eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

Impulses and Collisions

Worked Solutions

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Full worked solutions for the free Impulses and Collisions worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Mechanics 2 Impulses and Collisions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Mechanics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $-2\mathbf{i}+0.5\mathbf{j}=0.25\mathbf{v}-(1.5\mathbf{i}+0.25\mathbf{j})$, $\mathbf{v}=-2\mathbf{i}+3\mathbf{j}$: $\boxed{\sqrt{13}\approx 3.61\,\text{m s}^{-1}}$

A2 $13=2v_A+3v_B$; $v_B-v_A=0.4(4)=1.6$: $\boxed{v_A=1.64\,\text{m s}^{-1},\ v_B=3.24\,\text{m s}^{-1}}$

A3 Rebound $0.75(8)=6$; $0.3(6+8)$: $\boxed{\mathbf{4.2\,N s}}$

STRETCH

A4 $|m\mathbf{v}|=\sqrt{3^2+4^2}=5$: $\boxed{10\,\text{m s}^{-1}}$; $\tan^{-1}\frac{4}{3}$: $\boxed{53.1^\circ}$

A5 $\frac{1}{2}(2)(5^2)+\frac{1}{2}(3)(1^2)-\left[\frac{1}{2}(2)(1.64^2)+\frac{1}{2}(3)(3.24^2)\right]=26.5-18.436$: $\boxed{\mathbf{8.06\,J}}$

A6 $12-4=3v_P+2(3.5)$: $\boxed{v_P=\frac{1}{3}\,\text{m s}^{-1}}$ (same direction); $e=\dfrac{3.5-\frac{1}{3}}{4+2}$: $\boxed{e=\frac{19}{36}}$

A7 $2f>0.8$: $\boxed{f>0.4}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $2(7-3)$: $\boxed{\mathbf{8\,N s}}$ **(b)** $2(7+3)$: $\boxed{\mathbf{20\,N s}}$ **(c)** $0.5(3\mathbf{i}-4\mathbf{j})$: $\boxed{(1.5\mathbf{i}-2\mathbf{j})\,\mathbf{N s}}$ **(d)** $\boxed{\mathbf{2.5\,N s}}$

A reversal adds the speeds; a vector impulse is after minus before, component by component.

B2 **(a)** $\boxed{\textbf{both } 4\,\text{m s}^{-1}}$ **(b)** $\boxed{2\,\text{m s}^{-1},\ 8\,\text{m s}^{-1}}$ **(c)** $\boxed{3\,\text{m s}^{-1},\ 6\,\text{m s}^{-1}}$ **(d)** $36-(9+18)$: $\boxed{\mathbf{9\,J}}$

$e=0$: they move together; $e=1$: no energy is lost.

B3 **(a)** $\boxed{3\,\text{m s}^{-1}}$ **(b)** $0.5(3+6)$: $\boxed{\mathbf{4.5\,N s}}$ **(c)** $9-2.25$: $\boxed{\mathbf{6.75\,J}}$ **(d)** $e^2=\frac{1}{2}$: $\boxed{e=0.707}$

The KE after a wall impact is e^2 times the KE before.

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B4 **(a)** $\dfrac{2.5-1}{4}$: $\boxed{e=\frac{3}{8}}$ **(b)** $\dfrac{1+4}{5+3}$: $\boxed{e=\frac{5}{8}}$ **(c)** $5m_A-3m_B=-m_A+4m_B$: $\boxed{7:6}$

(d) $\boxed{A \text{ stops and } B \text{ moves on at } u}$

Speed of separation = $e\times$ speed of approach, with speeds, not signed velocities.

B5 **(a)** $0.2[(2\mathbf{i}+4\mathbf{j})-5\mathbf{i}]$: $\boxed{(-0.6\mathbf{i}+0.8\mathbf{j})\,\mathbf{N\,s}}$ **(b)** $\boxed{1\,\mathbf{N\,s}}$ **(c)** $\cos\theta=\dfrac{-0.6}{1}$: $\boxed{127^\circ}$

The impulse direction is not the new direction of motion.

