



Discrete Random Variables

Worked Solutions

Suitable for:
Pearson Edexcel International A-Level (iAL) — Statistics 1



Question 1 — Worked Solution

(a)

$$0.1 + 0.3 + k + 0.2 = 1 \Rightarrow k = 0.4$$

(b)

$$P(X > 2) = P(X = 3) + P(X = 4) = 0.4 + 0.2 = 0.6$$



Question 2 — Worked Solution

(a)

All probabilities sum to 1:

$$0.1 + a + b + 0.2 + 0.1 = 1 \Rightarrow a + b = 0.6 \quad \dots(1)$$

$$P(X \leq 1) = 0.35:$$

$$0.1 + a = 0.35 \Rightarrow a = 0.25$$

$$b = 0.6 - 0.25 = 0.35$$

$$\therefore a = 0.25 \quad b = 0.35$$

(b)

$$P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.25 + 0.35 + 0.20 = 0.80$$





Question 3 — Worked Solution

(a) $E(X)$

$$E(X) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 0 + 0.3 + 0.8 + 0.6 = 1.7$$

(b) $\text{Var}(X)$

$$E(X^2) = 0^2(0.1) + 1^2(0.3) + 2^2(0.4) + 3^2(0.2) = 0 + 0.3 + 1.6 + 1.8 = 3.7$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 3.7 - 1.7^2 = 3.7 - 2.89$$

$$\therefore = 0.81$$



Question 4 — Worked Solution

(a) Valid distribution

$$\sum P(X = x) = \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{10}{10} = 1 \quad \checkmark$$

(b) $E(X)$ and $\text{Var}(X)$

$$E(X) = \frac{1^2 + 2^2 + 3^2 + 4^2}{10} = \frac{1 + 4 + 9 + 16}{10} = \frac{30}{10} = 3$$

$$E(X^2) = \frac{1^3 + 2^3 + 3^3 + 4^3}{10} = \frac{1 + 8 + 27 + 64}{10} = \frac{100}{10} = 10$$

$$\therefore \text{Var}(X) = 10 - 3^2 = 10 - 9 = 1$$





Question 5 — Worked Solution

(a) $E(2X + 1) = 2E(X) + 1 = 2(3) + 1 = 7$

(b) $\text{Var}(2X + 1) = 4 \text{Var}(X) = 4(4) = 16$

(c) $E(3 - X) = 3 - E(X) = 3 - 3 = 0$

(d) $\text{Var}(3 - X) = (-1)^2 \text{Var}(X) = \text{Var}(X) = 4$





Question 6 — Worked Solution

(a)

$$E(2X - 3) = 2E(X) - 3 = 7 \Rightarrow 2E(X) = 10$$

$$\therefore E(X) = 5$$

(b)

$$\text{Var}(3X + 1) = 9 \text{Var}(X) = 36 \Rightarrow$$

$$\therefore \text{Var}(X) = 4$$





Question 7 — Worked Solution

(a) CDF $F(x)$

x	1	2	3	4	5
F(x)	0.15	0.40	0.70	0.90	1.00

(b)

$$P(X \leq 3) = F(3) = 0.70$$

(c)

$$P(1 < X \leq 4) = F(4) - F(1) = 0.90 - 0.15 = 0.75$$



Question 8 — Worked Solution

(a) Probability distribution

$P(X=x) = F(x) - F(x-1)$ for each x :

x	1	2	3	4
$P(X = x)$	0.20	0.30	0.30	0.20

(b) $E(X)$

$$E(X) = 1(0.2) + 2(0.3) + 3(0.3) + 4(0.2) = 0.2 + 0.6 + 0.9 + 0.8 = 2.5$$





Question 9 — Worked Solution

(a) Find n

$$E(X) = \frac{n+1}{2} = 5 \Rightarrow n+1 = 10 \Rightarrow n = 9$$

(b) $\text{Var}(X)$

$$\text{Var}(X) = \frac{n^2-1}{12} = \frac{81-1}{12} = \frac{80}{12} = \frac{20}{3}$$

