surface. Just before they collide directly, A has speed $2u$ and B has speed u. The coefficient of restitution between A and B is e

(a) Show that the speed of B immediately after the collision is $\frac{u(5+9e)}{4}$

(5)

Given that $e = \frac{1}{3}$

(b) find, in terms of m and u, the kinetic energy lost in the collision.

(3)

After the collision, B hits a fixed smooth vertical wall which is perpendicular to its direction of motion. The coefficient of restitution between B and the wall is $\frac{1}{4}$

(c) Find, in terms of m and u, the magnitude of the impulse exerted on B by the wall.

(2)

(a) Positive in A's direction. Before: A +2u, B −u. After: v_A, v_B.

Momentum: $3m(2u) + m(-u) = 3mv_A + mv_B$ M1

$5u = 3v_A + v_B$ A1

Restitution: $v_B - v_A = e(2u - (-u))$ M1

$v_B - v_A = 3eu$ A1

× 3 and add: $4v_B = 5u + 9eu \Rightarrow v_B = u(5 + 9e)/4$ ✓ A1*

(b) $e = 1/3$: $v_B = u(5 + 3)/4 = 2u$, $v_A = 2u - u = u$

KE before $= \frac{1}{2}(3m)(2u)^2 + \frac{1}{2}m u^2 = 6mu^2 + \frac{1}{2}mu^2 = 6.5mu^2$ M1

KE after $= \frac{1}{2}(3m)u^2 + \frac{1}{2}m(2u)^2 = 1.5mu^2 + 2mu^2 = 3.5mu^2$ A1

Lost $= 6.5mu^2 - 3.5mu^2$ $\boxed{3mu^2}$ A1

(c) B hits the wall at 2u and rebounds at ¼ × 2u = u/2 (direction reversed) M1

Impulse $= m(u/2 - (-2u)) = m(5u/2)$ $\boxed{5mu/2}$ A1

(Total for Question 6 is 10 marks)

DO NOT WRITE IN THIS AREA

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7.

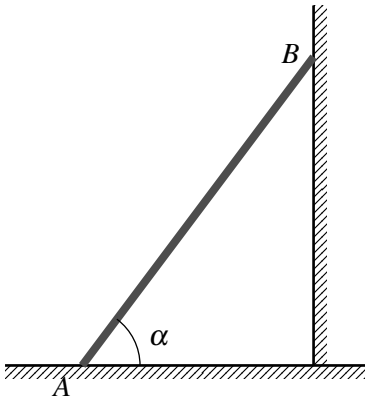

Figure 3

A uniform ladder AB, of mass 20 kg and length 6 m, has its end A on rough horizontal ground and its end B against a smooth vertical wall. The ladder rests in equilibrium in a vertical plane perpendicular to the wall, at an angle α to the ground, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 3. The coefficient of friction between the ladder and the ground is μ

(a) Show that $\mu \geqslant \frac{3}{8}$

(5)

The ladder is modelled as a uniform rod. Given that $\mu = 0.5$, a man of mass 80 kg climbs the ladder from A. The man is modelled as a particle.

(b) Find the distance the man can climb up the ladder before it is about to slip.

(5)

(a) $\sin\alpha = 0.8$, $\cos\alpha = 0.6$. Smooth wall: S horizontal. Ground: R and F.

Resolve: $R = 20g$, $F = S$ M1

Moments about A: $S \times 6\sin\alpha = 20g \times 3\cos\alpha \Rightarrow 4.8S = 36g$ M1

$S = 7.5g$, so $F = 7.5g$ A1

Equilibrium needs $F \leqslant \mu R$: $7.5g \leqslant \mu \times 20g$ M1

$\mu \geqslant 7.5/20 = 3/8$ ✓ A1*

(b) About to slip: $R = 20g + 80g = 100g$, $F = 0.5 \times 100g = 50g = S$ B1

Man x m from A. Moments about A: $S \times 6\sin\alpha = 20g \times 3\cos\alpha + 80g \times x\cos\alpha$ M1

$50g \times 4.8 = 20g \times 1.8 + 80g \times 0.6x$ A1

$240 = 36 + 48x \Rightarrow 48x = 204$ M1

$x = 4.25$ 4.25 m up the ladder A1

(Total for Question 7 is 10 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8.

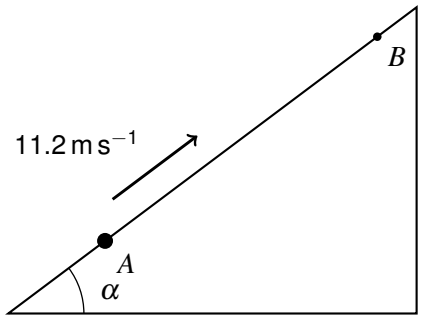

Figure 4

A particle of mass 2 kg is projected with speed 11.2 m s^{-1} from a point A up a line of greatest slope of a rough plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The particle comes to instantaneous rest at the point B, where $AB = 8$ m, as shown in Figure 4. The coefficient of friction between the particle and the plane is μ

(a) Use the work-energy principle to show that $\mu = \frac{1}{4}$

(5)

(b) Show that the particle does not remain at rest at B

(2)

(c) Use the work-energy principle to find the speed of the particle when it returns to A

(4)

(d) Find the total work done against friction as the particle moves from A to B and back to A

(2)

(a) sin α = 0.6, cos α = 0.8. R = 2g cos α = 15.68, so friction = 15.68μ — B1

Work-energy, A to B: KE lost = PE gained + work done against friction — M1

½ × 2 × 11.2² = 2g × (8 sin α) + 15.68μ × 8 — A1

125.44 = 94.08 + 125.44μ — A1

125.44μ = 31.36 ⇒ μ = 1/4 ✓ — A1*

(b) Down the slope: 2g sin α = 11.76 N. Greatest friction: ¼ × 15.68 = 3.92 N — M1

11.76 > 3.92, so friction cannot hold it: it slides back down. — A1

(c) Work-energy, B to A: PE lost = KE gained + work done against friction — M1

2g × 8 × 0.6 = ½ × 2 × v² + 3.92 × 8 ⇒ 94.08 = v² + 31.36 — A1

v² = 62.72 — A1

v = 7.919... 7.92 m s^{-1} — A1

(d) Friction 3.92 N over 8 m up and 8 m down: 3.92 × 16 — M1

= 62.72 62.7 J (check: ½ × 2 × (125.44 − 62.72) = 62.72 ✓) — A1

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END

★★★★★

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk