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A-LEVEL MATHEMATICS • DECISION 1

Critical Path Analysis

Worksheet

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Free Critical Path Analysis worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 7

D **Problem** solving page 8

E **Exam** practice page 13

Tap a section to jump straight to it

By the end of this worksheet you can: draw activity networks with dummies • early and late event times, critical paths and floats • *Stretch:* cascade charts, scheduling and lower bounds on workers.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Planning projects: activity networks with dummies, event times, floats, cascade charts and the workers needed.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: early event times

A (3) starts the project; B (4) and C (2) follow A; D (5) follows B and C. Find the earliest finish of D.

A ends at 3; B ends at 7, C at 5; D starts at 7

12

Your turn A1

P (5) starts the project; Q (2) and R (6) follow P; S (3) follows Q and R. Find the earliest finish of S.

Worked example A2: late event times

The project ends at 20. Activities X (4) and Y (6) both end at the final event and start at the same event. Find that event's late time.

Take the smallest of $20 - 4$ *and* $20 - 6$

14

Your turn A2

The project ends at 25. Activities U (3) and V (8) both end at the final event and start at the same event. Find that event's late time.

Worked example A3: total float

An activity of duration 5 starts at an event with early time 6 and ends at an event with late time 14. Find its total float.

$14 - 6 - 5 = 3$

Your turn A3

An activity of duration 7 starts at an event with early time 9 and ends at an event with late time 20. Find its total float.

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Worked example A4: the critical path

Activities with zero float are A, D, G and J, in that order. State the critical path, given durations 3, 5, 4, 6.

A–D–G–J, 18

Your turn A4

Activities with zero float are B, E and K, in that order, with durations 7, 9 and 5. State the critical path and its length.

Worked example A5: a lower bound for workers

The durations total 59 days and the critical path is 22 days. Find a lower bound for the workers.

$\frac{59}{22} = 2.68\ldots$, *round up*

3

Your turn A5

The durations total 75 days and the critical path is 24 days. Find a lower bound for the workers.

Worked example A6: a dummy

C follows A and B, but D follows A only. Explain the dummy needed.

logic dummy: end of A to start of C

Your turn A6

Activities E and F both follow G and precede H. Explain the dummy.

Worked example A7: must be in progress

An activity has earliest start 4, latest finish 10 and duration 5. When must it be in progress?

It must start by $10 - 5 = 5$ *and cannot finish before* $4 + 5 = 9$

$5 < t < 9$

Your turn A7

An activity has earliest start 2, latest finish 9 and duration 4. When must it be in progress?

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 Figure 1 shows an activity network; durations are in days.

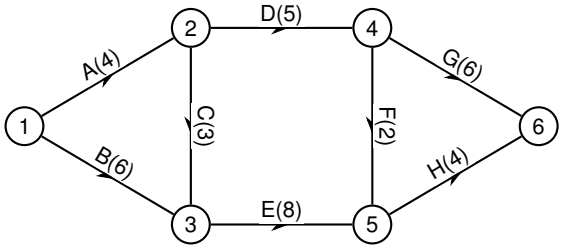


Figure 1

(a) Find the early event times.

(b) State the minimum completion time.

(c) Find the late event times.

(d) State the events where early and late times differ.

[4 marks]

B2 For the network in Figure 1,

(a) find the total float of each activity

(b) state the critical activities

(c) state the critical path

(d) explain why D is not critical.

[4 marks]

B3 Figure 2 shows an activity network with one dummy.

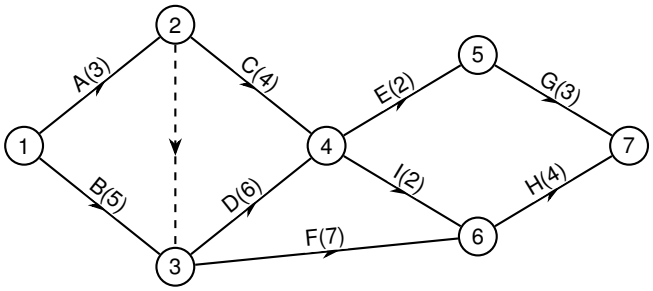


Figure 2

(a) State the activities that immediately precede C

(b) State those that immediately precede D

(c) Explain the purpose of the dummy.

