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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	3	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA14.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.

Marking a proof. A proof by contradiction earns its final mark only when it has all three parts: a clear assumption at the start (“Assume there is ...”), correct deductions in the right order, and a statement of the contradiction with a conclusion. A sketch or calculator values are not a proof.



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Chapter 6: Integration

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
	Parts with $u = x$, $\frac{dv}{dx} = \sec^2 x$: $x \tan x - \int \tan x dx$	M1
	Correct first application	A1
	$\int \tan x dx = \ln \sec x $ (or $-\ln \cos x $)	M1
	$[x \tan x - \ln \sec x]_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \ln \sqrt{2}$	dM1
	$\frac{1}{4}\pi - \frac{1}{2}\ln 2$	A1
		(5)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
	$x = u^2$, so $dx = 2u du$ (or $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$)	B1
	Full substitution: $\int \frac{2u}{u-1} du$	M1
	Limits $u = 2$ and $u = 3$	B1
	$\frac{2u}{u-1} = 2 + \frac{2}{u-1}$	M1
	$[2u + 2\ln(u-1)]_2^3 = (6 + 2\ln 2) - (4 + 0)$	M1
	$2 + 2\ln 2$, so $a = 2$, $b = 2$	A1
	Using the x limits 4 and 9 with an integral in u loses the last two marks (Oct 2022 report). Writing dx as du is M0.	
		(6)
Total for Question 2: 6 marks		



Question 3

8 marks

Part	Scheme	Marks
(a)	$5 \equiv A(x+2) + B(2x-1)$; substitutes $x = \frac{1}{2}$ or $x = -2$ $A = 2$ or $B = -1$ $f(x) = \frac{2}{2x-1} - \frac{1}{x+2}$	M1 A1 A1 (3)
(b)	Integrates to the form $p \ln(2x-1) + q \ln(x+2)$ $\ln(2x-1) - \ln(x+2)$ Substitutes the limits and combines: $\ln \frac{(4k-1)(k+2)}{(2k+2)(2k-1)}$ $9(4k-1)(k+2) = 14(2k+2)(2k-1) \Rightarrow 4k^2 - 7k - 2 = 0$ $(4k+1)(k-2) = 0$; $k = -\frac{1}{4}$ is rejected as $k > \frac{1}{2}$: $k = 2$ <i>The integral of $\frac{2}{2x-1}$ is $\ln(2x-1)$, not $2\ln(2x-1)$. The June 2023 report: many failed to reject the second root.</i>	M1 A1 M1 M1 A1 (5)
Total for Question 3: 8 marks		

Question 4

5 marks

Part	Scheme	Marks
	$V = \pi \int_1^2 \left(2 + \frac{1}{x}\right)^2 dx$ Squares to three terms: $4 + \frac{4}{x} + \frac{1}{x^2}$ Integrates: $4x + 4 \ln x - \frac{1}{x}$ $\pi \left[\left(8 + 4 \ln 2 - \frac{1}{2}\right) - (4 + 0 - 1) \right]$ $\pi \left(\frac{9}{2} + 4 \ln 2 \right)$ <i>Squaring as $4 + \frac{1}{x^2}$ (each term only) is the Oct 2024 error: M0. A missing π loses the final mark.</i>	B1 M1 M1 dM1 A1 (5)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
	Separates: $\int \sec^2 y \, dy = \int (2x + 1) \, dx$ Integrates both sides $\tan y = x^2 + x + c$ Uses $x = 0, y = \frac{\pi}{4}$ $c = 1$ $y = \arctan(x^2 + x + 1)$	B1 M1 A1 M1 A1 A1 (6)
Total for Question 5: 6 marks		



Question 6

11 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = 2 \sec^2 t$ $\text{Area} = \int y \frac{dx}{dt} dt = \int 4 \cos^2 t \times 2 \sec^2 t dt$ $x = 0 \Rightarrow t = 0, x = 2 \Rightarrow t = \frac{\pi}{4}: \int_0^{\frac{\pi}{4}} 8 dt, \text{ so } k = 8$ $\text{Area } \boxed{2\pi}$ <i>Oct 2024 report: replacing dx by dt without $\frac{dx}{dt}$ scores M0.</i>	B1 M1 B1 A1 (4)
(b)	$\cos^2 t = \frac{1}{\sec^2 t} = \frac{1}{1 + \tan^2 t} = \frac{1}{1 + \frac{x^2}{4}}$ $y = \frac{4}{1 + \frac{x^2}{4}} = \frac{16}{4 + x^2}^*$	M1 A1* (2)
(c)	$V = \pi \int y^2 \frac{dx}{dt} dt = \pi \int 16 \cos^4 t \times 2 \sec^2 t dt$ $= 32\pi \int_0^{\frac{\pi}{4}} \cos^2 t dt$ $\cos^2 t = \frac{1}{2}(1 + \cos 2t): 16\pi \left[t + \frac{1}{2} \sin 2t \right]$ $16\pi \left(\frac{\pi}{4} + \frac{1}{2} \right)$ $\boxed{\pi(4\pi + 8)}$	B1 M1 M1 dM1 A1 (5)
Total for Question 6: 11 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	Parts: $-e^{-x} \sin x + \int e^{-x} \cos x dx$ Correct first application Parts again in the same direction: $\int e^{-x} \cos x dx = -e^{-x} \cos x - \int e^{-x} \sin x dx$ $I = -e^{-x} \sin x - e^{-x} \cos x - I$, so $2I = \dots$ $-\frac{1}{2}e^{-x}(\sin x + \cos x) + c$ <i>Changing direction in the second application returns $I = I$ and scores M0 for the third mark.</i>	M1 A1 M1 M1 A1 (5)
(b)	$\left[-\frac{1}{2}e^{-x}(\sin x + \cos x)\right]_0^{\pi} = \frac{1}{2}e^{-\pi} + \frac{1}{2}$ $\frac{1}{2}(1 + e^{-\pi})$	M1 A1 (2)
(c)	$\int_{\pi}^{2\pi} = -\frac{1}{2}e^{-2\pi} - \frac{1}{2}e^{-\pi}$ Negative (below the axis), so area $R_2 = \frac{1}{2}e^{-\pi}(1 + e^{-\pi})$ $\frac{\text{area } R_1}{\text{area } R_2} = \frac{\frac{1}{2}(1 + e^{-\pi})}{\frac{1}{2}e^{-\pi}(1 + e^{-\pi})} = e^{\pi}$ *	M1 M1 A1* (3)
Total for Question 7: 10 marks		



