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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

Further Pure 2

Chapter 8: Polar Coordinates

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Inequalities** Rational and modulus inequalities, critical values, graphs	10	80	separate file	
2	**Series** Method of differences, partial fractions, telescoping sums	10	80	separate file	
3	**Complex Numbers** Exponential form, de Moivre's theorem, trigonometric identities, nth roots	10	80	separate file	
4	**Further Argand Diagrams** Loci, regions and transformations of the complex plane	10	80	separate file	
5	**First-Order Differential Equations** Integrating factors, substitutions, general and particular solutions	10	80	separate file	
6	**Second-Order Differential Equations** Auxiliary equations, complementary functions, particular integrals, substitutions	10	80	separate file	
7	**Maclaurin and Taylor Series** Higher derivatives, standard and compound series, Taylor series, series solutions	10	80	separate file	
8	**Polar Coordinates** Polar curves, areas, tangents parallel and perpendicular to the initial line	10	80	2	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 8 • KEY SKILLS

8 Polar Coordinates

Specification FP2: Polar coordinates • 10 questions • 80 marks • about 96 minutes

1 Converting

$x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$. Multiply through by r when the equation has $\cos\theta$ or $\sin\theta$ alone.

2 Sketching

Only values with $r \geqslant 0$ are drawn. Find r at $\theta = 0, \frac{\pi}{2}, \pi, \ldots$ and where $r = 0$ (the curve meets the pole).

3 Intersections

Set the two expressions for r equal, or use a given length OP; give the points as (r, θ)

4 Area

$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\,\mathrm{d}\theta$, with limits from where the region starts and ends, not automatically 0 to 2π

5 Squaring

Expand r^2 fully, then $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ before integrating.

6 Combining regions

Split at intersection points; subtract or add triangles, using $\frac{1}{2}r^2\sin\theta\cos\theta$ or $\frac{1}{2}\times$ base $\times$ height

7 Parallel tangents

Tangent parallel to the initial line: $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$, where $y = r\sin\theta$

8 Perpendicular tangents

Tangent perpendicular to the initial line: $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$, where $x = r\cos\theta$

9 Exact answers

Leave areas in terms of π and surds; reject angles that give $r < 0$ or lie outside the given interval.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Converting	•					•		•		
2 Sketching		•	•				•		•	
3 Intersections						•	•			
4 Area		•	•			•	•	•	•	•
5 Squaring		•	•			•	•			•
6 Combining regions						•	•		•	
7 Parallel tangents				•					•	•
8 Perpendicular tangents					•			•		
9 Exact answers	•	•	•	•	•	•	•	•	•	•
Interleaved: trigonometric identities and integration (P3, P4)		•	•	•	•	•	•		•	•
Interleaved: Newton–Raphson (FP1)								•		
Interleaved: circles, parabolas and coordinate geometry (P1, P2)	•					•		•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.
Must learn. Mark the region on the figure and decide its limits before integrating. Square r fully (three terms for $(a+b\cos\theta)^2$), and draw any triangle you need to add or subtract.

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1. **(a)** Find a Cartesian equation of the curve with polar equation $r = 4\cos\theta$, and describe the curve.

(3)

(b) Find a polar equation of the curve with Cartesian equation $y = x^2$, in the form $r = \mathrm{f}(\theta)$

(2)

(c) Find the polar coordinates (r, θ) of the point with Cartesian coordinates $\left(-1, \sqrt{3}\right)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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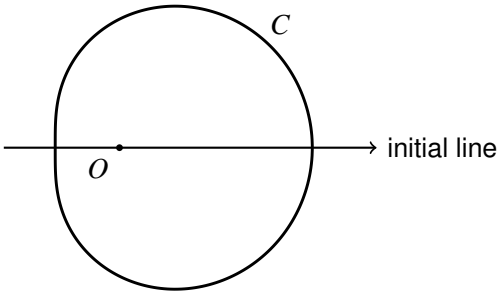

Figure 1

Figure 1 shows the curve C with polar equation $r = 2 + \cos\theta$, $0 \leqslant \theta < 2\pi$

(a) Show that $r^2 = \frac{9}{2} + 4\cos\theta + \frac{1}{2}\cos 2\theta$

(2)

(b) Hence find the exact area of the region enclosed by C.

(4)

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Question 2 continued

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3. The curve with polar equation $r = 3\sin 2\theta$ has a loop for $0 \leqslant \theta \leqslant \frac{\pi}{2}$

(a) Sketch this loop, stating the value of θ at which r is greatest.

(2)

(b) Find the exact area enclosed by the loop.

