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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 1 (WME01)

Mechanics 1 Practice Paper 1

Worked Solutions

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WME01/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
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2	Motion under gravity	8	3
3	Forces as vectors	9	4
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Question paper & mark scheme | Exam questions by topic | More practice papers

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1. Two particles, P and Q, have masses 2 kg and 3 kg respectively. They are moving towards each other in opposite directions along the same straight line on a smooth horizontal plane. Just before they collide, P has speed 6 m s^{-1} and Q has speed 2 m s^{-1}

In the collision, the direction of motion of P is reversed and its speed immediately after the collision is 1.5 m s^{-1}

(a) Find the speed of Q immediately after the collision, and state its direction of motion.

(4)

(b) Find the magnitude of the impulse exerted on Q by P in the collision.

(3)

(a) Take P's original direction as positive. Before: P +6, Q −2. After: P −1.5, Q v

Conservation of momentum: $2(6) + 3(-2) = 2(-1.5) + 3v$ M1

$12 - 6 = -3 + 3v \Rightarrow 3v = 9$ A1

$v = 3$ m s^{-1} A1

$v > 0$, so Q moves in the direction P was moving at first: its direction is reversed. B1

(b) Impulse on Q = change in momentum of Q = $3(3) - 3(-2)$ M1

$= 9 + 6$ (check with P: $2(6) - 2(-1.5) = 15$ ✓) A1

15 N s A1

(Total for Question 1 is 7 marks)

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2. A ball is projected vertically upwards with speed 14.7 m s^{-1} from a point O which is 24.5 m vertically above the sea. The ball is modelled as a particle moving freely under gravity.

(a) Find the greatest height of the ball above the sea.

(3)

(b) Find the time taken for the ball to reach the sea.

(3)

(c) Find the speed of the ball as it reaches the sea.

(2)

(a) Up positive, $a = -9.8$. At the highest point $v = 0$: $0 = 14.7^2 - 2 \times 9.8 \times s$ M1

$s = 216.09 \div 19.6 = 11.025$ m above O A1

Height above the sea $= 24.5 + 11.025 = 35.525$ 35.5 m A1

(b) The sea is 24.5 m below O, so $s = -24.5$: $-24.5 = 14.7t - 4.9t^2$ M1

$4.9t^2 - 14.7t - 24.5 = 0 \Rightarrow t^2 - 3t - 5 = 0 \Rightarrow t = (3 \pm \sqrt{29})/2$ M1

$t > 0$: $t = (3 + \sqrt{29})/2 = 4.19...$ 4.19 s A1

(c) $v^2 = u^2 + 2as = 14.7^2 + 2(-9.8)(-24.5) = 216.09 + 480.2 = 696.29$ M1

$v = 26.38...$ 26.4 m s^{-1} A1

(Total for Question 2 is 8 marks)

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3. *In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular horizontal unit vectors.*

A particle P of mass 2 kg moves on a smooth horizontal plane under the action of three forces

$$\mathbf{F}_1 = (2\mathbf{i} + 5\mathbf{j})\text{ N}, \qquad \mathbf{F}_2 = (3\mathbf{i} - \mathbf{j})\text{ N} \qquad \text{and} \qquad \mathbf{F}_3 = (\mathbf{i} + 4\mathbf{j})\text{ N}$$

(a) Find the magnitude of the acceleration of P

(4)

(b) Find the angle between the acceleration of P and the vector $\mathbf{i}$

(2)

At time $t = 0$, the velocity of P is $(-3\mathbf{i} + 2\mathbf{j})\text{ m s}^{-1}$

(c) Find the speed of P when $t = 2$

(3)

