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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 3 (WFM03)

Further Pure 3 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM03/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1 and 3 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

Rough grade guide (out of 75): A* **68** • A **62** • B **55** • C **48**

Based on recent WFM03 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	A double-angle identity, a quadratic in $\cosh x$, and both signs of x	6	Ch 1 Hyperbolic Functions	
2	Differentiating with $\operatorname{arsinh} x$; an exact integral from the result	7	Ch 3 Differentiation · Ch 4 Integration	
3	Completing the square to reach arsinh and arctan forms	7	Ch 4 Integration	
4	A 3×3 determinant with a parameter; an inverse matrix	8	Ch 6 Further Matrix Algebra	
5	Eccentricity, foci and directrices of an ellipse	8	Ch 2 Further Coordinate Systems	
6	A plane from three points; where a line meets it; the angle	9	Ch 5 Vectors	
7	A reduction formula by parts twice; an exact value	9	Ch 4 Integration	
8	Eigenvalues and eigenvectors; an orthogonal $\mathbf{P}$ with $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$	10	Ch 6 Further Matrix Algebra	
9	A tangent to a hyperbola; its asymptotes; a constant area	11	Ch 2 Further Coordinate Systems	
	Total	**75**		

Where did the marks go? FP3 has many printed results, and the marks there go on steps left out: a product rule not written, a boundary term not evaluated, an identity used without saying so. Elsewhere, look for a missing negative solution of $\cosh x = k$, a sign slip in a cofactor, or an eigenvector left unnormalised. Find the first line where your working differs from the scheme, then practise that chapter.

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Question 1

6 marks

Part	Scheme	Marks
	$\cosh 2x = 2\cosh^2 x - 1$: $4\cosh^2 x - 2 - 9\cosh x + 7 = 0$	M1
	$4\cosh^2 x - 9\cosh x + 5 = 0$	A1
	$(4\cosh x - 5)(\cosh x - 1) = 0$, $\cosh x = \frac{5}{4}$ or $\cosh x = 1$	M1 A1
	$\cosh x = \frac{5}{4}$: $x = \pm\ln\left(\frac{5}{4} + \sqrt{\frac{25}{16} - 1}\right) = \pm\ln 2$	M1
	$\boxed{x = 0,\ x = \ln 2,\ x = -\ln 2}$	A1
	cosh is even, so $\cosh x = \frac{5}{4}$ *gives two values of* x*. Giving only* $\ln 2$ *loses the final A1.*	(6)
		Total for Question 1: 6 marks

Question 2

7 marks

Part	Scheme	Marks
(a)	$\frac{d}{dx}(x\,\text{arsinh}\,x) = \text{arsinh}\,x + \frac{x}{\sqrt{1+x^2}}$ (product rule)	M1
	$\frac{d}{dx}\sqrt{1+x^2} = \frac{x}{\sqrt{1+x^2}}$	M1
	$\frac{dy}{dx} = \text{arsinh}\,x$	A1*
		(3)
(b)	$\left[x\,\text{arsinh}\,x - \sqrt{1+x^2}\right]_0^{3/4}$	M1
	$\text{arsinh}\,\frac{3}{4} = \ln\left(\frac{3}{4} + \frac{5}{4}\right) = \ln 2$, $\sqrt{1+\frac{9}{16}} = \frac{5}{4}$	M1
	$\frac{3}{4}\ln 2 - \frac{5}{4} - (0 - 1)$	A1
	$\boxed{\frac{3}{4}\ln 2 - \frac{1}{4}}$	A1
		(4)
		Total for Question 2: 7 marks