(d) State those that immediately precede H

[4 marks]

B4 The table shows the precedences for a project.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A, B
E	C
F	D, G
G	C

(a) State which activities can start at the beginning.

(b) Explain why a dummy is needed.

(c) Draw the activity network.

(d) State the activities that end at the final event.

[4 marks]

B5 For the network in Figure 1, activity G

(a) has earliest start

(b) has latest start

(c) and E must both be in progress during $13 < t < 14$. Explain why. **[3 marks]**

B6 For the network in Figure 1, each activity needs one worker.

(a) Find the total of the durations.

(b) Find a lower bound for the workers.

(c) Explain why the answer is rounded up.

(d) State whether more workers could ever be needed.

[4 marks]

STRETCH

B7 For the table in B4,

(a) state the type of dummy used

(b) explain why D needs it but C does not

(c) state the number of dummies needed

(d) explain what a uniqueness dummy is.

[4 marks]

B8 For the network in Figure 1,

(a) state the float of D

(b) give the completion time if D takes 3 days longer

(c) give it if D takes 6 days longer

(d) state the new critical activities.

[4 marks]

B9 For the network in Figure 1, two workers are available.

(a) State which activities start at time 0

(b) Give the time C can start.

(c) Give a schedule for both workers.

(d) State the completion time.

[4 marks]

B10 A new activity K, of duration k days, must follow D and precede H in Figure 1.

(a) State its earliest start.

(b) Find its float when $k = 5$

(c) Find the greatest k that causes no delay.

(d) Find the completion time when $k = 9$

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 In Figure 1, Amir gives the late time of event 4 as 15, the larger of $19 - 6$ and $15 - 2$. Explain his error.

[2 marks]

C2 Bea says E is on the critical path in Figure 1 "because it is the longest activity". Explain why her reason is wrong.

[2 marks]

C3 Two activities, P and Q, both follow M and both precede N. Cal draws them from the same start event to the same end event. Explain his error and how to correct it.

[3 marks]

C4 Dev draws a cascade chart for Figure 1 showing only the bars for the activities. Explain what is missing and why it matters.

[3 marks]

STRETCH

C5 Asked which activities must be in progress during day 14 of Figure 1, Eve answers for $14 < t < 15$. Explain her error.

[3 marks]

C6 A project has durations totalling 50 days and a critical path of 16 days. Fay says 3 workers is a lower bound. Explain her error.

[3 marks]

C7 Gus says that delaying D in Figure 1 by its total float of 4 days delays the whole project. Explain why he is wrong.

[3 marks]

C8 **(a)** Explain why the total of the durations divided by the critical path length, rounded up, is a lower bound for the number of workers.

(2)

(b) Explain why every activity on the critical path has zero total float, and why the critical path is the longest path.

(3)

[5 marks]

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D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The table shows the precedences for a project.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	A, B, C
F	C
G	D
H	E, F
I	D
J	G, H

Draw the activity network, using activity on arc and exactly two dummies.

[3 marks]

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D2 Figure 3 shows the activity network for a project. The durations are in days.

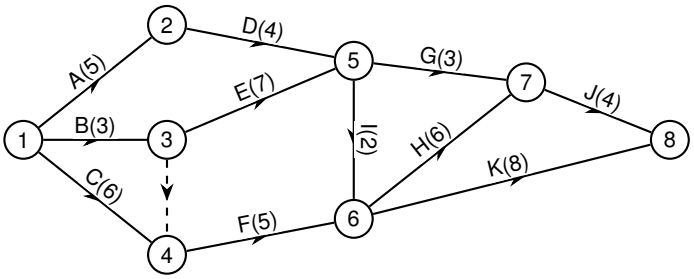


Figure 3

(a) Calculate the early event times and the late event times.

(3)

(b) State the minimum completion time of the project.

(1)

[4 marks]

D3 Figure 4 shows an activity network with the early and late event times. Durations are in hours and each activity needs one worker.