(4)

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Question 3 continued

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4. The curve C has polar equation $r = 1 + \cos\theta$, $0 \leqslant \theta \leqslant \pi$

(a) Show that, at points where the tangent to C is parallel to the initial line, $2\cos^2\theta + \cos\theta - 1 = 0$

(4)

(b) Hence find the polar coordinates of the point on C, other than the pole, at which the tangent is parallel to the initial line.

(2)

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Question 4 continued

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5. For $0 \leqslant \theta \leqslant \frac{2\pi}{3}$, a limacon is traced out by $r = 1 + 2\cos\theta$

(a) By writing $x = r\cos\theta$, prove that $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\sin\theta(1 + 4\cos\theta)$

(3)

(b) Locate, in polar coordinates, every point of this arc where the tangent is perpendicular to the initial line, with angles to 3 significant figures.

(3)

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Question 5 continued

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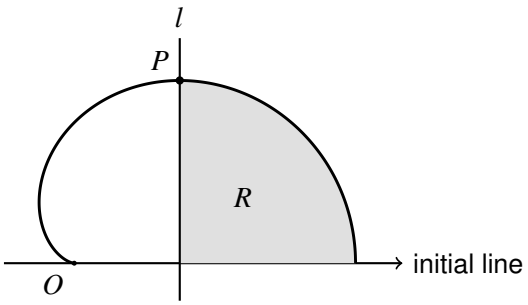

Figure 2

Figure 2 shows part of the curve C with polar equation $r = 3(1+\cos\theta)$, $0 \leqslant \theta \leqslant \pi$, and the line l with polar equation $r = \frac{9}{4}\sec\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The line l meets C at the point P. The region R, shaded, is bounded by C, l and the initial line.

(a) Show that P has polar coordinates $\left(\frac{9}{2}, \frac{\pi}{3}\right)$

(3)

(b) Find the exact area of R, giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational.

(7)

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Question 6 continued

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Question 6 continued

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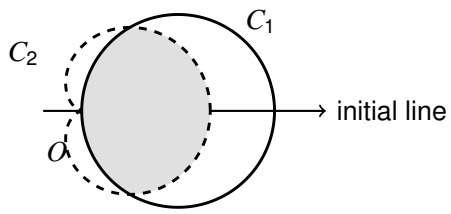

Figure 3

Figure 3 shows the curves C_1 and C_2 with polar equations $r = 3\cos\theta$, $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$, and $r = 1 + \cos\theta$, $0 \leqslant \theta < 2\pi$, respectively. The shaded region lies inside both curves.

(a) Find the polar coordinates of the points, other than the pole, where the curves meet.

(3)

(b) Find the exact area of the shaded region.

(7)

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Question 7 continued

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8. The curve C is the spiral with polar equation $r = \theta$, $0 \leqslant \theta \leqslant \pi$

(a) Find the exact area of the region bounded by C and the initial line produced backwards through O.

(3)

(b) Show that, at a point where the tangent to C is perpendicular to the initial line, $\theta \tan \theta = 1$

(3)

(c) Use the Newton–Raphson process once, with $\theta_0 = 0.85$, applied to $f(\theta) = \cos\theta - \theta \sin\theta$, to find this value of θ to 3 decimal places, and find the x-coordinate of the point.

(4)

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Question 8 continued

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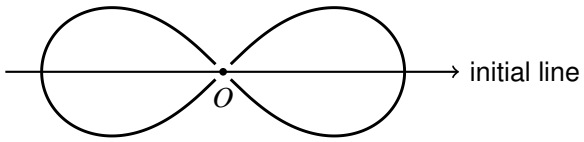

Figure 4

Figure 4 shows the curve with polar equation $r^2 = 4\cos 2\theta$

(a) Explain why there are no points on the curve for $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$

(2)

(b) Find the total area enclosed by the curve.

(4)

(c) Find the polar coordinates of the point on the curve, with $0 < \theta < \frac{\pi}{4}$, at which the tangent is parallel to the initial line.

(4)

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Question 9 continued

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10. The curve C has polar equation $r = a + 2\cos\theta$, $0 \leqslant \theta < 2\pi$, where $a > 2$. The area enclosed by C is 18π

(a) Show that $a = 4$

(3)

(b) Show that the tangents to C parallel to the initial line touch C where $\cos\theta = \frac{\sqrt{3}-1}{2}$, and find the value of r at these points in the form $p + \sqrt{q}$

(5)

(c) Find, to 3 significant figures, the distance between these two tangents.

(2)

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TOTAL FOR CHAPTER 8 IS 80 MARKS

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