(a) Resultant $= F_1 + F_2 + F_3 = (2 + 3 + 1)i + (5 - 1 + 4)j = 6i + 8j$ M1

$F = ma$: $6i + 8j = 2a \Rightarrow a = 3i + 4j$ A1

$|a| = \sqrt{3^2 + 4^2} = \sqrt{25}$ M1

5 m s^{-2} A1

(b) $\tan\theta = 4/3 \Rightarrow \theta = 53.13...°$ $53.1°$ M1 A1

(c) $v = u + at = (-3i + 2j) + 2(3i + 4j)$ M1

$= 3i + 10j$ A1

speed $= \sqrt{3^2 + 10^2} = \sqrt{109} = 10.44...$ 10.4 m s^{-1} A1

(Total for Question 3 is 9 marks)

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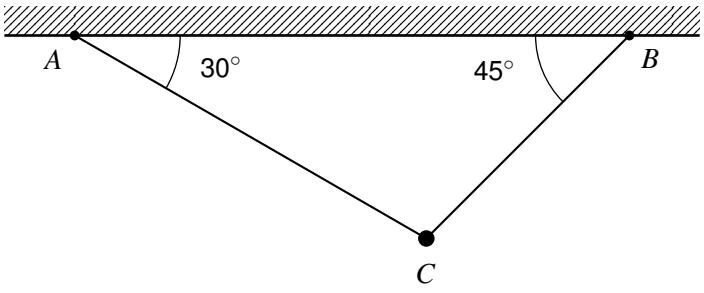

Figure 1

A particle of mass 3 kg is attached at C to two light inextensible strings, AC and BC. The other ends of the strings are attached to points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC at 30° to the ceiling and BC at 45° to the ceiling, as shown in Figure 1.

(a) Find the tension in AC and the tension in BC

(6)

Each string will break if its tension exceeds 40 N. The particle is replaced by a heavier particle, and the strings stay at the same angles.

(b) Find the greatest mass of the particle for which neither string breaks.

(3)

(a) Let T_A and T_B be the tensions in AC and BC.

Resolve horizontally (→): $T_B \cos 45° - T_A \cos 30° = 0$ M1

$T_A \cos 30° = T_B \cos 45° \Rightarrow T_B = 1.2247... T_A$ A1

Resolve vertically (↑): $T_A \sin 30° + T_B \sin 45° - 3g = 0$ M1

$T_A \sin 30° + T_B \sin 45° = 3g = 29.4$ A1

$T_A(0.5 + 1.2247... \times 0.7071...) = 29.4 \Rightarrow 1.3660...T_A = 29.4$ M1

$T_A = 21.5$ N $T_B = 26.4$ N A1

(b) Both tensions are proportional to the mass, and BC has the larger one,

so BC reaches 40 N first: $T_B = (26.36.../3) \times m = 8.786...m$ M1

$8.786...m = 40 \Rightarrow m = 40 \div 8.786...$ M1

$= 4.552...$ 4.55 kg (then AC has 32.7 N < 40 ✓) A1

(Total for Question 4 is 9 marks)

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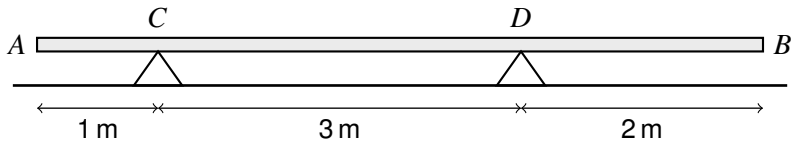

Figure 2

A uniform plank AB has length 6 m and mass 30 kg. It rests in a horizontal position on two supports, at C and D, where AC = 1 m and DB = 2 m, as shown in Figure 2. A man of mass 70 kg stands on the plank at the point that is 2 m from A

The plank is modelled as a uniform rod and the man as a particle.

(a) Find the magnitude of the reaction on the plank at C and at D

(4)

The man now walks along the plank towards B

(b) Find the distance from B of the man when the plank is about to tilt.

(4)

(c) State how you have used the fact that the plank is uniform.

(1)

(a) Plank's weight 30g acts at its midpoint, 3 m from A; the man (70g) is 2 m from A.