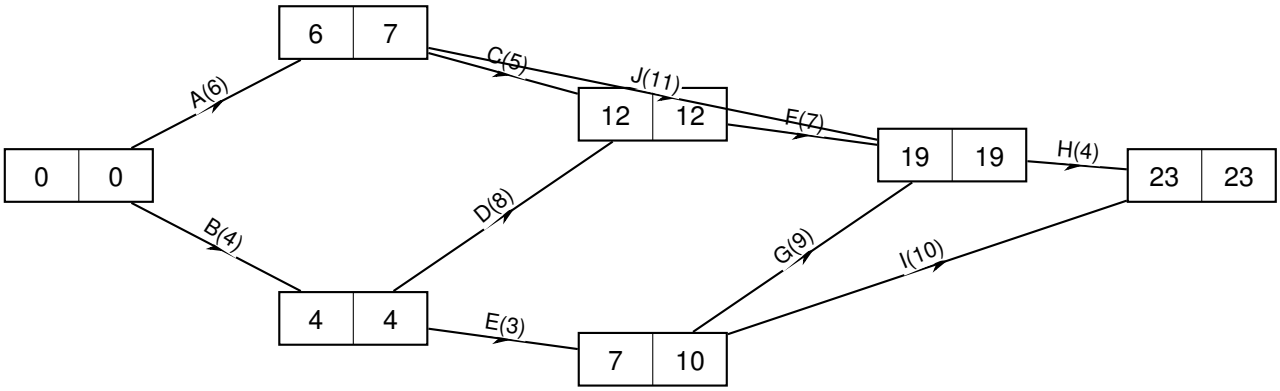


Figure 4

(a) Calculate the total float on activity G, making the numbers you use clear.

(1)

(b) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

[3 marks]

D4 Figure 5 shows an activity network; durations are in days. Each activity needs one worker and the project is completed in the minimum time.

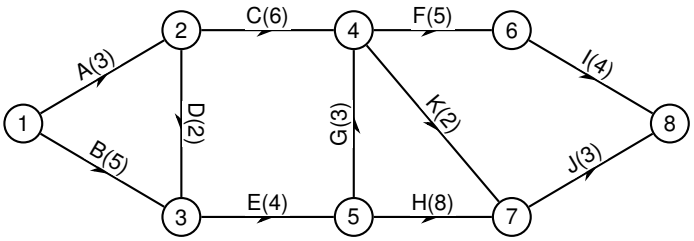


Figure 5

Determine which activities must be in progress during $10 < t < 11$, giving reasons, and hence state the least number of workers needed at that time. **[3 marks]**

STRETCH

D5 Figure 6 shows a cascade chart for a project, with the floats shown dashed. Time is in days.

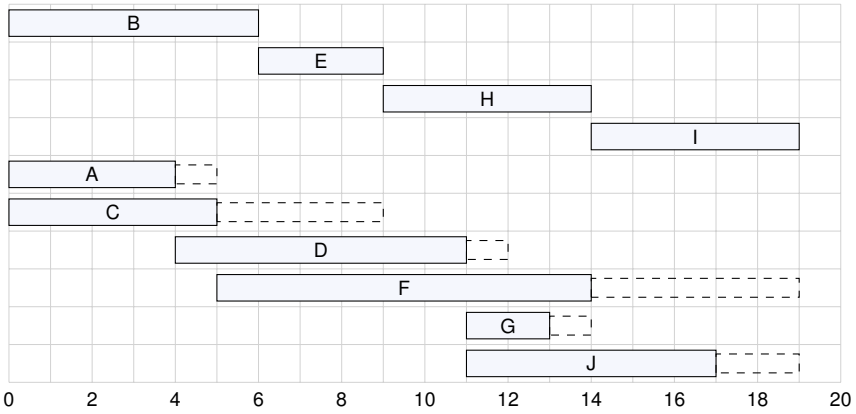


Figure 6

(a) State the minimum completion time and the critical activities.

(2)

(b) State the total floats of D and J, and the latest time at which D could start without delaying the project.

(2)

[4 marks]

D6 Figure 7 shows an activity network with two dummies.

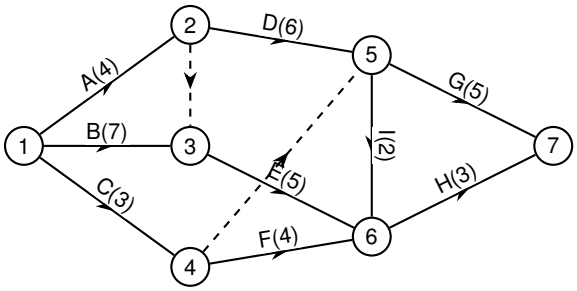


Figure 7

(a) Write down the immediately preceding activities for D, E, G, H and I

(3)

(b) Explain why each dummy is needed.

