eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 1 (WMA11)

Pure 1 Practice Paper 2

Mark Scheme

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA11/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1, 2, 4, 5, 6 and 8, and parts (b) and (c) of Question 9, say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A **49** • B **42** • C **35**

Based on recent WMA11 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Question paper | Exam questions by topic | More practice papers

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Surds and rationalising inside a right-angled triangle	7	Ch 1 Algebraic Expressions Ch 6 Trigonometric Ratios	
2	Forming and solving linear and quadratic inequalities from a context	8	Ch 3 Equations and Inequalities Ch 2 Quadratics	
3	Reading a sine graph; counting intersections with $y = k$	8	Ch 6 Trigonometric Ratios Ch 4 Graphs and Transformations	
4	Parallel and perpendicular lines; area of a trapezium	8	Ch 5 Straight-line Graphs	
5	Constants from f' and f''; a quadratic inequality	9	Ch 8 Differentiation Ch 3 Equations and Inequalities	
6	Tangency with the discriminant; area of a triangle	8	Ch 3 Equations and Inequalities Ch 2 Quadratics Ch 5 Straight-line Graphs	
7	Segment and triangle areas in radians; arc length	8	Ch 7 Radians Ch 6 Trigonometric Ratios	
8	Integration with an unknown constant; normal; parallel lines	9	Ch 9 Integration Ch 8 Differentiation Ch 5 Straight-line Graphs	
9	Reciprocal graph, asymptotes, intersections and a shift	10	Ch 4 Graphs and Transformations Ch 3 Equations and Inequalities	
	Total	**75**		

Where did the marks go? This paper gives fewer steps than a real paper, so the first marks of each question test whether you can plan the route yourself. If you lost those, write down the method before any algebra next time. If you lost the final marks, look for the condition you missed: a width that must be positive, an end point of the graph, or the $k = 0$ case.

★★★★★

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 1

7 marks

Part	Scheme	Marks
(a)	$AC^2 = (2+\sqrt{3})^2 + (2\sqrt{3}-1)^2 = (7+4\sqrt{3}) + (13-4\sqrt{3})$	M1
	$AC = \sqrt{20} = \boxed{2\sqrt{5}\,\text{cm}}$	A1
		(2)
(b)	$\frac{1}{2}(2+\sqrt{3})(2\sqrt{3}-1) = \frac{1}{2}(4\sqrt{3}-2+6-\sqrt{3})$	M1
	$\boxed{2+\frac{3}{2}\sqrt{3}}\,\text{cm}^2$	A1
		(2)
(c)	$\tan\theta = \dfrac{AB}{BC} = \dfrac{2+\sqrt{3}}{2\sqrt{3}-1}$	B1
	Multiplies by $\dfrac{2\sqrt{3}+1}{2\sqrt{3}+1}$: $\dfrac{4\sqrt{3}+2+6+\sqrt{3}}{12-1}$	M1
	$\boxed{\tan\theta = \dfrac{8+5\sqrt{3}}{11}}$, so $a = 8$, $b = 5$, $c = 11$	A1
	θ is the angle at C, so the opposite side is AB. Using $\frac{BC}{AB}$ finds $\tan BAC$ instead and scores B0 M1 A0.	(3)
		Total for Question 1: 7 marks

Question 2

8 marks

Part	Scheme	Marks
(a)	$2(2x+1)+2(x-2) < 56$, i.e. $6x-2 < 56$	M1
	$x < \dfrac{29}{3}$ *	A1*
		(2)
(b)	$(2x+1)(x-2) > 52$	M1
	$2x^2 - 3x - 54 > 0$	A1
	Solves $2x^2 - 3x - 54 = 0$: $(2x+9)(x-6) = 0$, so $x = 6$ or $x = -\frac{9}{2}$	M1
	$x > 6$ or $x < -\frac{9}{2}$	A1
	Combines with $x < \frac{29}{3}$ and with $x > 2$, as the width $x-2$ must be positive	M1
	$\boxed{6 < x < \dfrac{29}{3}}$	A1
	The common error is to give $x > 6$ alone, forgetting the perimeter condition, or to keep $x < -\frac{9}{2}$, which gives a negative width.	(6)
		Total for Question 2: 8 marks