Moments about C: $R_D \times 3 = 30g \times 2 + 70g \times 1$ (C is 1 m from A) M1

$R_D = 130g/3 = 424.6...$ A1

Resolve vertically: $R_C + R_D = 100g \Rightarrow R_C = 170g/3 = 555.3...$ M1

$R_C = 555$ N $R_D = 425$ N A1

(b) About to tilt about D: the plank leaves C, so $R_C = 0$ B1

Moments about D: $30g \times 1 = 70g \times x$ (x = distance of the man beyond D) M1

$x = 3/7$ m A1

distance from B = $2 - 3/7 = 11/7 = 1.571...$ 1.57 m A1

(c) Uniform, so the weight of the plank acts at its midpoint, 3 m from A. B1

(Total for Question 5 is 9 marks)

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6. A train travels along a straight horizontal track from station A to station B, a distance of 5 km. It starts from rest at A and accelerates uniformly for 40 s until its speed is 20 m s^{-1}. It then travels at 20 m s^{-1} for T seconds, before decelerating uniformly at 0.5 m s^{-2} until it comes to rest at B

(a) Sketch a speed-time graph for the motion of the train from A to B

(2)

(b) Show that $T = 210$

(3)

(c) Find the total time taken to travel from A to B

(1)

(d) Find the time taken for the train to travel the first 2.5 km of the journey.

(2)

A signal is beside the track, 300 m before B

(e) Find the time taken by the train to travel from the signal to B

(2)

(a) Speed rises to 20 in 40 s, steady for T s, then falls to 0 in 20 ÷ 0.5 = 40 s.



B1 B1

(b) Deceleration time = 20 ÷ 0.5 = 40 s B1

Distance = area: ½ × 40 × 20 + 20T + ½ × 40 × 20 = 5000 (5 km = 5000 m) M1

400 + 20T + 400 = 5000 ⇒ 20T = 4200 ⇒ T = 210 ✓ A1*

(c) 40 + 210 + 40 290 s B1

(d) 400 m in the first 40 s; then 2500 − 400 = 2100 m at 20: 2100 ÷ 20 = 105 s M1

40 + 105 145 s A1

(e) It stops in the last 300 m: 300 = ½ × 0.5 × t² (think of it in reverse from rest) M1

t² = 1200 ⇒ t = 34.64... 34.6 s A1

(Total for Question 6 is 10 marks)

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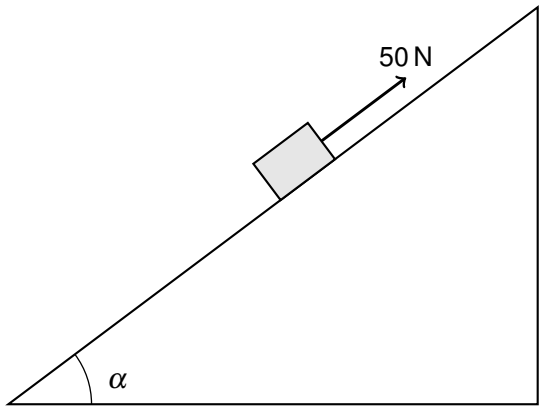

Figure 3

A box of mass 5 kg is pulled up a line of greatest slope of a rough plane by a force of magnitude 50 N. The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, and the force acts parallel to the line of greatest slope, as shown in Figure 3. The box is modelled as a particle, and it moves up the plane with acceleration 2.16 m s^{-2}

(a) Show that the coefficient of friction between the box and the plane is 0.25

(5)

The box starts from rest. After 5 s the force is removed.

(b) Find the further distance that the box moves up the plane before coming to rest.

(4)

(c) Determine whether the box will then remain at rest, giving a reason for your answer.