(1)

[4 marks]

D7 Figure 8 shows an activity network; durations are in days. Each activity needs one worker.

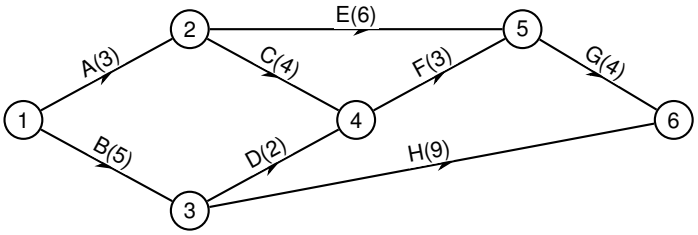


Figure 8

(a) Calculate a lower bound for the number of workers needed to finish in the minimum time.

(1)

(b) Show that the project cannot be finished in the minimum time with this many workers.

(3)

[4 marks]

D8 The table shows the precedences and durations, in hours, of the activities in a project. The duration of D is x hours.

Activity	Immediately preceding activities	Duration
A	–	8
B	–	5
C	–	6
D	A, B	x
E	B	9
F	C, D	4
G	E	3
H	E	7
I	F, G	5
J	C, D	6

(a) Draw the activity network, using activity on arc and the minimum number of dummies.

(3)

(b) Given that A–D–F–I is the only critical path, determine the range of possible values of x

(2)

[5 marks]

E Exam practice

Exam-style questions in the style of WDM11. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A project is modelled by the activity network in Figure 9. The activities are represented by the arcs, and the number in brackets on each arc is the time, in days, to complete the activity. Each activity requires exactly one worker, and the project is to be completed in the shortest possible time. The manager wants to know which activities can be delayed, and how many workers must be hired.

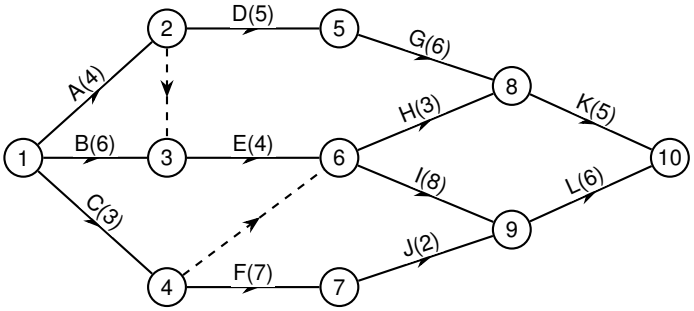


Figure 9

(a) Complete the diagram on the answer-book page to show the early event times and the late event times.

(4)

(b) Calculate the total float on activity D. You must make the numbers used in your calculation clear, and explain what this float means for the manager.

(1)

(c) Draw a cascade (Gantt) chart for the project on the grid on the answer-book page.

(3)

(d) Calculate a lower bound for the number of workers, and use your cascade chart to determine the minimum number of workers needed. You must make specific reference to times and activities.

(2)

(Total for Question E1 is 10 marks)

Answer book: Question E1

Work in the space on this page, as you would in the WDM11 answer book.

