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A-LEVEL MATHEMATICS • FURTHER PURE 1

Complex Numbers

Worksheet

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Free Complex Numbers worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

By the end of this worksheet you can: calculate with complex numbers • divide using the conjugate • use the Argand diagram • find modulus and argument • *Stretch:* roots of cubics and quartics, unknown coefficients, and geometry on the Argand diagram.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Arguments in every quadrant, unknown coefficients, and areas and perimeters on the Argand diagram.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: arithmetic

$z = 4 - 3\text{i}$ and $w = 1 + 2\text{i}$. Find zw and $z - 2w$

Expand, then use $\text{i}^2 = -1$

$zw = 4 + 8\text{i} - 3\text{i} - 6\text{i}^2 = 10 + 5\text{i}$; $z - 2w = 4 - 3\text{i} - 2 - 4\text{i}$

$10 + 5\text{i},\ 2 - 7\text{i}$

Your turn A1

$z = 5 + 2\text{i}$ and $w = 3 - \text{i}$. Find zw and $2z + w$

Worked example A2: division

Express $\dfrac{7 + \text{i}}{1 - 2\text{i}}$ in the form $a + b\text{i}$

Multiply top and bottom by the conjugate $1 + 2\text{i}$

$$\frac{(7 + \text{i})(1 + 2\text{i})}{1 + 4} = \frac{5 + 15\text{i}}{5}$$

$1 + 3\text{i}$

Your turn A2

Express $\dfrac{5 + 15\text{i}}{2 + \text{i}}$ in the form $a + b\text{i}$

Worked example A3: modulus and argument

Find the modulus and argument of $z = -2 - 2\text{i}$

Sketch: the third quadrant, so subtract from $-\pi$

$|z| = \sqrt{4 + 4}$; $\arg z = -\pi + \dfrac{\pi}{4}$

$2\sqrt{2},\ -\frac{3\pi}{4}$

Your turn A3

Find the modulus and argument of $z = -\sqrt{3} + \text{i}$

STRETCH

Worked example A4: a quadratic with complex roots

Solve $z^2 - 4z + 13 = 0$

$$z = \frac{4 \pm \sqrt{16 - 52}}{2} = \frac{4 \pm 6\text{i}}{2}$$

$2 \pm 3\text{i}$

Your turn A4

Solve $z^2 + 2z + 10 = 0$

Worked example A5: equating parts

Find z such that $z + 2z^* = 9 - 4\text{i}$

Let $z = a + b\text{i}$, then equate real and imaginary parts

$3a - b\text{i} = 9 - 4\text{i}$

$z = 3 + 4\text{i}$

Your turn A5

Find z such that $3z - z^* = 4 + 12\text{i}$

Worked example A6: a cubic from one complex root

$2 - \text{i}$ is a root of $z^3 - 6z^2 + 13z - 10 = 0$. Find the other roots.

The conjugate is a root, so $(z - 2 + \text{i})(z - 2 - \text{i}) = z^2 - 4z + 5$ is a factor

$z^3 - 6z^2 + 13z - 10 = (z^2 - 4z + 5)(z - 2)$

$2 + \text{i},\ 2$

Your turn A6

$-1 + 2\text{i}$ is a root of $z^3 + 5z^2 + 11z + 15 = 0$. Find the other roots.

Worked example A7: products and quotients

$|z| = 4$, $\arg z = \frac{\pi}{3}$, $|w| = 2$, $\arg w = \frac{\pi}{6}$. Find zw in the form $a + b\text{i}$

Multiply the moduli, add the arguments

$|zw| = 8$, $\arg(zw) = \frac{\pi}{2}$

8i

Your turn A7

$|z| = 6$, $\arg z = \frac{3\pi}{4}$, $|w| = 3$, $\arg w = \frac{\pi}{4}$. Find $\frac{z}{w}$ in the form $a + b\text{i}$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 $z = 3 + 4\text{i}$. Find, in the form $a + b\text{i}$,

(a) z^2

(b) zz^*

(c) $z + z^*$

(d) $z^2 - 6z$

[4 marks]

B2 Express in the form $a + b\text{i}$

(a) $\dfrac{10}{1+3\text{i}}$

(b) $\dfrac{10\text{i}}{1+3\text{i}}$

(c) $\dfrac{1+3\text{i}}{1-3\text{i}}$

(d) $\dfrac{(2+\text{i})^2}{3-4\text{i}}$

[4 marks]