(1)

(a) $\tan\alpha = 3/4$, so $\sin\alpha = 3/5 = 0.6$ and $\cos\alpha = 4/5 = 0.8$ B1

Perpendicular to the plane: $R = 5g\cos\alpha = 5 \times 9.8 \times 0.8 = 39.2$ M1

Along the plane, up positive: $50 - F - 5g\sin\alpha = 5 \times 2.16 \Rightarrow 50 - F - 29.4 = 10.8$ M1

$F = 9.8$ A1

Moving, so $F = \mu R$: $\mu = 9.8 \div 39.2 = 0.25$ ✓ A1*

(b) After 5 s: $v = u + at = 0 + 2.16 \times 5 = 10.8$ m s^{-1} M1

Force removed, still moving up: weight component and friction both act down the plane:

$5g\sin\alpha + 0.25 \times 39.2 = 5a \Rightarrow 29.4 + 9.8 = 5a$ M1

$a = 7.84$ m s^{-2} (deceleration) A1

$0 = 10.8^2 - 2 \times 7.84 \times s \Rightarrow s = 116.64 \div 15.68 = 7.43...$ 7.44 m A1

(c) $5g\sin\alpha = 29.4$ N > greatest friction $\mu R = 9.8$ N, so it slides back down. B1

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Question 7 continued

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(Total for Question 7 is 10 marks)

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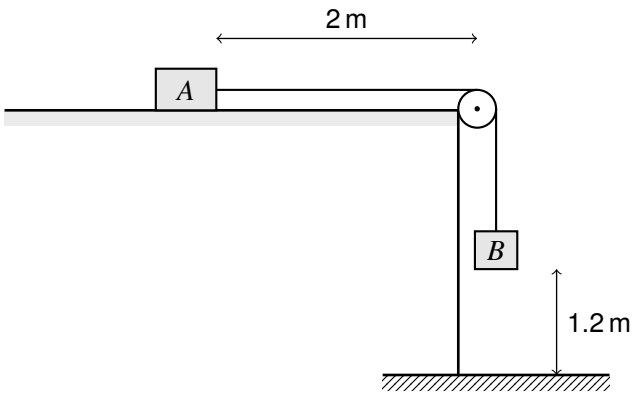

Figure 4

A particle A of mass 3 kg rests on a rough horizontal table. It is attached to one end of a light inextensible string, which passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a particle B of mass 2 kg, which hangs freely 1.2 m above the floor. A is 2 m from the pulley, as shown in Figure 4. The coefficient of friction between A and the table is $\frac{1}{3}$

The system is released from rest with the string taut.

(a) Find the acceleration of A and the tension in the string while B is falling.

(6)

(b) Find the speed of B just before it hits the floor.

(2)

B does not rebound when it hits the floor.

(c) Show that A comes to rest before it reaches the pulley.

(4)

(d) State how you have used the fact that the string is inextensible.

(1)

(a) A: $R = 3g$, so friction $F = \mu R = (1/3) \times 3g = g = 9.8$ N M1

Equation of motion for A (towards the pulley): $T - F = 3a$ M1

$T - g = 3a$ A1

Equation of motion for B (down): $2g - T = 2a$ M1

Add: $2g - g = 5a \Rightarrow a = g/5$ A1

$a = 1.96 \text{ m s}^{-2}$ $T = 2g - 2(1.96) = 15.68$ $T = 15.7$ N A1

(b) B falls 1.2 m from rest: $v^2 = 0 + 2 \times 1.96 \times 1.2 = 4.704$ M1

$v = 2.168...$ 2.17 m s^{-1} A1

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Question 8 continued

(c) B is on the floor, so the string is slack: only friction acts on A M1

deceleration = $F/3 = g/3 = 3.266\ldots$ m s^{-2} A1

$0 = 4.704 - 2 \times 3.266\ldots \times s \Rightarrow s = 0.72$ m M1

A has already moved 1.2 m: $1.2 + 0.72 = 1.92$ m < 2 m, so A stops first ✓ A1*

(d) Inextensible: A and B have the same acceleration (and speed) while the string is taut. B1

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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