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3

8 marks

Part	Scheme	Marks
(a)	$\boxed{a = 4}$, from the maximum value	B1
	$\sin(30 + b)^\circ = 1 \Rightarrow 30 + b = 90$, so $\boxed{b = 60}$	B1
		(2)
(b)	$\boxed{P\left(0, 2\sqrt{3}\right)}$, since $4\sin 60^\circ = 2\sqrt{3}$	B1
	$\boxed{Q(210, -4)}$	B1
		(2)
(c)	Uses the maximum 4, the minimum -4 and the value $2\sqrt{3}$ at both ends of the curve	M1
	Identifies $k = 2\sqrt{3}$ as a special case: the line meets the curve three times, at $x = 0$, 60 and 360	B1
	$\boxed{-4 < k < 2\sqrt{3}}$	A1
	$\boxed{2\sqrt{3} < k < 4}$	A1
	Most candidates give $-4 < k < 4$. At $k = \pm 4$ the line touches the curve once, and the end points of the domain make $k = 2\sqrt{3}$ give three points.	
		(4)
		Total for Question 3: 8 marks

Question 4

8 marks

Part	Scheme	Marks
(a)	Gradient of $l_1 = \dfrac{1-5}{6-(-2)} = -\dfrac{1}{2}$; $y - 7 = -\frac{1}{2}(x - 14)$	M1
	$\boxed{x + 2y - 28 = 0}$	A1
		(2)
(b)	Line AD has gradient 2: $y - 5 = 2(x + 2)$, i.e. $y = 2x + 9$	M1
	Solves with l_2: $x + 2(2x + 9) = 28 \Rightarrow x = 2$	M1
	$\boxed{D(2, 13)}$	A1
		(3)
(c)	Finds at least two of $AB = \sqrt{80} = 4\sqrt{5}$, $DC = \sqrt{180} = 6\sqrt{5}$, $AD = \sqrt{80} = 4\sqrt{5}$	M1
	Area $= \frac{1}{2}(AB + DC) \times AD = \frac{1}{2}\left(4\sqrt{5} + 6\sqrt{5}\right) \times 4\sqrt{5}$	M1
	$\boxed{\text{Area} = 100}$	A1
	The height of the trapezium is AD, because AD is perpendicular to both parallel sides. Splitting into a rectangle and a triangle is also fine.	
		(3)
		Total for Question 4: 8 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5

9 marks

Part	Scheme	Marks
(a)	$f'(x) = 6x^2 + 2ax + b$	**M1**
	$f''(x) = 12x + 2a$, and $24 + 2a = 18$	**M1**
	Gradient of $4x + y = 7$ is -4, so $f'(1) = 6 + 2a + b = -4$	**M1**
	$\boxed{a = -3,\ b = -4}$	**A1**
		(4)
(b)	$6x^2 - 6x - 4 = -4 \Rightarrow 6x(x-1) = 0$, so $x = 0$ (the other is $x = 1$)	**M1**
	$\boxed{(0, 3)}$	**A1**
		(2)
(c)	$6x^2 - 6x - 4 < 8$, i.e. $x^2 - x - 2 < 0$	**M1**
	Critical values from $(x-2)(x+1) = 0$: $x = 2$, $x = -1$	**M1**
	$\boxed{-1 < x < 2}$	**A1**
	The gradient is $f'(x)$, not $f(x)$: solving $f(x) < 8$ scores M0.	
		(3)
		Total for Question 5: 9 marks

Question 6

8 marks

Part	Scheme	Marks
(a)	$x^2 - 2x + 7 = kx - 2$	**M1**
	$x^2 - (2+k)x + 9 = 0$	**A1**
	Tangent, so $b^2 - 4ac = 0$: $(2+k)^2 - 36 = 0$	**M1**
	$\boxed{k = 4 \text{ or } k = -8}$	**A1**
	Solves the repeated root for each: $x^2 - 6x + 9 = 0$ and $x^2 + 6x + 9 = 0$	**M1**
	$\boxed{(3, 10)}$ and $\boxed{(-3, 22)}$	**A1**
		(6)
(b)	Both tangents pass through $(0, -2)$: area of the triangle with vertices $(0, -2)$, $(3, 10)$ and $(-3, 22)$, by any complete method	**M1**
	$\boxed{\text{Area} = 54}$	**A1**
	For example, the line joining the contact points meets the y-axis at $(0, 16)$, so the area is $\frac{1}{2} \times 18 \times 3 + \frac{1}{2} \times 18 \times 3 = 54$.	
		(2)
		Total for Question 6: 8 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7