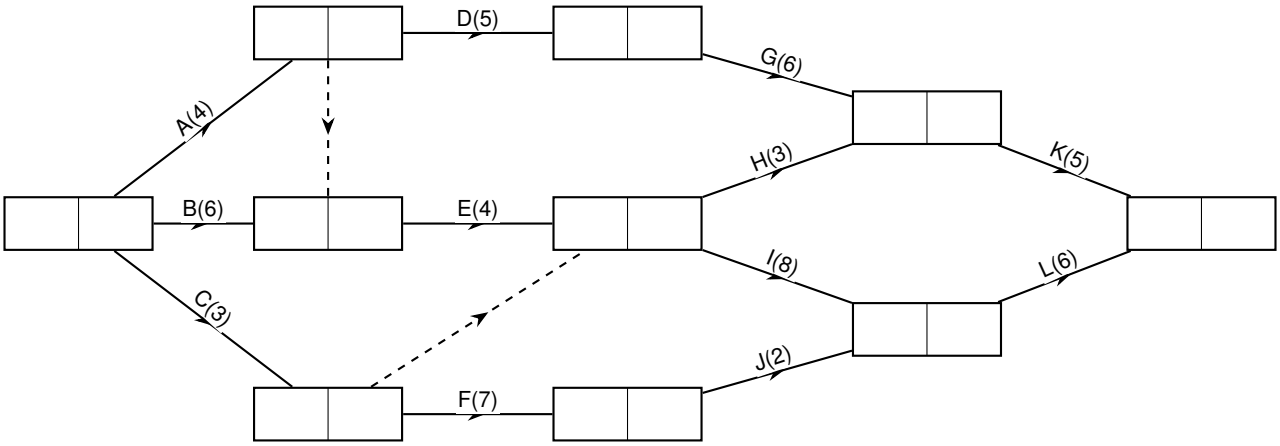
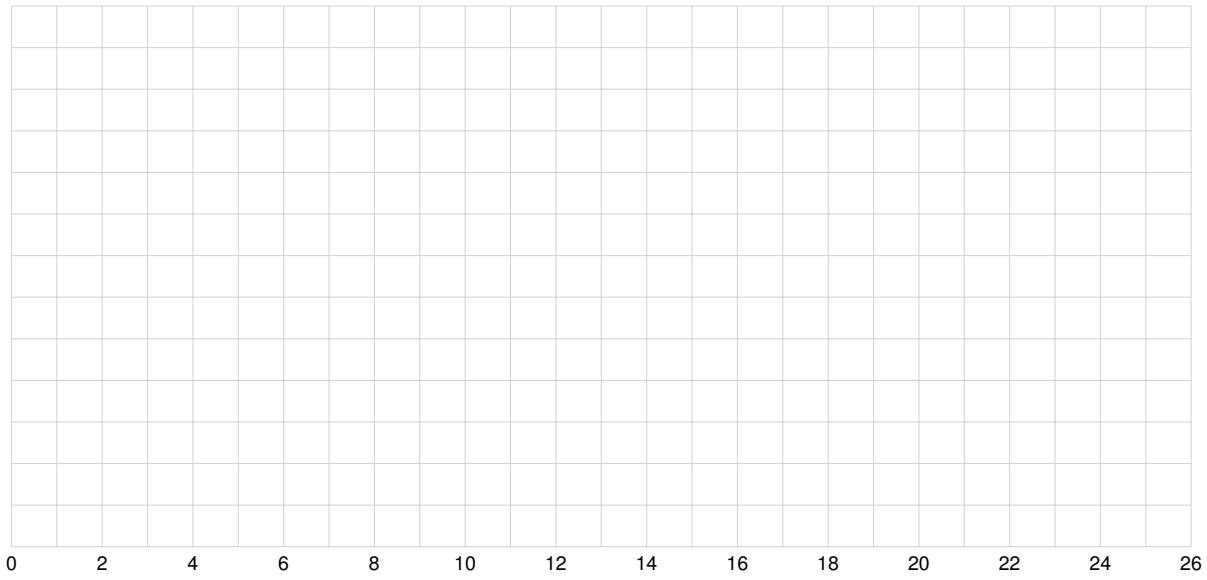


Diagram 1



Grid 1

E2 Figure 10 shows the activity network for a building project. The number in brackets on each arc is the duration, in weeks, of the activity. Each activity requires one team, and the project must be completed in the shortest possible time. The builder wants to employ as few teams as possible without delaying the project.

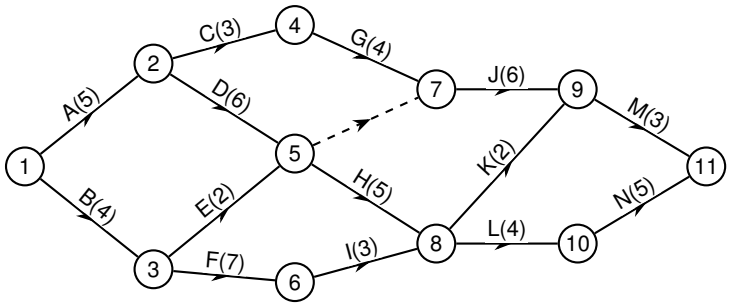


Figure 10

(a) Write down the immediately preceding activities for each of the activities H, J, K and L

(2)

(b) Complete the diagram on the answer-book page to show the early and late event times.

(3)

(c) State the critical activities, and hence the minimum completion time of the project.

(1)

(d) Calculate the total float on activity J

(1)

(e) Calculate a lower bound for the number of teams needed. You must show your working.

