eClassroom

A-LEVEL MATHEMATICS • DECISION 1

Algorithms on Graphs

Worked Solutions

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Full worked solutions for the free Algorithms on Graphs worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 3

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Tap a section to jump straight to it

Full worked solutions for every question on the Decision 1 Algorithms on Graphs worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Decision 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 PR 2 ✓, ST 3 ✓, QS 4 ✓, QR 5 ✓ (4 arcs reached): **weight 14**

A2 PQ 2, QR 3, RS 4: **weight 9**

A3 DC 2, CB 5, BA 3: **weight 10**

STRETCH

A4 S 0, A 3, B 7 (9 improved to 7), C 9 (11 improved to 9), T 14: **S–A–B–T, 14**

A5 $14 - 5 = 9$: BT; $9 - 9 = 0$: SB (AT: $14 - 11 \neq 4$): **S–B–T**

A6 **11**

A7 $13 + 7$: **20**

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** CD 3, AC 4, BC 6, AD 7, AB 9, BE 10 **(b)** **AD and AB** **(c)** $3 + 4 + 6 + 10$: **23** **(d)** **4**

Stop once there are $n - 1$ arcs.

B2 **(a)** **(b)** **EB 10, BC 6, CD 3, CA 4** **(c)** also **23** **(d)** the minimum spanning tree is unique as all weights differ

Prim grows one tree from the start.

B3 **(a)** **CA 9** **(b)** **CA 9, CE 11, CB 12, BD 8** **(c)** **40** **(d)** **D**

Search only the numbered columns.

B4 **(a)** S, A, B, C, D, T **(b)** 0, 5, 8, 14, 18, 23 **(c)** 27, 23 **(d)** S–A–B–C–D–T, 23

A working value is written only when it improves.

B5 **(a)** $14 - 8 = 6 = BC$ **(b)** $23 - 14 = 9 \neq 13$ **(c)** 18

final(Y) − final(X) = weight of XY

B6 **(a)** 9 **(b)** an arc that would complete a cycle is not needed **(c)** it only ever joins a new vertex to the tree **(d)** the shortest distance from the start to every vertex

What each algorithm guarantees.

STRETCH

B7 **(a)** DE 16, EC 19, CB 15, CA 18, AF 20 **(b)** 88 **(c)** D, E, C, B, A, F numbered 1 to 6 **(d)** AF

The order of the arcs is the evidence.

B8 **(a)** S, R, P, Q, U, T **(b)** Q: 14, 13; U: 21, 18; T: 24, 21 **(c)** S–P–Q–U–T **(d)** 21

Values improve as better routes appear.

B9 **(a)** 18 **(b)** S–R–T, 24 **(c)** $4 + 20$: 24 **(d)** they already give the shortest distance to every vertex

Reuse the final values.

B10 **(a)** A–C–B–E **(b)** BE 10 **(c)** $x < 10$ **(d)** $23 - 10 + 5$: 18

A cheaper arc replaces the largest arc on the cycle it makes.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 He added the rejected arcs AD 7 and AB 9 ($23 + 16 = 39$); only accepted arcs count: 23

C2 Prim on a matrix starts at the given vertex: cross out row C, number column C, and choose the least entry in that column: CA 9

C3 That is nearest neighbour; Prim chooses the least arc from *any* vertex already in the tree, so it can branch.

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C4 Every working value must be shown in order: **27, then 23**

STRETCH

C5 The final value at U, 18, is already the shortest distance from S to U.

C6 The shortest first arc need not lead to the shortest route; trace back from T, using arcs with final(Y) − final(X) equal to the weight.

C7 With AE 5 the tree changes from weight 23 to **18**: AE replaces BE, the largest arc on the cycle A–C–B–E–A.

