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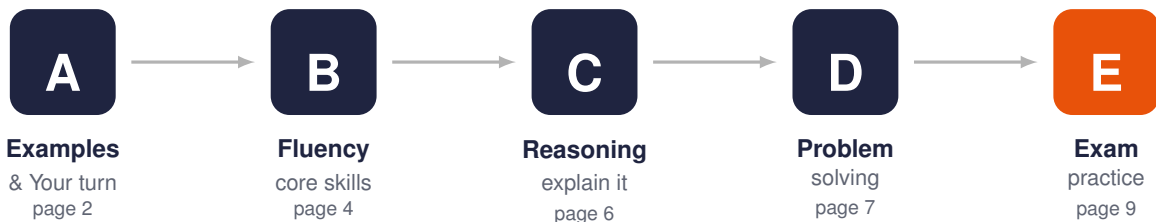
A-LEVEL MATHEMATICS • STATISTICS 1

The Normal Distribution

Worksheet

Pearson Edexcel International A Level (WST01) & A Level (9MA0) • Calculator and statistical tables allowed: show every step

Free The Normal Distribution worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: standardise to find probabilities • use the Φ table and the percentage points table • find values from probabilities • find an unknown mean or standard deviation • *Stretch*: two unknowns, conditional probabilities, repeated selections and outliers.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Table values quoted in full, signs that matter, unknown means and standard deviations, and conditionals on a curve.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	21
Stretch marks	16	14	17	12
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE*Secure the method***Worked example A1: standardising**

$X \sim N(50, 4^2)$. Find $P(X < 56)$

$$Z = \frac{X - \mu}{\sigma}, \text{ then read } \Phi$$

$$P\left(Z < \frac{56 - 50}{4}\right) = \Phi(1.5)$$

0.9332

Your turn A1

$X \sim N(120, 10^2)$. Find $P(X < 108)$

Worked example A2: an upper tail

$X \sim N(30, 5^2)$. Find $P(X > 37)$

$$P(Z > z) = 1 - \Phi(z)$$

$$z = \frac{37 - 30}{5} = 1.4: 1 - 0.9192$$

0.0808

Your turn A2

$X \sim N(200, 15^2)$. Find $P(X > 191)$

Worked example A3: a range

$X \sim N(80, 6^2)$. Find $P(77 < X < 89)$

$$\Phi(z_2) - \Phi(z_1), \text{ with } \Phi(-z) = 1 - \Phi(z)$$

$$\Phi(1.5) - \Phi(-0.5) = 0.9332 - 0.3085$$

0.6247

Your turn A3

$X \sim N(15, 2^2)$. Find $P(13 < X < 18)$



STRETCH

Worked example A4: a percentage point

$X \sim N(60, 8^2)$ and $P(X > a) = 0.05$. Find a

Percentage points table: 5% gives $z = 1.6449$

$$a = 60 + 1.6449 \times 8$$

$$a = 73.16$$

Your turn A4

$X \sim N(400, 25^2)$ and $P(X < b) = 0.1$. Find b

Worked example A5: an unknown σ

$X \sim N(50, \sigma^2)$ and $P(X > 58) = 0.2$. Find σ

$$\frac{58 - 50}{\sigma} = 0.8416$$

$$\sigma = 9.51$$

Your turn A5

$X \sim N(\mu, 4^2)$ and $P(X < 30) = 0.025$. Find μ

Worked example A6: two unknowns

$P(X < 20) = 0.1$ and $P(X > 44) = 0.05$. Find μ and σ

$$\mu - 1.2816\sigma = 20, \quad \mu + 1.6449\sigma = 44$$

$$\text{Subtract: } 2.9265\sigma = 24$$

$$\sigma = 8.20, \quad \mu = 30.5$$

Your turn A6

$P(X < 12) = 0.3$ and $P(X > 30) = 0.01$. Find μ and σ

Worked example A7: a conditional

$X \sim N(100, 10^2)$. Find $P(X > 115 | X > 100)$

The first event lies inside the second, so divide.

$$\frac{1 - \Phi(1.5)}{0.5} = \frac{0.0668}{0.5}$$

$$0.1336$$

Your turn A7

$X \sim N(40, 5^2)$. Find $P(X < 32 | X < 40)$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** $X \sim N(25, 4^2)$. Find