(2)

(f) Schedule the activities on the grid on the answer-book page, using the minimum number of teams, so that the project is completed in the minimum time.

(3)

(Total for Question E2 is 12 marks)

Answer book: Question E2

Work in the space on this page, as you would in the WDM11 answer book.

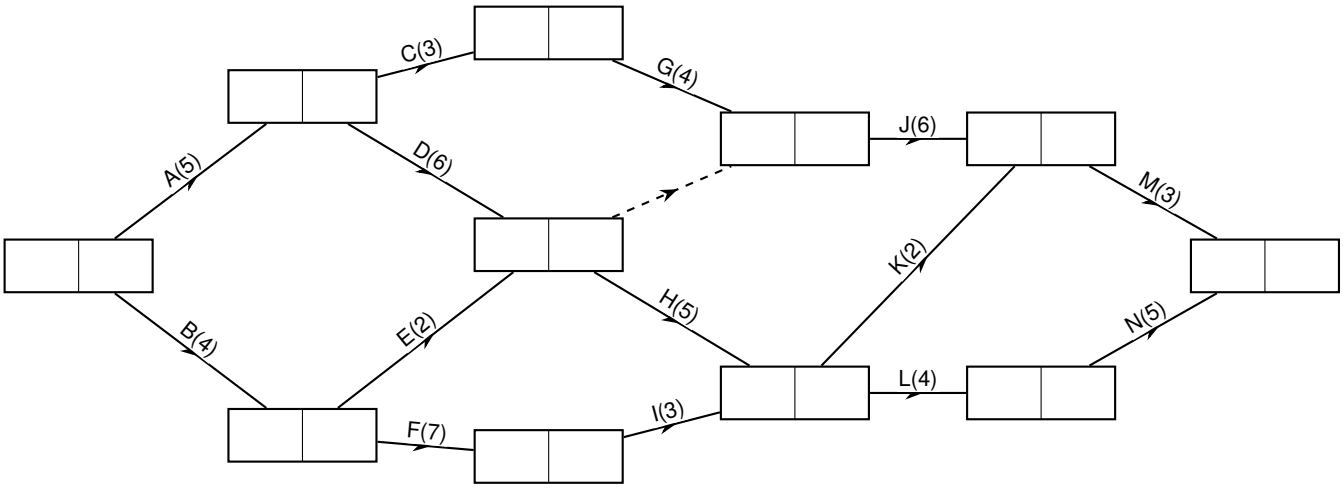
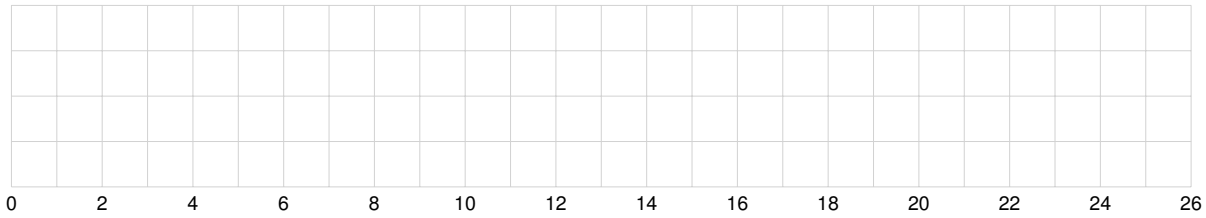


Diagram 2



Grid 2 (one row for each team)

STRETCH

E3 Figure 11 shows the activity network for a project. The duration of C is y days and the duration of H is x days; the other durations are shown in brackets. Some of the early and late event times are shown on the answer-book page. Each activity requires one worker, and the project is to be completed in the shortest possible time.

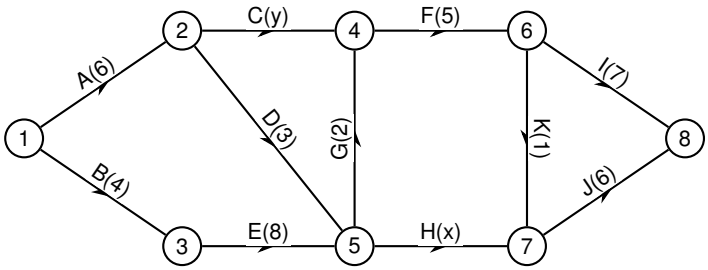


Figure 11

It is given that B–E–H–J is a critical path, and that the total float on activity C is 5 days.