B3 Find the modulus and the argument of

(a) $1 + \text{i}$

(b) $-1 + \text{i}$

(c) $-1 - \text{i}$

(d) $1 - \text{i}$

[4 marks]

B4 Find the modulus and the argument, in radians to 3 significant figures where necessary, of

(a) $3 + 4\text{i}$

(b) $-3 + 4\text{i}$

(c) $-3 - 4\text{i}$

(d) 4i

[4 marks]

B5 Solve each equation, giving the roots in the form $a + b\text{i}$

(a) $z^2 + 9 = 0$

(b) $z^2 - 6z + 10 = 0$

(c) $2z^2 + 2z + 5 = 0$

[3 marks]

B6 Write each number in the form $a + b\text{i}$, using exact values.

(a) $4\left(\cos\frac{\pi}{2} + \text{i}\sin\frac{\pi}{2}\right)$

(b) $2\left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right)$

(c) $6\left(\cos\left(-\frac{\pi}{6}\right) + \text{i}\sin\left(-\frac{\pi}{6}\right)\right)$

(d) $\sqrt{2}\left(\cos\frac{3\pi}{4} + \text{i}\sin\frac{3\pi}{4}\right)$

[4 marks]

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STRETCH

B7 Find the complex number z in each case.

(a) $z+3z^* = 8+6\text{i}$

(b) $2z+z^* = 6+\text{i}$

(c) $\text{i}z+2z^* = 1-4\text{i}$

(d) $z^2 = 3+4\text{i}$

[4 marks]

B8 $\text{f}(z) = z^3-3z^2+4z-4$

(a) Show that $z = 2$ is a root of $\text{f}(z) = 0$

(b) Find the quadratic factor of $\text{f}(z)$

(c) Find the complex roots.

(d) Find the sum of all three roots.

[4 marks]

B9 Find the polynomial, with real coefficients and leading coefficient 1, that has the given roots.

(a) $3\pm 2\text{i}$

(b) $1\pm 4\text{i}$

(c) 2 and $1\pm\text{i}$

(d) $\pm\text{i}$ and $2\pm\text{i}$

[4 marks]

B10 $|z| = 3$, $\arg z = \dfrac{\pi}{6}$, $|w| = 2$ and $\arg w = -\dfrac{\pi}{3}$. Find

(a) $|zw|$

(b) $\arg(zw)$

(c) $\left|\dfrac{z}{w}\right|$

(d) $\arg\left(\dfrac{z}{w}\right)$

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Amir finds the argument of $-4+4\text{i}$ as $\tan^{-1}\left(\frac{4}{-4}\right) = -\frac{\pi}{4}$. Explain his error, and find the argument.

[2 marks]

C2 Bea writes $(2+3\text{i})^2 = 4+9 = 13$. Explain her error, and find $(2+3\text{i})^2$ **[2 marks]**

C3 Cal says that because $z^3 - 2z^2 + 4z - 8 = 0$ has real coefficients and the roots 2 and 2i, its third root must be real. Explain his error, and find the third root. **[3 marks]**

C4 The roots of $(z+2)(z^2+9) = 0$ are shown on an Argand diagram. Dev plots the root -2 at the point $(0, -2)$ Explain his error, and give the points representing all three roots. **[3 marks]**

STRETCH

C5 $z = 4\text{i}$ and $w = 1+\text{i}$. Eve says $\arg\left(\frac{z}{w}\right) = \frac{\arg z}{\arg w} = \frac{\pi/2}{\pi/4} = 2$

Explain her error, and find $\arg\left(\frac{z}{w}\right)$ **[3 marks]**

C6 $1+\text{i}$ is a root of $z^2 - (3+\text{i})z + 2 + 2\text{i} = 0$. Fay says that $1-\text{i}$ must be the other root.

Explain her error, and find the other root. **[3 marks]**

C7 Gus says that $z = 3\left(\cos\frac{\pi}{3} - \text{i}\sin\frac{\pi}{3}\right)$ is in modulus-argument form with argument $\frac{\pi}{3}$

Explain his error, and write z correctly in modulus-argument form. **[3 marks]**

C8 $z = a + b\text{i}$, where a and b are real and not both zero.