8 marks

Part	Scheme	Marks
(a)	Area $S_1 = \frac{1}{2} \times 6^2(\theta - \sin\theta) = 18\theta - 18\sin\theta$	M1
	Area $S_2 = \frac{1}{2} \times 6 \times 6\sin(\pi - \theta) = 18\sin\theta$	M1
	$18\theta - 18\sin\theta = 18\sin\theta \Rightarrow \theta = 2\sin\theta$ *	A1*
	The angle BOC is $\pi - \theta$, and $\sin(\pi - \theta) = \sin\theta$ must be used or seen.	(3)
(b)	Arc $AC = 6 \times 1.895 = 11.37$	M1
	Chord AC: $AC^2 = 6^2 + 6^2 - 2 \times 6 \times 6\cos 1.895$, so $AC = 9.74\ldots$ (or $AC = 12\sin 0.9475$)	M1
	Perimeter = 21.1 cm	A1
		(3)
(c)	Semicircle minus both shaded regions: $18\pi - 2 \times 18\sin 1.895$	M1
	$22.4\ \text{cm}^2$	A1
	Finding the two unshaded parts separately (triangle AOC and the segment on BC) is also fine.	(2)
		Total for Question 7: 8 marks

Question 8

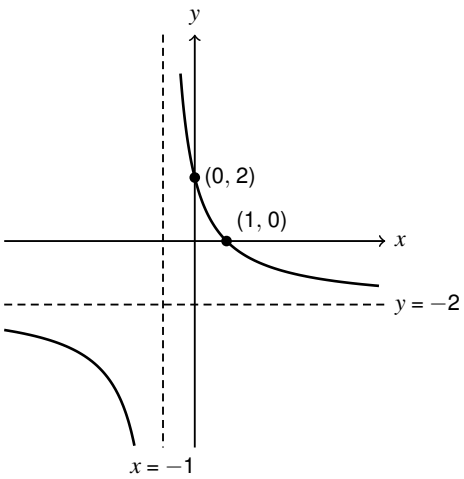
9 marks

Part	Scheme	Marks
(a)	$f'(4) = 3 \times 2 + \frac{k}{16} = 5$	M1
	$k = -16$	A1
		(2)
(b)	Integrates $3x^{\frac{1}{2}} - 16x^{-2}$: $2x^{\frac{3}{2}} + 16x^{-1}$ $(+c)$	M1 A1
	Uses $(1, -2)$: $2 + 16 + c = -2$, so $f(x) = 2x^{\frac{3}{2}} + \frac{16}{x} - 20$	A1
		(3)
(c)	$f(4) = 16 + 4 - 20 = 0$, so $P(4, 0)$; the normal at P is $y = -\frac{1}{5}(x - 4)$	M1
	Solves with l: $-\frac{1}{5}(x - 4) = 5x + 6 \Rightarrow x = -1$, giving $(-1, 1)$	M1
	Distance from $(4, 0)$ to $(-1, 1)$: $\sqrt{5^2 + 1^2}$	M1
	$\sqrt{26}$	A1
	The normal at P is perpendicular to both parallel lines, so it gives the shortest distance. Any other correct method, e.g. the perpendicular from a point on l to the tangent $y = 5x - 20$, is fine.	(4)
		Total for Question 8: 9 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9

10 marks

Part	Scheme	Marks
(a)	 Correct shape: two branches, top right and bottom left of the asymptotes Asymptotes $x = -1$ and $y = -2$ Meets the axes at (1, 0) and (0, 2)	B1 B1 B1 B1 (4)
(b)	$\frac{4}{x+1} - 2 = kx + 2$, multiplied through by $x+1$ $kx^2 + (k+4)x = 0$ $x = 0$ or $x = -\frac{k+4}{k}$; only one point when the second root is also 0, or when there is no x^2 term $\boxed{k = -4 \text{ or } k = 0}$ *$k = -4$ makes the line a tangent at $(0, 2)$. With $k = 0$ the equation is $4x = 0$: the line $y = 2$ meets the curve once. Using the discriminant $(k+4)^2 = 0$ alone misses $k = 0$ and scores A0.*	M1 A1 M1 A1 (4)
(c)	Recognises that the horizontal asymptote must become the x-axis: $-2 + c = 0$ $\boxed{c = 2}$	M1 A1 (2)
		Total for Question 9: 10 marks

TOTAL FOR PAPER: 75 MARKS