(a) Determine the values of x and y, making your method clear.

(3)

(b) Complete the diagram on the answer-book page to show all the early and late event times.

(3)

(c) Draw a cascade chart for the project on the grid on the answer-book page.

(4)

(d) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities.

(4)

(Total for Question E3 is 14 marks)

Answer book: Question E3

Work in the space on this page, as you would in the WDM11 answer book.

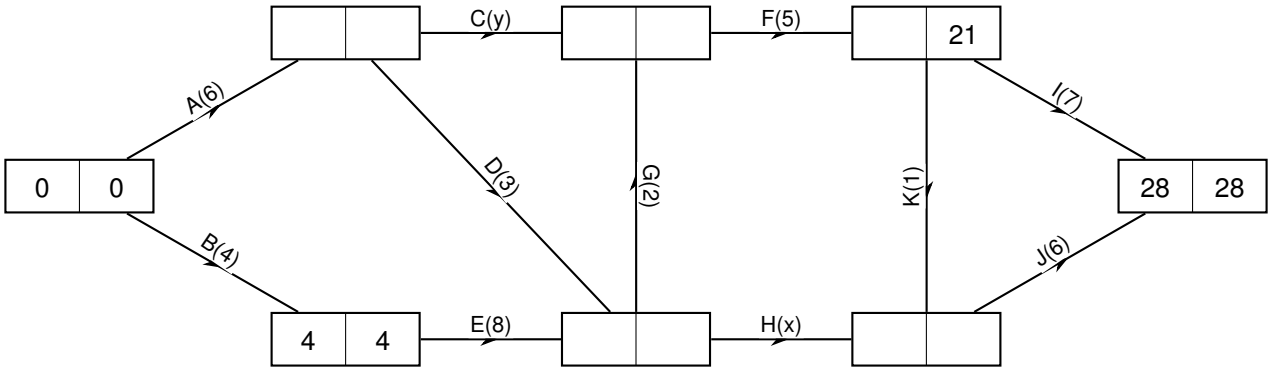
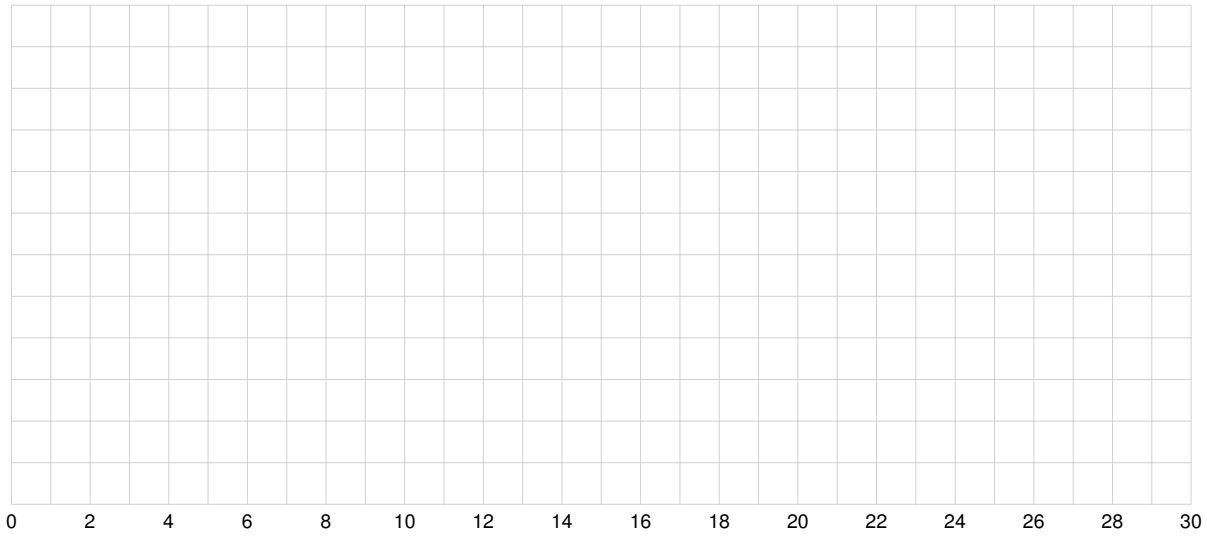


Diagram 3



Grid 3

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