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Question 3

7 marks

Part	Scheme	Marks
(a)	$x^2+4x+13=(x+2)^2+9$	M1
	$\int \frac{\mathrm{d}x}{\sqrt{(x+2)^2+9}} = \operatorname{arsinh}\frac{x+2}{3}$	A1
	$\operatorname{arsinh}\frac{4}{3} - \operatorname{arsinh}0$	M1
	$= \ln\left(\frac{4}{3}+\sqrt{\frac{16}{9}+1}\right) = \ln\left(\frac{4}{3}+\frac{5}{3}\right) = \ln 3$	A1*
		(4)
(b)	$4x^2+4x+5=(2x+1)^2+4$	M1
	$\int \frac{\mathrm{d}x}{(2x+1)^2+4} = \frac{1}{4}\arctan\frac{2x+1}{2}$	A1
	$\boxed{\frac{1}{4}\left(\arctan\frac{3}{2}-\arctan\frac{1}{2}\right)}$ $\left(=\frac{1}{4}\arctan\frac{4}{7}\right)$	A1
		(3)
		Total for Question 3: 7 marks

Question 4

8 marks

Part	Scheme	Marks
(a)	$1(2k-1)-2(0-3)+k(0-3k)$	M1
	$\boxed{-3k^2+2k+5}$	A1
		(2)
(b)	$3k^2-2k-5=0,\ (3k-5)(k+1)=0$	M1
	$\boxed{k=-1,\ k=\frac{5}{3}}$	A1
		(2)
(c)	$k=2$: $\det\mathbf{M} = -12+4+5=-3$	B1
	Matrix of cofactors $\begin{pmatrix} 3 & 3 & -6 \\ -2 & -4 & 5 \\ -2 & -1 & 2 \end{pmatrix}$ (at least six correct)	M1
	Transpose and divide by $\det\mathbf{M}$	M1
	$\boxed{\mathbf{M}^{-1} = \frac{1}{3}\begin{pmatrix} -3 & 2 & 2 \\ -3 & 4 & 1 \\ 6 & -5 & -2 \end{pmatrix}}$	A1
		(4)
		Total for Question 4: 8 marks

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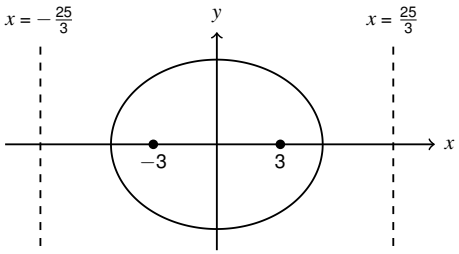
Question 5

8 marks

Part	Scheme	Marks
(a)	$b^2 = a^2(1-e^2)$: $16 = 25(1-e^2)$	M1
	$\boxed{e = \frac{3}{5}}$	A1
		(2)
(b)	Foci $(\pm ae, 0)$: $\boxed{(3, 0) \text{ and } (-3, 0)}$	B1
	Directrices $x = \pm\frac{a}{e}$	M1
	$\boxed{x = \frac{25}{3} \text{ and } x = -\frac{25}{3}}$ 	A1
		(3)
(c)	$\frac{16}{25} + \frac{p^2}{16} = 1$, $p = \frac{12}{5}$	M1
	$PS = \sqrt{1^2 + \left(\frac{12}{5}\right)^2}$ or $PS = e\left(\frac{25}{3} - 4\right)$	M1
	$PS = \frac{13}{5}$, $PS' = \frac{37}{5}$, $PS + PS' = \frac{50}{5} = 10$	A1
		(3)
		Total for Question 5: 8 marks

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Question 6

9 marks

Part	Scheme	Marks
(a)	$\overrightarrow{AB} = (2, 1, -1),\ \overrightarrow{AC} = (1, -1, 2)$	M1
	$\overrightarrow{AB} \times \overrightarrow{AC}$	M1
	$= (1, -5, -3)$	A1
	$\boxed{x - 5y - 3z = -5}$	A1
		(4)
(b)	$(2+t) - 5(2-t) - 3(1) = -5$	M1
	$6t - 11 = -5,\ t = 1$	A1
	$\boxed{(3, 1, 1)}$	A1
		(3)
(c)	$\sin\theta = \dfrac{\lvert(1,-5,-3)\cdot(1,-1,0)\rvert}{\sqrt{35}\sqrt{2}} = \dfrac{6}{\sqrt{70}}$	M1
	$\boxed{45.8^\circ}$	A1
	Using $\cos\theta$ gives the angle with the normal, 44.2°: M1 only if it is then subtracted from 90°.	
		(2)
	Total for Question 6: 9 marks	