(a) $P(X < 29)$

(b) $P(X > 22)$

(c) $P(23 < X < 31)$

(d) $P(X = 25)$

[4 marks]**B2** $Z \sim N(0, 1^2)$. Find

(a) $P(Z < 1.23)$

(b) $P(Z > -0.85)$

(c) $P(-1.2 < Z < 0.4)$

(d) $P(|Z| > 2)$

[4 marks]**B3** $X \sim N(70, 12^2)$. Find**(a)** the value exceeded by 10%**(b)** the value below which 5% lie**(c)** the quartiles**(d)** the interquartile range**[4 marks]****B4** **(a)** $X \sim N(40, \sigma^2)$ and $P(X < 35) = 0.3$. Find σ **(b)** $X \sim N(\mu, 3^2)$ and $P(X > 50) = 0.025$. Find μ **(c)** $X \sim N(40, 6^2)$. Find k with $P(40 - k < X < 40 + k) = 0.9$ **(d)** For any normal X , find $P(X > \mu + \sigma)$ **[4 marks]**

CORE

*continued: symmetry and context***B5** $X \sim N(\mu, \sigma^2)$. Find

(a) $P(X > \mu)$

(b) $P(\mu - \sigma < X < \mu + \sigma)$

(c) $P(X < \mu - 2\sigma)$

[3 marks]

B6 The lengths of 400 bolts are modelled by $N(18, 1.5^2)$ mm.

(a) Find $P(\text{a bolt is longer than 20 mm})$

(b) Estimate how many are longer than 20 mm

(c) Find $P(\text{a bolt is shorter than 16 mm})$

(d) Estimate how many are between 16 and 20 mm

[4 marks]

STRETCH

B7 $X \sim N(\mu, \sigma^2)$, with $P(X < 28) = 0.2$ and $P(X > 52) = 0.1$

(a) Form two equations

(b) Find σ

(c) Find μ

(d) Find $P(X > 45)$

[4 marks]

B8 $X \sim N(60, 8^2)$. Find

(a) $P(X > 70)$

(b) $P(X > 64)$

(c) $P(X > 70 | X > 64)$

(d) $P(X < 64 | X > 56)$

[4 marks]

B9 Bags of flour have weights $N(500, 3^2)$ g. A bag is under weight if it weighs less than 495 g.

(a) Find $P(\text{a bag is under weight})$

(b) Find $P(\text{two bags are both not under weight})$

(c) Find $P(\text{exactly one of three is under weight})$

(d) Find $P(\text{at least one of three is under weight})$

[4 marks]

B10 $X \sim N(50, 6^2)$. An outlier is greater than $Q_3 + 1.5(Q_3 - Q_1)$ or less than $Q_1 - 1.5(Q_3 - Q_1)$

(a) Find Q_3

(b) Find Q_1

(c) Find the upper outlier limit

(d) Find $P(\text{a value is an outlier})$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

C1 $X \sim N(50, 16)$. Ama writes $P(X < 54) = \Phi\left(\frac{54 - 50}{16}\right) = \Phi(0.25)$

Explain her error, and find $P(X < 54)$

[2 marks]

C2 $X \sim N(50, 4^2)$. Ben writes $P(X < 44) = \Phi(1.5) = 0.9332$

Explain his error, and find $P(X < 44)$

[2 marks]

C3 $X \sim N(70, 12^2)$. To find the value below which 10% lie, Cal writes $70 + 1.2816 \times 12 = 85.4$

Explain his error, and find the correct value.

[3 marks]

C4 $X \sim N(400, 20^2)$ and $P(X > a) = 0.05$. Dee uses $z = 1.64$ and gives $a = 432.8$

Explain why this would lose marks, and give the correct value of a

[3 marks]

STRETCH

C5 $X \sim N(\mu, 5^2)$ and $P(X < 42) = 0.2$. Eli writes $\frac{42 - \mu}{5} = 0.8416$, so $\mu = 37.8$

Explain his error, and find μ

[3 marks]

C6 Each of 4 items has probability 0.1 of being faulty, independently. Faye writes $P(\text{exactly 2 faulty}) = 0.1^2 \times 0.9^2$

Explain her error, and give the correct probability.

[3 marks]

C7 $X \sim N(60, 8^2)$. Gus writes $P(X > 70 | X > 64) = P(X > 70) \times P(X > 64)$

Explain his error, and find the correct value.

[3 marks]

C8 (a) Show that, for any normal variable X and any $k > 0$, $P(\mu - k\sigma < X < \mu + k\sigma) = 2\Phi(k) - 1$

(3)

(b) Hence find the proportion of any normal population within 2 standard deviations of the mean.

(2)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** *In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.*

The battery life, B hours, of a phone on a full charge is modelled by a normal distribution with mean 32 hours and standard deviation 4 hours.

- (a) Find the probability that, on a randomly chosen full charge, the battery lasts at least 38 hours.

(2)

- (b) A shop tests 50 of these phones. Estimate how many of them would last at least 38 hours on a full charge.