Question 8

12 marks

Part	Scheme	Marks
(a)	$1 \equiv A(10 - P) + BP$; substitutes $P = 0$ or $P = 10$ $A = \frac{1}{10}$ or $B = \frac{1}{10}$ $\frac{1}{10P} + \frac{1}{10(10 - P)}$	M1 A1 A1 (3)
(b)	Separates: $\int \frac{20}{P(10 - P)} dP = \int dt$ Integrates using (a) to the form $p \ln P + q \ln(10 - P)$ $2 \ln P - 2 \ln(10 - P) = t + c$ $t = 0, P = 2: c = 2 \ln \frac{2}{8} = -2 \ln 4$ Combines and exponentiates: $\frac{P}{10 - P} = \frac{1}{4} e^{\frac{1}{2}t}$ $P = \frac{10e^{\frac{1}{2}t}}{4 + e^{\frac{1}{2}t}} = \frac{10}{1 + 4e^{-\frac{1}{2}t}}$ * <i>The integral of $\frac{1}{10 - P}$ with respect to P is $-\ln(10 - P)$: the sign is often lost. Omitting $+c$ is a critical error (June 2023 report).</i>	B1 M1 A1 M1 M1 A1* (6)
(c)	$e^{-\frac{1}{2}t} \rightarrow 0$: 10 000 fish	B1 (1)
(d)	$8 = \frac{10}{1 + 4e^{-\frac{1}{2}t}} \Rightarrow e^{-\frac{1}{2}t} = \frac{1}{16}$ $t = 2 \ln 16 = 8 \ln 2$ years	M1 A1 (2)
Total for Question 8: 12 marks		



Question 9

12 marks

Part	Scheme	Marks
(a)	$V = \pi \int_0^k x e^{-x} dx$ $\text{Parts: } -x e^{-x} + \int e^{-x} dx$ $-x e^{-x} - e^{-x}$ $\text{Limits: } (-k e^{-k} - e^{-k}) - (0 - 1)$ $V = \pi (1 - (k+1)e^{-k}) *$	B1 M1 A1 dM1 A1* (5)
(b)	π	B1 (1)
(c)	$1 - (\beta + 1)e^{-\beta} = 0.9 \Rightarrow (\beta + 1)e^{-\beta} = 0.1$ $e^{\beta} = 10(\beta + 1), \text{ so } \beta = \ln(10\beta + 10) *$	M1 A1* (2)
(d)	$\beta_2 = 3.9120$ $\beta_3 = 3.8943$	M1 A1 (2)
(e)	$h(\beta) = \beta - \ln(10\beta + 10): h(3.8895) = -0.00018, h(3.8905) = 0.00062$ Sign change and h continuous, so $\beta = 3.890$ (3 d.p.)	M1 A1 (2)
Total for Question 9: 12 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	Parts: $\frac{1}{2}x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$ $\frac{1}{2}x \sin 2x$ $\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$	M1 A1 A1 (3)
(b)	Separates: $\int y \, dy = \int x \cos^2 x \, dx$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, so $x \cos^2 x = \frac{1}{2}x + \frac{1}{2}x \cos 2x$ Integrates using (a) $\frac{1}{2}y^2 = \frac{1}{4}x^2 + \frac{1}{4}x \sin 2x + \frac{1}{8} \cos 2x + c$ $x = 0, y = 1: \frac{1}{2} = \frac{1}{8} + c, c = \frac{3}{8}$ $y^2 = \frac{1}{2}x^2 + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + \frac{3}{4} *$	B1 M1 M1 A1 M1 A1* (6)
(c)	$y^2 = \frac{\pi^2}{8} + 0 - \frac{1}{4} + \frac{3}{4} = \frac{\pi^2 + 4}{8}$ $y = \sqrt{\frac{\pi^2 + 4}{8}}$ (positive, as $y > 0$)	M1 A1 (2)
Total for Question 10: 11 marks		