B6 **(a)** Together at $4\,\text{m s}^{-1}$; $20-16$: $\boxed{4\,\mathbf{J}}$ **(b)** $\boxed{0\,\mathbf{J}}$ **(c)** $v_A=1$, $v_B=3$; $16-10$: $\boxed{6\,\mathbf{J}}$

(d) $e=0$: both $2\,\text{m s}^{-1}$, loss $16-8=8$; $\dfrac{6}{8}$: $\boxed{\frac{3}{4}}$, which is $1-e^2$

The KE lost is proportional to $1-e^2$

STRETCH

B7 **(a)** $\dfrac{\sqrt{2^2+3^2}}{0.4}$: $\boxed{9.01\,\text{m s}^{-1}}$ **(b)** $\dfrac{3-2}{0.4}$: $\boxed{2.5\,\text{m s}^{-1}\ \textbf{(reversed)}}$ **(c)** $\dfrac{\sqrt{19}}{0.4}$: $\boxed{10.9\,\text{m s}^{-1}}$

(d) $2^2=2^2+3^2+2(2)(3)\cos\theta$, $\cos\theta=-\frac{3}{4}$: $\boxed{139^\circ}$

Every case is the triangle of momenta $m\mathbf{u}+\mathbf{I}=m\mathbf{v}$.

B8 **(a)** $\boxed{\frac{1}{3}u(1-2e)}$ **(b)** $\boxed{\frac{1}{3}u(1+e)}$ **(c)** $\boxed{e>\frac{1}{2}}$

(d) $\frac{1}{2}mu^2-\frac{1}{2}m\left[\frac{u(1-2e)}{3}\right]^2-\frac{1}{2}(2m)\left[\frac{u(1+e)}{3}\right]^2=\frac{1}{3}(1-e^2)mu^2$

Work in terms of e once, then read off every case.

B9 **(a)** $\boxed{4f\,\text{m s}^{-1}}$ **(b)** $1(4f+4)$: $\boxed{4(1+f)\,\mathbf{N\,s}}$ **(c)** $4f>1$: $\boxed{f>\frac{1}{4}}$

(d) Away from the wall: $2(1)+1(2)=2v_A+v_B$, $v_A-v_B=\frac{1}{2}(2-1)$: $\boxed{v_A=1.5,\ v_B=1\,\text{m s}^{-1}}$

After the wall both move the same way: B must be faster to catch A

B10 **(a)** $\boxed{\frac{1}{4}u,\ \frac{3}{4}u}$ **(b)** $\boxed{\frac{3}{16}u,\ \frac{9}{16}u}$ **(c)** $\frac{1}{4}u>\frac{3}{16}u$: $\boxed{A \text{ catches } B}$

(d) $\dfrac{\left(\frac{1}{4}\right)^2+\left(\frac{3}{16}\right)^2+\left(\frac{9}{16}\right)^2}{1}$: $\boxed{\frac{53}{128}}$

Each impact is worked separately with the speeds just before it.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 A magnitude is a number, not a vector: $\sqrt{0.6^2+2.1^2}$: **2.18 N s**

C2 The ball's direction is reversed, so the change of velocity is $2-(-5)$: $0.4(7)$: **2.8 N s**

C3 They approach at $5+2=7$ and separate at $1+3=4$ (speeds, not signed velocities): $e=\frac{4}{7}$

C4 He has ignored the kinetic energy that B gains: $25-(1+16)$: **8 J**

STRETCH

C5 Along the motion the impulse component is $3\cos 60°$, and across it $3\sin 60°$

$0.5v=\sqrt{(2+1.5)^2+\left(\frac{3\sqrt{3}}{2}\right)^2}$: $8.72\,\text{m s}^{-1}$

C6 Both move the same way at the same speed, so the gap never closes: **no further collision**. They collide again only if B's speed after the wall is *greater* than $1.5\,\text{m s}^{-1}$

C7 The sphere's mass is $2m$, and the change of direction gives $3u+u$: $2m(4u)$: $8mu$