(1)

[3 marks]

- D2** The time, T minutes, that a commuter takes to travel to work is modelled by $T \sim N(42, 6^2)$

- (a) Find the probability that a journey takes between 39 and 48 minutes.

(2)

- (b) The probability that a journey takes at least d minutes is 0.85. Use the percentage points table to find the value of d

(2)

[4 marks]

- D3** The mass, Y grams, of a large cake is normally distributed with mean 800 grams and standard deviation σ grams. The 90th percentile of the masses is 825.63 grams.

- (a) Show that $\sigma = 20$, to the nearest gram.

(2)

- (b) Find the probability that a randomly chosen cake has a mass of less than 780 grams.

(1)

[3 marks]

- D4** The times taken by runners to complete a qualifying race are modelled by a normal distribution with mean 55 minutes and standard deviation 5 minutes. Only the fastest 30% of runners qualify for the final.

- (a) Use standardisation to find the longest time a runner can take and still qualify.

(2)

- (b) Find the proportion of runners who take less than 50 minutes.

(1)

[3 marks]



STRETCH

- D5** The daily rainfall, W mm, in a town in the rainy season is modelled by $W \sim N(\mu, \sigma^2)$. Given that $P(W > 92) = 0.0228$ and $P(W < 56) = 0.3085$
- (a) Use the tables to form two equations in μ and σ (2)
- (b) Hence find the value of μ and the value of σ (2)
- [4 marks]
- D6** The lengths, X mm, of screws made by a machine are modelled by $X \sim N(200, 8^2)$. A screw longer than 210 mm is rejected. A random sample of 5 screws is taken.
- Find the probability that exactly 2 of the 5 screws are rejected. You must show your working clearly. [4 marks]
- D7** The time, T minutes, of a train journey is normally distributed with mean μ and standard deviation σ . Given that $P(\mu - 6 < T < \mu + 6) = 0.8$
- (a) Find the value of σ (2)
- (b) Find $P(T > \mu + 3 | T > \mu - 3)$ (2)
- [4 marks]
- D8** The random variables X and Y are such that $X \sim N(40, 5^2)$ and $Y \sim N(90, 10^2)$. Given that $P(X > k) = P(Y < 2k) = p$, where k and p are constants,
- (a) find the value of k (3)
- (b) find the value of p (2)
- [5 marks]



E**Exam practice**

Full-size exam questions in the style of WST01. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 A bakery makes large numbers of loaves of bread each day. The weight, W grams, of a loaf is modelled by a normal distribution with $W \sim N(805, 10^2)$. A loaf is chosen at random from the bakery's daily output.

(a) Using standardisation, show that the probability that the loaf weighs less than 820 grams is 0.9332

(2)

(b) Find the probability that the loaf weighs between 790 grams and 820 grams.

(2)

(c) Find the value of k such that $P(805 - k < W < 805 + k) = 0.9$

(2)

The lightest 20% of loaves are rejected.

(d) Find the maximum weight of a loaf that is rejected. You must show your working clearly.

(2)

(e) Three loaves are chosen at random from the bakery's output. Find the probability that at least one of the three loaves is rejected.

(2)

(Total for Question E1 is 10 marks)



E2 A botanist measures the leaves of a particular species of plant. The lengths, L mm, of the leaves are normally distributed with $L \sim N(30, 2.4^2)$

(a) Find the probability that a randomly selected leaf is shorter than 27 mm.

(2)

(b) Find the upper quartile and the lower quartile of L

(2)

An outlier is defined as a value greater than $Q_3 + 1.5(Q_3 - Q_1)$ or smaller than $Q_1 - 1.5(Q_3 - Q_1)$

(c) Find the two outlier limits.

(2)

(d) Using standardisation, show that the probability that a randomly selected leaf is not an outlier is 0.993 to 3 decimal places.

(2)

(e) Given that a randomly selected leaf is not an outlier, find the probability that it is longer than 33 mm. You must show your working clearly.

(3)

(Total for Question E2 is 11 marks)



STRETCH

E3 A machine in a factory fills bottles with fruit juice. The volume of juice, R ml, in a randomly chosen bottle is normally distributed with mean 300 ml and standard deviation k ml. Given that $P(R < r) = 0.2$ and $P(R > 2r - 220) = 0.025$, where r is a constant,

(a) use the percentage points table to form two equations in r and k

(3)

(b) find the value of r and the value of k

(3)

(c) find the probability that a randomly chosen bottle filled by this machine contains more than 320 ml.

(2)

Four bottles are chosen at random.

(d) Find the probability that exactly one of the four bottles contains more than 320 ml.

(2)

(e) A bottle contains more than 300 ml. Find the probability that it contains more than 320 ml.

(2)

(Total for Question E3 is 12 marks)

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