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GCSE MATHEMATICS • ALGEBRA

Sequences

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • Calculator allowed

Full worked solutions for the free Sequences worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Sequences worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

FOUNDATION

A1 The terms go down by 2, so start with $-2n$: $-2, -4, -6, \dots$ Add 13 $13 - 2n$

A2 $7n + 5 = 60$, so $7n = 55$ and $n = 7.857\dots$, not a whole number no

A3 Divide by 2 each time: 5 and 2.5 8th term $= 80 \times \left(\frac{1}{2}\right)^7 = \text{0.625}$

HIGHER

A4 First differences 5, 7, 9, 11. Second difference 2, so start with n^2
Sequence $-n^2$: 1, 3, 5, 7, 9, which is $2n - 1$ $n^2 + 2n - 1$

A5 Multiply by $\sqrt{2}$ each time. 9th term $= 5 \times (\sqrt{2})^8 = 5 \times 16 = \text{80}$

A6 Terms: $a, b, a + b, a + 2b, 2a + 3b, 3a + 5b$ $a + 2b = 11$ and $3a + 5b = 29$
 $3a + 6b = 33$, so $b = 4$ and $a = 3$ $a = 3$, $b = 4$

A7 $80 - 3n < 0$ gives $n > 26.7$, so $n = 27$ $80 - 81 = \text{−1}$

B**Fluency solutions**

One mark for each correct part. The notes in italics explain why each sequence was chosen.

FOUNDATION

B1 (a) 19 (b) 18 (c) 27 (d) 8 *Add 5; subtract 4; multiply by 3; add the two before.*

B2 (a) 5 (b) 32 (c) 302 (d) $3n + 2 = 62$, $n = 20$: 20th

B3 (a) $3n + 2$ (b) $5n$ (c) $4n - 7$ (d) $23 - 3n$



- B4** (a) $n = 10$: **yes, 10th** (b) $n = 12.75$: **no** (c) $n = 25$: **yes, 25th** (d) $n = 50.25$: **no**

A number is a term only if its position is a whole number.

- B5** (a) **250** (b) **5** (c) $2 \times 5^5 =$ **6250**

- B6** (a) **21** (b) 6, 11, 16: add 5 each time: **$5n+1$** (c) **101** (d) $5n+1 = 150$ gives $n = 29.8$, so **no**

HIGHER

- B7** (a) **n^2+1** (b) **$3n^2$** (c) **n^2-1** (d) second difference 2; 4, 10, 18, 28 minus n^2 is 3, 6, 9, 12: **n^2+3n**

- B8** (a) second difference 4; remainder 4, 7, 10, 13: **$2n^2+3n+1$** (b) second difference -2 ; remainder 11, 20, 29, 38: **$-n^2+9n+2$**

(c) remainder 5, 5, 5: **$2n^2+5$** (d) remainder 2, 1, 0, -1 : **n^2-n+3** Halve the second difference to get the n^2 coefficient.

- B9** (a) $(\sqrt{2})^7 =$ **$8\sqrt{2}$** (b) $27 \times \left(\frac{1}{3}\right)^5 =$ **$\frac{1}{9}$** (c) $2 \times (-3)^4 =$ **162** (d) $(\sqrt{3})^6 =$ **27**

- B10** (a) $n > 11.75$: **51** (b) $n > 14.3$: **-5** (c) $n^2 > 49.5$, $n = 8$: **129** (d) $2^n > 333.3$, $n = 9$: **1536**

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

- C1** The terms go up by 3, so the n th term starts with $3n$, not n . $3n$ gives 3, 6, 9, so add 2 **$3n+2$**

- C2** **No**. The differences are 3, 5, 7, which are not the same. These are the square numbers.

- C3** $4n+2 = 2(2n+1)$, which is 2 times a whole number, so it is even. $4n+3 = 2(2n+1)+1$, one more than an even number, so it is odd.



- C4** (a) $3 \times 20 - 5 = 55$, as required (b) $3n - 5 = 3(n - 2) + 1$, which is 1 more than a multiple of 3, so it is never a multiple of 3

HIGHER

- C5** Half the second difference is the n^2 coefficient, so it starts n^2 . Then 2, 6, 12, 20 minus n^2 is 1, 2, 3, 4, which is n $n^2 + n$

- C6** The terms are $5 \times 2^{n-1}$. $1000 \div 5 = 200$, and 200 is not a power of 2, so 1000 is **not** a term.