C8 **(a)** $mu=mv_A+kmv_B$ and $v_B-v_A=eu$; adding $k\times$(second): $u-keu=(1+k)v_A$

$v_A=\frac{u(1-ke)}{1+k}$, and $v_B=v_A+eu$: $v_B=\frac{u(1+e)}{1+k}$

(b) $v_A=0$ only if $ke=1$; then $e=\frac{1}{k}$, and $e\leqslant 1$ needs $k\geqslant 1$

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** $-\mathbf{i}+3\mathbf{j} = 0.5\mathbf{v} - (2\mathbf{i}-\mathbf{j})$, $\mathbf{v} = 2\mathbf{i}+4\mathbf{j}$: $\sqrt{20}$: $\boxed{4.47\,\text{m s}^{-1}}$

(b) $(4\mathbf{i}-2\mathbf{j})\cdot(2\mathbf{i}+4\mathbf{j}) = 8-8 = 0$: $\boxed{90^\circ}$

D2 **(a)** Positive in P's direction: $2m(3u) - 3m(2u) = -2mx + 3my$, so $2x = 3y$

$x+y = \frac{1}{2}(3u+2u)$: $\boxed{P\text{: } 1.5u,\ Q\text{: } u}$

(b) $3m(u+2u)$: $\boxed{9mu}$

D3 **(a)** $\mathbf{I} = (\lambda-4)\mathbf{i}+\lambda\mathbf{j}$; $(\lambda-4)^2+\lambda^2 = 10$, so $\lambda^2-4\lambda+3 = 0$, $\lambda = 1$ or 3

The speed increases, so $\lambda\sqrt{2} > 4$: $\boxed{\lambda = 3}$

(b) $\boxed{\mathbf{I} = (-\mathbf{i}+3\mathbf{j})\,\text{N s}}$

D4 **(a)** $1\,[(-\mathbf{i}+2\mathbf{j}) - (2\mathbf{i}+3\mathbf{j})]$: $\boxed{(-3\mathbf{i}-\mathbf{j})\,\text{N s}}$

(b) B receives $(3\mathbf{i}+\mathbf{j})$: $2\mathbf{v}_B = 2(-\mathbf{i}+\mathbf{j}) + (3\mathbf{i}+\mathbf{j})$: $\boxed{(0.5\mathbf{i}+1.5\mathbf{j})\,\text{m s}^{-1}}$

STRETCH

D5 **(a)** $mu = mv_P + 3mv_Q$, $v_Q - v_P = eu$: $v_P = \frac{1}{4}u(1-3e)$; reversed, so $e > \frac{1}{3}$: $\boxed{\frac{1}{4}u(3e-1)}$

(b) $v_Q = \frac{1}{4}u(1+e)$; after the wall $\frac{1}{8}u(1+e)$; need $\frac{1}{8}u(1+e) > \frac{1}{4}u(3e-1)$, $e < \frac{3}{5}$: $\boxed{\frac{1}{3} < e < \frac{3}{5}}$

D6 **(a)** $v_A = \frac{1}{4}u(3-e)$, $v_B = \frac{3}{4}u(1+e)$; $\frac{1}{2}(3m)u^2 - \frac{1}{2}(3m)v_A^2 - \frac{1}{2}mv_B^2 = \frac{3}{8}(1-e^2)mu^2$, as required

(b) $m \times \frac{3}{4}u(1+e) = \frac{6}{5}mu$: $\boxed{e = \frac{3}{5}}$

D7 **(a)** $7^2 = 3^2 + I^2 + 2(3)I\cos 60^\circ$, so $I^2+3I-40 = 0$, $(I+8)(I-5) = 0$: $\boxed{I = 5}$

(b) $\dfrac{\sin\theta}{5} = \dfrac{\sin 120^\circ}{7}$: $\boxed{38.2^\circ}$

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D8 **(a)** $2mu = 2mv_A + mv_B$, $v_B - v_A = \frac{1}{2}u$: $\boxed{A\text{: } \frac{1}{2}u,\ B\text{: } u}$