(a) Show that $zz^* = |z|^2$

(2)

(b) Show that $\frac{1}{z} = \frac{z^*}{|z|^2}$, and hence that $\left|\frac{1}{z}\right| = \frac{1}{|z|}$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The complex number z is defined by $z = 2 + \mathrm{i}$, and z^* is the complex conjugate of z

(a) Determine $z^2 + z^*$ in the form $a + b\mathrm{i}$

(1)

(b) Express $\dfrac{50}{z^*}$ in the form kz, where k is a positive integer.

(2)

[3 marks]

D2 The complex number z_1 has modulus 4 and argument $\dfrac{2\pi}{3}$. The complex number z_2 has modulus 3. Given that the product $z_1 z_2$ is a positive real number, and that arguments are measured in radians,

(a) state the value of $|z_1 z_2|$

(1)

(b) determine z_2 in the form $a + b\mathrm{i}$, giving exact values.

(3)

[4 marks]

D3 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The complex number z is given by $z = (1 + \mathrm{i})(\lambda - 3\mathrm{i})$, where λ is a real constant.

(a) Write z in the form $x + y\mathrm{i}$, where x and y are in terms of λ

(2)

(b) Given that z is purely imaginary, find the value of λ and the value of z

(1)

[3 marks]

D4 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Given that the complex number $1 + 2\mathrm{i}$ is a root of the cubic equation $z^3 + z^2 - z + 15 = 0$, use algebra to determine the other two roots of the equation, giving each root in its simplest form. **[3 marks]**

STRETCH

D5 $\mathrm{f}(z) = z^3 + pz^2 + qz + 26$, where p and q are real constants. Given that the complex number $3 - 2\mathrm{i}$ is a root of the equation $\mathrm{f}(z) = 0$, use the fact that the coefficients of $\mathrm{f}(z)$ are real to determine the value of p, the value of q and the real root of the equation. **[4 marks]**

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D6 The complex numbers z and w are such that $w = (1+2\text{i})z$. Given that $w = -1+8\text{i}$, and showing your working clearly,

(a) determine z in the form $a+b\text{i}$, where a and b are integers

(2)

(b) determine $\arg w$, giving your answer in radians to 3 significant figures.

(2)

[4 marks]

D7 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The cubic equation $z^3 - 5z^2 + 7z + 13 = 0$, which has real coefficients, has the real root -1

(a) Determine the other two roots of the equation.

(2)

(b) The points representing the three roots of the equation on an Argand diagram are the vertices of a triangle. Determine the exact area of this triangle.

(2)

[4 marks]

D8 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The roots of the equation $z^2 - 4z + 8 = 0$ are represented on an Argand diagram by the points P and Q. The numbers -2 and 6 are represented by the points R and S

(a) Determine the roots of the equation.

(2)

(b) Show these four points on a single Argand diagram, and determine the perimeter of the quadrilateral $RPSQ$, giving your answer as a simplified surd.

(3)

[5 marks]

E Exam practice

Exam-style questions in the style of WFM01. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 $f(z) = 2z^3 + Az^2 + Bz + C$, where A, B and C are integers.

Given that $z = 1 + 2i$ is a root of the equation $f(z) = 0$

(a) write down the other complex root of the equation.

(1)

Given also that $z = \frac{1}{2}$ is a root of the equation $f(z) = 0$

(b) determine the value of A, the value of B and the value of C

(3)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

(Total for Question E1 is 6 marks)

E2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The complex numbers z_1 and z_2 are given by $z_1 = 2 + \mathrm{i}$ and $z_2 = 1 - 3\mathrm{i}$

(a) Determine the exact value of $|z_1 z_2|$

(2)

(b) Express $\frac{z_1}{z_2}$ in the form $a + b\mathrm{i}$, where a and b are rational.

(2)

(c) Determine $\arg\left(\frac{z_1}{z_2}\right)$, giving your answer in radians to 3 significant figures.

(2)

(d) State the quadrant of the Argand diagram in which $\frac{z_1}{z_2}$ lies.

(1)

(Total for Question E2 is 7 marks)

STRETCH

E3 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

$f(z) = z^4 + Pz^3 + 7z^2 + Qz + 26$, where P and Q are real constants.

Given that $2 + 3i$ is a root of the equation $f(z) = 0$

(a) write down another root of the equation $f(z) = 0$

(1)

(b) Hence determine a quadratic factor, with real coefficients, of $f(z)$

(2)

(c) Determine the value of P and the value of Q

(3)

(d) Determine the other roots of the equation $f(z) = 0$

(2)

(e) Show all four roots on a single Argand diagram.

(1)

(Total for Question E3 is 9 marks)

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