



eClassroom

A-LEVEL MATHEMATICS • PURE 4

Differentiation

Worked Solutions

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Differentiation worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 4 Differentiation worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $\frac{dy}{dx} = \frac{3t^2 - 3}{2} = \frac{9}{2}$ at $t = 2$; the point is $(7, 2)$: $y - 2 = \frac{9}{2}(x - 7)$

$$2y = 9x - 59$$

A2 $2x - 2y - 2x \frac{dy}{dx} + 8y \frac{dy}{dx} = 0$; at $(-2, 1)$: $-6 + 12 \frac{dy}{dx} = 0$: $\frac{dy}{dx} = \frac{1}{2}$

A3 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$: $10 = 100\pi \frac{dr}{dt}$: $\frac{dr}{dt} = \frac{1}{10\pi} \text{ cms}^{-1}$

STRETCH

A4 $\frac{dy}{dx} = \frac{2 \cos 2t}{-4 \sin t}$; at $t = \frac{\pi}{3}$: $\frac{-1}{-2\sqrt{3}}$: **gradient $\frac{\sqrt{3}}{6}$, normal gradient $-2\sqrt{3}$**

A5 $2^y \ln 2 \frac{dy}{dx} + 2x = 3$; at $(1, 1)$: $2 \ln 2 \frac{dy}{dx} = 1$: $\frac{dy}{dx} = \frac{1}{2 \ln 2}$

A6 $\frac{dr}{dt} = \frac{k}{r}$ and $0.5 = \frac{k}{10}$, so $k = 5$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi r \times \frac{5}{r}$: $\frac{dA}{dt} = 10\pi$, **a constant**

A7 $\frac{dy}{dx} = -\frac{2x-y}{2y-x} = 0$ when $y = 2x$: $x^2 - 2x^2 + 4x^2 = 3$, so $x = \pm 1$

$$(1, 2) \text{ and } (-1, -2)$$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\frac{2}{t}$ (b) t^2 (c) $-\cot t$ (d) $\frac{e^t + te^t}{e^t} : 1+t$

Always $\frac{dy}{dt}$ over $\frac{dx}{dt}$, then simplify.

B2 (a) $-\frac{x}{y}$ (b) $\frac{1}{2-3y^2}$ (c) $\frac{4-y}{x+2y}$ (d) $-\frac{2xy}{e^y + x^2}$

Every y term gains $\frac{dy}{dx}$; products such as xy and x^2y need the product rule.

B3 (a) $\frac{6t^2}{2t} = 3t : 3$ (b) $\frac{-2\sin 2t}{2\cos t} = -2\sin t : -1$ (c) $-\frac{x}{y} : -\frac{1}{3}$ (d) $-\frac{x^2}{y^2} : -4$

Use the parameter value for a parametric curve, and both coordinates for an implicit one.

B4 (a) $\frac{dy}{dx} = -\frac{x}{2y} = -\frac{1}{4} : x+4y=9$ (b) gradient 4: $y=4x-2$
 (c) $\frac{dy}{dx} = \frac{3t}{2} = 3$ at $(4, 8) : y=3x-4$ (d) gradient $-\frac{1}{3} : x+3y=28$

The normal gradient is -1 divided by the tangent gradient.

B5 (a) $2\pi(5)(0.2) : 2\pi$ (b) $12 = 3(2)^2 \frac{dx}{dt} : 1$ (c) $8\pi(3)(0.1) : 2.4\pi$

Differentiate the formula, then multiply by the known rate (or divide to find the unknown one).

B6 (a) $5^x \ln 5$ (b) $3 \ln 2 \times 2^{3x}$ (c) $3^x + x3^x \ln 3$ (d) $4^x \ln 4$; at $x = \frac{1}{2} : 4 \ln 2$

$a^x = e^{x \ln a}$, so $\frac{d}{dx}(a^x) = a^x \ln a$

STRETCH

B7 (a) $\frac{1+t^{-2}}{1-t^{-2}} : \frac{t^2+1}{t^2-1}$ (b) $t^2 = 1 : (2, 0), (-2, 0)$
 (c) $\frac{dy}{dx} = (2t-2)t = 0$ at $t = 1 : (0, -1)$ (d) $\frac{3\cos 3t}{\cos t} : -3$

A vertical tangent has $\frac{dx}{dt} = 0$; a stationary point has $\frac{dy}{dt} = 0$



B8 (a) $\frac{dy}{dx} = -\frac{2xy}{x^2 + 3y^2}$: $-\frac{1}{2}$ (b) $\frac{1}{y} \frac{dy}{dx} + y + x \frac{dy}{dx} = 0$: $-\frac{1}{3}$

(c) $\frac{dy}{dx} = \frac{3-x}{y}$; vertical when $y = 0$: $(0, 0), (6, 0)$ (d) $\ln y = x \ln x$: $x^x(\ln x + 1)$, gradient 1

Each part needs a different first step: product rule, chain rule on $\ln y$, the denominator, logarithms.

