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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 3

Further Pure 3

Chapter 6: Further Matrix Algebra

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM03/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM03 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Hyperbolic Functions** Definitions, graphs, inverse functions, identities and equations	10	80	separate file	
2	**Further Coordinate Systems** Ellipses and hyperbolas, foci and directrices, tangents, normals and loci	10	80	separate file	
3	**Differentiation** Hyperbolic and inverse trigonometric and hyperbolic functions	10	80	separate file	
4	**Integration** Substitutions, reduction formulae, arc length and surface area	10	80	separate file	
5	**Vectors** Vector product, triple products, lines and planes, distances and angles	10	80	separate file	
6	**Further Matrix Algebra** 3×3 matrices, inverses, eigenvalues and eigenvectors, diagonalisation	10	80	2	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 6 • KEY SKILLS

6 Further Matrix Algebra

Specification FP3: Further matrix algebra • 10 questions • 80 marks • about 96 minutes

1 3×3 determinants

Expand along a row or column with zeros. Singular means det = 0; $\det(\mathbf{AB}) = \det\mathbf{A}\det\mathbf{B}$

2 Inverse

Minors, then cofactors (sign pattern), transpose to get the adjugate, then divide by the determinant.

3 Products

$(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$ and $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$: the order reverses.

4 Linear equations

Write $\mathbf{Mx} = \mathbf{c}$; if $\det\mathbf{M} \neq 0$ the three planes meet at the single point $\mathbf{x} = \mathbf{M}^{-1}\mathbf{c}$

5 Transformations

The image of $\mathbf{a} + t\mathbf{b}$ is $\mathbf{Ma} + t\mathbf{Mb}$; the point with image $\mathbf{c}$ is $\mathbf{M}^{-1}\mathbf{c}$

6 Eigenvalues

Solve $\det(\mathbf{M} - \lambda\mathbf{I}) = 0$, not $\det\mathbf{M} = 0$. If one eigenvalue is given, substitute it straight away.

7 Eigenvectors

Solve $(\mathbf{M} - \lambda\mathbf{I})\mathbf{x} = \mathbf{0}$ (equivalently $\mathbf{Mx} = \lambda\mathbf{x}$). Normalise by dividing by the magnitude.

8 Diagonal form

For symmetric $\mathbf{A}$, the columns of $\mathbf{P}$ are normalised, orthogonal eigenvectors; $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$ with eigenvalues in the same order.

9 Repeated eigenvalues

Choose two eigenvectors for the repeated value that are perpendicular to each other (check the scalar product is 0).

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 3×3 determinants	•	•			•	•	•	•		•
2 Inverse		•	•				•	•		
3 Products	•		•							
4 Linear equations							•			
5 Transformations								•		
6 Eigenvalues				•	•	•			•	•
7 Eigenvectors				•	•	•			•	•
8 Diagonal form									•	•
9 Repeated eigenvalues										•
Interleaved: 2×2 matrices and transformations (FP1)	•	•	•					•		
Interleaved: planes and lines (Chapter 5)							•	•		
Interleaved: vectors, scalar product and cubic equations (P1, P4)				•	•	•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM03 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Use any zeros when expanding determinants. For eigenvalues, form $\det(\mathbf{M} - \lambda\mathbf{I})$; for eigenvectors, solve $(\mathbf{M} - \lambda\mathbf{I})\mathbf{x} = \mathbf{0}$; respect any condition such as $k < 0$.

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1.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ k & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

(a) Find $\det \mathbf{M}$ in terms of k, and the value of k for which $\mathbf{M}$ is singular.

(4)

(b) Given that $k = 5$, find $\det\left(\mathbf{M}^2\right)$.

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

Find $\mathbf{A}^{-1}$, showing the matrix of cofactors.

(6)

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Question 2 continued

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3. **(a)** Given that $\mathbf{A}$ and $\mathbf{B}$ are non-singular 3×3 matrices, prove that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

(2)

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(b) Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$, and hence find $(\mathbf{AB})^{-1}$.

(4)

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Question 3 continued

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4.

$$\mathbf{M} = \begin{pmatrix} 4 & 2 & 0 \\ 1 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

(a) Show that the eigenvalues of $\mathbf{M}$ are 1, 2 and 5

(3)

(b) Find an eigenvector corresponding to the eigenvalue 5

(3)

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Question 4 continued

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5.

$$\mathbf{M} = \begin{pmatrix} 1 & k & 2 \\ 0 & 3 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

Given that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$,

(a) find the corresponding eigenvalue and the value of k,

(4)

(b) find $\det \mathbf{M}$ for this value of k.

(2)

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Question 5 continued

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6.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & k \\ 0 & 3 & 0 \\ k & 0 & 2 \end{pmatrix}, \qquad k < 0$$

Given that 4 is an eigenvalue of $\mathbf{M}$,

(a) find the value of k,

(3)

(b) find the other two eigenvalues of $\mathbf{M}$,

(3)

(c) find a normalised eigenvector corresponding to the eigenvalue 4,

(2)

(d) explain, without calculating the determinant, why $\mathbf{M}$ is singular.

(2)

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Question 6 continued

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Question 6 continued

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7. Three planes have equations

$$x+y+z=6, \qquad 2x-y+z=3, \qquad x+2y-z=2$$

(a) Write the equations in the form $\mathbf{Mx}=\mathbf{c}$, and find $\mathbf{M}^{-1}$.

(5)

(b) Hence find the point where the three planes meet.

(3)

(c) Explain the significance of $\det \mathbf{M} \neq 0$ for the three planes.

(2)

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Question 7 continued

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Question 7 continued

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8. The transformation T of three-dimensional space is represented by $\mathbf{T} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(a) Find $\mathbf{T}^{-1}$.

(5)

(b) The line l has equation $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{1}$. Find a vector equation of the image of l under T.

(3)

(c) Find the point whose image under T is $(4, 2, 3)$.

(2)

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Question 8 continued

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9.

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

(a) Find the eigenvalues of $\mathbf{A}$.

(3)

(b) Find a normalised eigenvector for each eigenvalue.

(3)

(c) Write down an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} = \mathbf{D}$

(3)

(d) Explain why $\mathbf{P}^{-1} = \mathbf{P}^{\mathrm{T}}$

(1)

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Question 9 continued

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10.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

(a) Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$, and state its eigenvalue.

(2)

(b) Show that the other eigenvalue of $\mathbf{A}$ is 1, repeated.

(3)

(c) Find two perpendicular eigenvectors for the eigenvalue 1, and hence write down an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$

(4)

(d) Use $\mathbf{D}$ to write down $\det \mathbf{A}$.

(1)

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Question 10 continued

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TOTAL FOR CHAPTER 6 IS 80 MARKS

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