(b) B returns at $\frac{1}{2}u$ while A still moves towards the wall, so **they move towards each other and collide**

(c) Towards the wall positive: $2m\left(\frac{1}{2}u\right) - m\left(\frac{1}{2}u\right) = 2mv_A + mv_B$, $v_B - v_A = \frac{1}{2}u$: $v_A = 0$, $v_B = \frac{1}{2}u$

$m\left(\frac{1}{2}u - \left(-\frac{1}{2}u\right)\right)$: $\boxed{mu}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** Impulse on P: $2m(x+3u) = \frac{36}{5}mu$ *M1 A1*

$\boxed{x = \frac{3}{5}u}$ *A1**

(b) $2m(3u) - 3mu = -2m\left(\frac{3}{5}u\right) + 3my$ *M1*

$\boxed{y = \frac{7}{5}u}$ *A1*

(c) $e = \dfrac{\frac{3}{5}u + \frac{7}{5}u}{3u+u}$ *M1*

$\boxed{e = \frac{1}{2}}$ *A1*

(d) $\frac{1}{2}(2m)(3u)^2 + \frac{1}{2}(3m)u^2 - \left[\frac{1}{2}(2m)\left(\frac{3}{5}u\right)^2 + \frac{1}{2}(3m)\left(\frac{7}{5}u\right)^2\right]$ *M1 A1*

$\frac{21}{2}mu^2 - \frac{33}{10}mu^2$: $\boxed{\frac{36}{5}mu^2}$ *A1*

E2 **(a)** $2mu = 2mv_A + mv_B$ and $v_B - v_A = eu$ *M1 M1*

$3v_B = 2u + 2eu$: $\boxed{v_B = \frac{2}{3}(1+e)u}$ *A1**

(b) $v_A = v_B - eu = \frac{1}{3}(2-e)u > 0$ as $e \leqslant 1$: $\boxed{A \text{ is not reversed}}$ *B1*

(c) After the wall B's speed is $f \times \frac{2}{3}(1+e)u$ *M1*

$\frac{2}{3}f(1+e)u = \frac{1}{3}(2-e)u$ *M1*

$\boxed{f = \dfrac{2-e}{2(1+e)}}$ *A1**

(d) $v_B = \frac{5}{6}u$, $f = \frac{7}{10}$: $m\left(\frac{5}{6}u\right)\left(1 + \frac{7}{10}\right)$ *M1*

$\boxed{\frac{17}{12}mu}$ *A1*

(e) After the wall B moves back towards A, which still moves towards the wall: **yes, they collide** *B1*

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E3 **(a)** Positive in P's direction: $4mu - kmu = -mu + kmv_Q$ *M1 A1*

$\boxed{v_Q = \dfrac{(5-k)u}{k}}$; Q reverses, so $v_Q > 0$: $\boxed{k < 5}$ *A1**

(b) $e = \dfrac{u + \dfrac{(5-k)u}{k}}{4u+u}$ *M1 A1*

$= \dfrac{5u/k}{5u}$: $\boxed{e = \dfrac{1}{k}}$ *A1**

(c) $0 < e \leqslant 1$ gives $k \geqslant 1$; with $k < 5$: $\boxed{1 \leqslant k < 5}$ *M1 A1*

(d) $k = 2$: Q moves at $\frac{3}{2}u$ towards the wall and P at u away from Q *B1*

After the wall Q needs $\frac{3}{2}fu > u$ *M1*

$\boxed{\frac{2}{3} < f \leqslant 1}$ *A1*

(e) Before: $\frac{1}{2}m(4u)^2 + \frac{1}{2}(2m)u^2 = 9mu^2$; after: $\frac{1}{2}mu^2 + \frac{1}{2}(2m)\left(\frac{3}{2}u\right)^2 = \frac{11}{4}mu^2$ *M1*

$\dfrac{9 - \frac{11}{4}}{9}$: $\boxed{\frac{25}{36}}$ *A1*