B9 (a) $6 = 2\pi(9) \frac{dr}{dt}$: $\frac{1}{3\pi}$ (b) $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$: $-\frac{3}{8}$

(c) $\frac{dy}{dx} = \frac{2x}{1+x^2} \times 2$: 2 (d) $10 \cos \theta \times 0.1$: 0.5

In (b) the two rates are linked through the equation, not a formula for one variable.

B10 (a) $\frac{dT}{dt} = -k(T - 20)$ (b) $\frac{dN}{dt} = k\sqrt{N}$

(c) $4\pi r^2 \frac{dr}{dt} = -k \times 4\pi r^2$: $\frac{dr}{dt} = -k$ (d) $3 \frac{dh}{dt} = 0.6 - 0.2h$: $\frac{dh}{dt} = \frac{0.6 - 0.2h}{3}$

"Proportional to" needs a constant; a falling quantity needs a minus sign; rate in minus rate out.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 y^3 differentiates to $3y^2 \frac{dy}{dx}$, not $3y^2$. Then $2x + 3y^2 \frac{dy}{dx} = 0$: $\frac{dy}{dx} = -\frac{2x}{3y^2}$

C2 $3xy$ is a product: its derivative is $3y + 3x \frac{dy}{dx}$. So $2x + 3y + 3x \frac{dy}{dx} = 0$: $\frac{dy}{dx} = -\frac{2x+3y}{3x}$

C3 He divided $\frac{dx}{dt}$ by $\frac{dy}{dt}$. Correctly $\frac{dy}{dx} = \frac{4-3t^2}{2t}$: gradient $\frac{1}{2}$ at $t = 1$

C4 She left out $\frac{dr}{dt}$: $\frac{dV}{dt} = 4\pi r^2 \times \frac{dr}{dt} = 36\pi \times 0.1$: $3.6\pi \text{ cm}^3 \text{ s}^{-1}$



STRETCH

C5 He used the tangent gradient $\frac{2}{3}$ (the normal needs $-\frac{3}{2}$), and an equation needs “= 0”

$$y - 3 = -\frac{3}{2}(x - 2): \boxed{3x + 2y - 12 = 0}$$

C6 A stationary point needs the numerator to be 0: $x - 2y = 0$, used with the curve's equation.

points with $2x - y = 0$ have vertical tangents (the gradient is undefined there)

C7 $x8^{x-1}$ is the rule for x^n , not a^x : $\frac{d}{dx}(8^x) = 8^x \ln 8$

$$\text{At } x = \frac{1}{3}: 2 \ln 8: \boxed{6 \ln 2}$$

C8 (a) $\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3p}{2}$ at (p^2, p^3) : $y - p^3 = \frac{3p}{2}(x - p^2)$, so $\boxed{2y = 3px - p^3}$

(b) Substitute (s^2, s^3) : $2s^3 - 3ps^2 + p^3 = 0$, which is $(s - p)^2(2s + p) = 0$

$s = p$ is the point of contact (a repeated root), so the tangent meets C again at $\boxed{s = -\frac{p}{2}}$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $3x^2 + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{3x^2 + y}{x + 2y}$

$$\text{At } P(1, 3): -\frac{3+3}{1+6}: \boxed{-\frac{6}{7} \text{ as required}}$$

D2 (a) $\frac{dy}{dx} = \frac{3 \cos t}{-4 \sin t} = -\frac{3}{4}$ at $t = \frac{\pi}{4}$; $P\left(2\sqrt{2}, \frac{3\sqrt{2}}{2}\right)$

$$y - \frac{3\sqrt{2}}{2} = -\frac{3}{4}(x - 2\sqrt{2}), \text{ so } 4y - 6\sqrt{2} = -3x + 6\sqrt{2}: \boxed{3x + 4y = 12\sqrt{2}}$$

(b) $A(4\sqrt{2}, 0), B(0, 3\sqrt{2})$: $\frac{1}{2} \times 4\sqrt{2} \times 3\sqrt{2}: \boxed{12}$