C8 **(a)** a spanning tree connects n vertices with no cycles: each arc added joins one new vertex, so $n-1$ arcs

(b) when a vertex is labelled its value is the least of all working values; any other route reaches it through an unlabelled vertex with a value at least as large, plus a positive arc, so it cannot be shorter

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 DF 7 ✓, DE 8 ✓, BE 11 ✓, AD 12 ✓, AB 16 ×, EF 17 ×, CE 18 ✓: **cost 56 thousand pounds**

D2 **(a)** **ST 17, SR 22, RQ 19, RP 24, TU 26**

(b) **108 km**

D3 **(a)** $27-11=16$: DE; $16-16=0$: AD: **A–D–E** (CE: $27-19\neq 12$)

(b) **25 m**; no other final value changes (E was labelled last)

D4 **(a)** **DC 4, DE 10, EF 5, EG 12, GB 8, BA 6**

(b) **45**; Kruskal gives the same weight, 45

STRETCH

D5 **(a)** JK 12, KL 9, LM 11, MN 14: **46**

(b) below 46, JL must be in the tree with KL, LM, MN: $x+34=41$: $x=7$

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D6 **(a)** order A, D, B, G, E, H, C, J, F; E: 19, 13; H: 27, 16: **A–D–E–J, 30 min**

(b) **A–D–E–H–J, 34 min**

D7 **(a)** TU 4 ✓, XP 5 ✓, PQ 6 ✓, TZ 8 ✓, UP 10 ✓, RY 14 ✓, XT 16 ×, XR 17 ✓, QR 20 ×, QY 22 ×, UZ 23 ×, ST 26 ✓: **90**

(b) QS 15 makes a cycle with Q–P–U–T–S whose largest arc is ST 26, so QS replaces ST: $90 - 26 + 15$: **79**

D8 **(a)** order A, C, B, E, D, F, G; B: 21, 20; D: 33, 31; G: 54, 43: **A–C–F–G, 43 km**

(b) E has final value 28 (labelled before G), so $28 + x < 43$: $x < 15$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** order of labelling S, B, A, D, E, G, C, F, T *M1*

final values S 0, B 11, A 22, D 26, E 30, G 34, C 36, F 39, T 47 *A1 A1*

working values: F 43, 39; G 36, 34; T 58, 47; one value at each other vertex *A1 A1*

(b) **S–A–C–F–T, 47 minutes** *B1*

traced back from T using final(Y) − final(X) = arc: $47 - 8 = 39$, $39 - 3 = 36$, $36 - 14 = 22$, $22 - 22 = 0$ *B1*

(c) S to G is 34 (S–B–E–G); G to T directly is 24, other routes from G are longer *M1 A1*

S–B–E–G–T, 58 minutes *A1*

E2 **(a)** GJ 3, CD 4, CE 5, DG 6, AC 7, GH 9, AB 13 accepted; EG 14, BC 15, DE 18 rejected; FH 19 accepted *M1 A1 A1*

(b) **66 km** *B1*

(c) **AC 7, CD 4, CE 5, DG 6, GJ 3, GH 9, AB 13, HF 19** *M1 A1 A1*

(d) **66 km** *B1*

every arc of the tree is a direct road that is also the shortest distance, so both give the same minimum connector *B1*

(e) **39 km** *B1*

B–C–D–F $(15 + 4 + 20)$ *M1 A1*

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STRETCH

E3 **(a)** order A, C, E, B, D, F, H, G *M1*

final values A 0, C 21, E 25, B 26, D 28, F 35, H 48, G 49 *A1 A1*

working values B 28, 26; D 32, 28; F 54, 35; G 51, 49 *A1*

A–C–E–D–F–H, 4800 m *A1*

(b) DE 3, CE 4, BC 5, DF 7 accepted; CD 11 rejected; FH 13, FG 14 accepted; GH 15, BD 18 rejected; AC 21 accepted *M1 A1*

weight 67 *A1*

(c) BD makes a cycle with B–C–E–D in the tree, whose largest arc is BC 5 *M1 A1*

$x < 5$ *A1*

(d) B has final value 26 and D has 28 (neither uses BD); the walk uses BD if $26 + x < 28$ *M1 A1*

$x < 2$, **so** $x = 1$ *A1*