(c) $P(X > 4)$

$$P(X > 4) = \frac{9-4}{9} = \frac{5}{9}$$





Question 10 — Worked Solution

(a) Show $b - a = 10$

$n = b - a + 1$ values. For discrete uniform:

$$\text{Var}(X) = \frac{n^2 - 1}{12} = 10 \Rightarrow n^2 - 1 = 120 \Rightarrow n^2 = 121 \Rightarrow n = 11$$

$$n = b - a + 1 = 11 \Rightarrow b - a = 10 \checkmark$$

(b) Find a and b

$$E(X) = \frac{a + b}{2} = 7 \Rightarrow a + b = 14 \quad \dots(1)$$

$$b - a = 10 \quad \dots(2)$$

$$(1) + (2): 2b = 24 \Rightarrow b = 12. \quad (1) - (2): 2a = 4 \Rightarrow a = 2$$

$$\therefore a = 2 \quad b = 12$$



Question 11 — Worked Solution

(a) Show $k = 1/18$

$$\begin{aligned}\sum P &= k(0^2 + 1) + k(1^2 + 1) + k(2^2 + 1) + k(3^2 + 1) \\ &= k(1 + 2 + 5 + 10) = 18k = 1 \Rightarrow k = \frac{1}{18} \checkmark\end{aligned}$$

(b) $E(X)$ and $\text{Var}(X)$

Probabilities: $P(0)=1/18$, $P(1)=2/18$, $P(2)=5/18$, $P(3)=10/18$

$$E(X) = 0 \cdot \frac{1}{18} + 1 \cdot \frac{2}{18} + 2 \cdot \frac{5}{18} + 3 \cdot \frac{10}{18} = \frac{0 + 2 + 10 + 30}{18} = \frac{42}{18} = \frac{7}{3}$$

$$E(X^2) = 0 + \frac{2}{18} + \frac{20}{18} + \frac{90}{18} = \frac{112}{18} = \frac{56}{9}$$

$$\text{Var}(X) = \frac{56}{9} - \left(\frac{7}{3}\right)^2 = \frac{56}{9} - \frac{49}{9} = \frac{7}{9}$$



Question 12 — Worked Solution

(a) Show $p + 3q = 0.8$

$$E(X) = 0(0.1) + 1(p) + 2(0.3) + 3(q) + 4(0.1) = p + 0.6 + 3q + 0.4 = p + 3q + 1 = 1.8$$

$$\Rightarrow p + 3q = 0.8 \quad \checkmark$$

(b) Find p and q

$$\Sigma p = 1: 0.1 + p + 0.3 + q + 0.1 = 1 \Rightarrow p + q = 0.5 \quad \dots(1)$$

$$p + 3q = 0.8 \quad \dots(2)$$

$$(2) - (1): 2q = 0.3 \Rightarrow q = 0.15, p = 0.35$$

$$\therefore p = 0.35 \quad q = 0.15$$

(c) $\text{Var}(X)$

$$E(X^2) = 0 + 0.35 + 1.2 + 1.35 + 1.6 = 4.5$$

$$\text{Var}(X) = 4.5 - (1.8)^2 = 4.5 - 3.24 = 1.26 = \frac{63}{50}$$

(d)

$$E(3X - 2) = 3(1.8) - 2 = 5.4 - 2 = 3.4 = \frac{17}{5}$$

$$\text{Var}(3X - 2) = 9 \text{Var}(X) = 9 \times \frac{63}{50} = \frac{567}{50} = 11.34$$





Question 13 — Worked Solution

(a) Show $a + 3b = 1.1$

$$E(X) = a + 0.4 + 3b + 0.8 + 0.5 = a + 3b + 1.7 = 2.8$$

$$\Rightarrow a + 3b = 1.1 \quad \checkmark$$

(b) Find a and b

$$\Sigma p = 1: a + 0.2 + b + 0.2 + 0.1 = 1 \Rightarrow a + b = 0.5 \quad \dots(1)$$

$$a + 3b = 1.1 \quad \dots(2)$$

$$(2) - (1): 2b = 0.6 \Rightarrow b = 0.3, a = 0.2$$

$$\therefore a = 0.2 \quad b = 0.3$$

(c) $\text{Var}(X)$

$$E(X^2) = 0.2 + 0.8 + 2.7 + 3.2 + 2.5 = 9.4$$

$$\text{Var}(X) = 9.4 - (2.8)^2 = 9.4 - 7.84 = 1.56 = \frac{39}{25}$$

(d) $P(X > E(X))$

$$E(X) = 2.8, \text{ so } P(X > 2.8) = P(X=3) + P(X=4) + P(X=5)$$

$$= 0.3 + 0.2 + 0.1 = 0.6$$