D3 $V = 6\pi h^2 - \frac{\pi h^3}{3}$, so $\frac{dV}{dh} = 12\pi h - \pi h^2 = 20\pi$ at $h = 2$

$$\frac{dh}{dt} = \frac{10\pi}{20\pi}: \boxed{\frac{1}{2} \text{ cm per second}}$$

D4 $4^x \ln 4 + 2y \frac{dy}{dx} = 5 \frac{dy}{dx}$, so $\frac{dy}{dx} = \frac{4^x \ln 4}{5 - 2y}$

At $(1, 1)$: $\frac{4 \ln 4}{3} = \frac{8 \ln 2}{3}$: $\boxed{\frac{8}{3} \ln 2 \text{ as required}}$

STRETCH

D5 (a) $\frac{dx}{dt} = \frac{1}{t+2}$, $\frac{dy}{dt} = 2t - 2$: $\boxed{\frac{dy}{dx} = 2(t-1)(t+2)}$

(b) $\frac{dy}{dx} = 0$ at $t = 1$ ($t = -2$ is excluded): $\boxed{M(\ln 3, -1)}$

For $-2 < t < 1$ the gradient is negative, for $t > 1$ positive, and x increases with t : $\boxed{\text{a minimum}}$

D6 (a) $2x - 2y - 2x \frac{dy}{dx} + 6y \frac{dy}{dx} = 0$: $\boxed{\frac{dy}{dx} = \frac{x-y}{x-3y}}$

(b) $\frac{x-y}{x-3y} = 1$ gives $y = 0$, then $x^2 = 8$: $\boxed{A(2\sqrt{2}, 0), B(-2\sqrt{2}, 0)}$

D7 (a) Cross-section area $\frac{1}{2} \times 2h \times h = h^2$, times length 3: $\boxed{V = 3h^2}$

(b) $\frac{dV}{dt} = 6h \frac{dh}{dt}$, so $0.06 = 1.2 \frac{dh}{dt}$: $\boxed{0.05 \text{ m per minute}}$

D8 (a) $A = \frac{1}{2}(4^2)(\theta - \sin \theta) = 8(\theta - \sin \theta)$: $\boxed{\frac{dA}{d\theta} = 8(1 - \cos \theta)}$

(b) $\frac{dA}{dt} = 8 \left(1 - \frac{1}{2}\right) \times 0.05$: $\boxed{0.2 \text{ cm}^2 \text{ per second}}$

(c) $0.4(1 - \cos \theta) = 0.6$, so $\cos \theta = -\frac{1}{2}$: $\boxed{\theta = \frac{2\pi}{3}}$



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $3y^2 \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 2x$

M1 B1: product rule on $2xy$

$$\frac{dy}{dx} = \frac{2x - 2y}{3y^2 + 2x}$$

A1

(b) At P: $\frac{dy}{dx} = \frac{2-4}{12+2} = -\frac{1}{7}$, so the normal gradient is 7

M1

$$y - 2 = 7(x - 1): \quad 7x - y - 5 = 0$$

A1

E2 (a) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3$

B1

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}, \text{ so } 9\pi = 9\pi \frac{dh}{dt}$$

M1

$$\frac{dh}{dt} = 1 \text{ cm per second}$$

A1

(b) $l = \sqrt{r^2 + h^2} = h\sqrt{2}$, so $S = \pi h \times h\sqrt{2} = \sqrt{2}\pi h^2$

B1

$$\frac{dS}{dt} = 2\sqrt{2}\pi h \frac{dh}{dt} = 2\sqrt{2}\pi(3)(1)$$

M1

$$6\sqrt{2}\pi \text{ cm}^2 \text{ per second}$$

A1

STRETCH

E3 (a) $\frac{dy}{dx} = \frac{2}{2t} = \frac{1}{t}$ at $(t^2, 2t)$

M1

$$y - 2t = \frac{1}{t}(x - t^2), \text{ so } ty - 2t^2 = x - t^2$$

dM1

$$ty = x + t^2$$

A1*

(b) $2y = x + 4$ and $-\frac{1}{2}y = x + \frac{1}{4}$ give $R\left(-1, \frac{3}{2}\right)$

M1 A1

gradients $\frac{1}{2}$ and -2 , product -1 : perpendicular

B1

(c) Perpendicular: $\frac{1}{p} \times \frac{1}{q} = -1$, so $pq = -1$; subtracting the tangents, $(p - q)y = p^2 - q^2$, $y = p + q$

M1

$$x = py - p^2 = pq: \quad x = -1, \text{ so } R \text{ lies on } x = -1$$

A1