Question 7

9 marks

Part	Scheme	Marks
(a)	$I_n = [x^n \sin x]_0^{\pi/2} - n\int_0^{\pi/2} x^{n-1}\sin x\,\mathrm{d}x$	M1
	$= \left(\frac{\pi}{2}\right)^n - n\int_0^{\pi/2} x^{n-1}\sin x\,\mathrm{d}x$	A1
	$\int_0^{\pi/2} x^{n-1}\sin x\,\mathrm{d}x = \left[-x^{n-1}\cos x\right]_0^{\pi/2} + (n-1)\int_0^{\pi/2} x^{n-2}\cos x\,\mathrm{d}x$	M1
	The boundary term is 0 at both limits (for $n \geqslant 2$), leaving $(n-1)I_{n-2}$	A1
	$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$	A1*
		(5)
(b)	$I_0 = \int_0^{\pi/2} \cos x\,\mathrm{d}x = 1$	B1
	$I_2 = \frac{\pi^2}{4} - 2$	M1
	$I_4 = \frac{\pi^4}{16} - 12\left(\frac{\pi^2}{4} - 2\right)$	M1
	$\boxed{\frac{\pi^4}{16} - 3\pi^2 + 24}$	A1
		(4)
	Total for Question 7: 9 marks	

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Question 8

10 marks

Part	Scheme	Marks
(a)	$\det(\mathbf{A}-\lambda\mathbf{I}) = (3-\lambda)\left[(1-\lambda)(-2-\lambda)-4\right]$ (expanding along row 2)	**M1**
	$= (3-\lambda)(\lambda^2+\lambda-6)$, which is 0 when $\lambda = 3$	**A1**
	$\lambda^2+\lambda-6 = (\lambda+3)(\lambda-2)$	**M1**
	$\boxed{\lambda = 2,\ \lambda = -3}$	**A1**
		(4)
(b)	Solving $(\mathbf{A}-\lambda\mathbf{I})\mathbf{v} = \mathbf{0}$ for one eigenvalue	**M1**
	Two correct	**A1**
	$\boxed{\lambda = 3\text{: } (0,1,0);\ \lambda = 2\text{: } (2,0,1);\ \lambda = -3\text{: } (1,0,-2)}$	**A1**
		(3)
(c)	Normalise: divide $(2,0,1)$ and $(1,0,-2)$ by $\sqrt{5}$	**M1**
	$\boxed{\mathbf{P} = \begin{pmatrix} 0 & \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix}}$	**A1**
	$\boxed{\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}}$, in the same order as the columns of $\mathbf{P}$	**A1**
		(3)
		Total for Question 8: 10 marks

Question 9

11 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = \frac{3\cosh t}{4\sinh t}$	M1 A1
	$y - 3\sinh t = \frac{3\cosh t}{4\sinh t}(x - 4\cosh t)$	M1
	$4y\sinh t - 12\sinh^2 t = 3x\cosh t - 12\cosh^2 t$, and $\cosh^2 t - \sinh^2 t = 1$ gives $3x\cosh t - 4y\sinh t = 12$	A1*
		(4)
(b)	$y = \frac{3}{4}x$: $3x(\cosh t - \sinh t) = 12$	M1
	$\cosh t - \sinh t = e^{-t}$, so $x = 4e^{t}$	M1
	$\boxed{A\left(4e^{t}, 3e^{t}\right)}$	A1
	$\boxed{B\left(4e^{-t}, -3e^{-t}\right)}$	A1
		(4)
(c)	Area $= \frac{1}{2}\left\|x_A y_B - x_B y_A\right\|$	M1
	$= \frac{1}{2}\left\|4e^{t}\left(-3e^{-t}\right) - 4e^{-t}\left(3e^{t}\right)\right\|$	M1
	$= \frac{1}{2}\left\|-12-12\right\| = 12$, which does not depend on t	A1*
		(3)
		Total for Question 9: 11 marks

TOTAL FOR PAPER: 75 MARKS

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