- C7** $((n+1)^2 + (n+1)) - (n^2 + n) = 2n + 2 = 2(n+1)$, which is even for every n

- C8** (a) A: 5, 8, 11, 14, 17, 20, 23, 26, 29, ... B: 4, 9, 14, 19, 24, 29, ... **14**, **29**, **44**

(b) They go up by 15, the lowest common multiple of 3 and 5: $15k - 1$

(c) $15k - 1 < 100$ gives $k \leq 6$: 14, 29, 44, 59, 74, 89 **6** numbers

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

- D1** Week n : $5n + 15$ $5n + 15 > 100$ gives $n > 17$ Week **18** (£105)

- D2** $6n + 1 = 50 - n$ $7n = 49$ $n = 7$ the term is **43**

- D3** White tiles: 8, 10, 12, ..., so $2n + 6$ $2n + 6 = 50$ gives $n = 22$ Pattern **22**

- D4** Terms: 2, x , $x + 2$, $2x + 2$, $3x + 4$ $3x + 4 = 25$ $x = 7$

HIGHER

- D5** Second difference 2 and the remainder is $2n$, so the n th term is $n^2 + 2n$

(a) $100 + 20 =$ **120** (b) $n^2 + 2n = 399$, so $(n+1)^2 = 400$ and $n+1 = 20$ Pattern **19**



D6 After n bounces the height is 2×0.75^n After 4 bounces: 0.633 m After 5 bounces: 0.475 m
First below 0.5 m after bounce **5**

D7 From the 5th term to the 12th is 7 steps: $51 - 23 = 28$, so the difference is 4
1st term = $23 - 4 \times 4 = 7$ **$4n + 3$**

D8 (a) $4 + 2b + c = 10$ and $25 + 5b + c = 40$ Subtract: $3b = 9$ **$b = 3$** , **$c = 0$**
(b) $n^2 + 3n = n(n + 3)$. If n is odd, $n + 3$ is even; if n is even, n is even. So the product is always even.

E**Exam practice solutions**

Marks are shown on the right of each line.

FOUNDATION *Foundation and Higher crossover*

E1 (a) **21** B1
(b) Goes up by 4: $4n$ M1
 $4n + 1$ A1

E2 $4n + 1 = 150$ gives $n = 37.25$ M1
Not a whole number, so **no** C1

E3 (a) 4, 7, 10, ... : **31** B1
(b) **$3n + 1$** B1
(c) $3n + 1 = 100$ Pattern **33** B1

E4 (a) **18** and **29** B1
(b) $a, b, a + b, a + 2b, \mathbf{2a + 3b}$ B1

E5 (a) **162** B1
(b) Multiply by 3 B1
(c) $2 \times 3^7 = \mathbf{4374}$ B1

E6 (a) 17, 14, 11 B1
(b) $20 - 3n < 0$ gives $n > 6.7$ M1
 $n = 7$: **-1** A1



HIGHER

E7	First differences 5, 9, 13, 17; second difference 4 So $2n^2$ Sequence $-2n^2$: 2, 1, 0, -1, -2, which is $3 - n$ $2n^2 - n + 3$	M1 A1 M1 A1
E8	Common ratio $\sqrt{2}$ n th term $= (\sqrt{2})^n$ $(\sqrt{2})^{10} = 32$	M1 M1 A1
E9	$n + 1 < 0.52(2n + 1)$ $n + 1 < 1.04n + 0.52$, so $0.48 < 0.04n$ and $n > 12$ $n = 13$: $\frac{14}{27} = 0.5185\dots$ $\frac{14}{27}$	M1 M1 A1
E10	(a) First differences 6, 10; second difference 4, so $2n^2$ Sequence $-2n^2$: 3, 3, 3 $2n^2 + 3$ (b) $2n^2$ is always even, so $2n^2 + 3$ is always odd (c) $2n^2 + 3 = 101$, $n^2 = 49$ the 7th term	M1 M1 A1 C1 B1

