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Vectors

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 4

Question 1 — Worked Solution

$$|\mathbf{a}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

Unit vector:

$$\hat{\mathbf{a}} = \frac{1}{\sqrt{14}}(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \frac{2\mathbf{i} + 3\mathbf{j} - \mathbf{k}}{\sqrt{14}}$$

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Question 2 — Worked Solution

$$\sqrt{\lambda^2 + 4 + 9} = \sqrt{17} \Rightarrow \lambda^2 = 4$$

$$\therefore \lambda = \pm 2$$

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Question 3 — Worked Solution

(a)

$$\overrightarrow{AB} = B - A = (4-1)\mathbf{i} + (-1-2)\mathbf{j} + (3-(-1))\mathbf{k} = 3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

(b)

$$|\overrightarrow{AB}| = \sqrt{9+9+16} = \sqrt{34}$$

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Question 4 — Worked Solution

$$\mathbf{m} = \frac{\mathbf{p}+\mathbf{q}}{2} = \frac{1}{2}[(3+1)\mathbf{i} + (-1+3)\mathbf{j} + (2-4)\mathbf{k}]$$

$$\therefore = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$$

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Question 5 — Worked Solution

$$\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

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Question 6 — Worked Solution

Direction vector: PQ = Q – P = (2, 2, –4)

$$\mathbf{r} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}$$

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Question 7 — Worked Solution

(a) t = 2

$$(1 + 2 \times 2,\ 2 + (-1) \times 2,\ 0 + 3 \times 2) = (5,\ 0,\ 6)$$

Point: (5, 0, 6)

(b) Is (7, –1, 9) on the line?

1+2t=7 ⇒ t=3. Check j: 2–3=–1 ✓. Check k: 3(3)=9 ✓

Yes — (7, –1, 9) lies on the line at t = 3.

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Question 8 — Worked Solution

Direction vectors: $\mathbf{d}_1$=(2,–1,4) and $\mathbf{d}_2$=(6,–3,12)

$$(6,\,-3,12) = 3\times(2,\,-1,4)\ \Rightarrow\ \mathbf{d}_2 = 3\mathbf{d}_1$$

Direction vectors are proportional (×3) ⇒ lines are parallel.

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Question 9 — Worked Solution

Step 1: Not parallel

$\mathbf{d}_1$=(1,1,0), $\mathbf{d}_2$=(1,0,1) — not proportional ✓

Step 2: Test for intersection

(1+s, s, 1) = (2+t, 1, t)

j: s=1 **k:** t=1

i: 1+1=2+1 ⇒ 2=3 — CONTRADICTION

Not parallel and do not intersect ⇒ lines are skew. ■

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Question 10 — Worked Solution

(2+s, 3–s, –1+2s) = (4–t, 1+t, 1)

k: –1+2s=1 ⇒ s=1. **j:** 2=1+t ⇒ t=1. **i:** 3=3 ✓

$$\text{Point: } (2+1,\ 3-1,\ -1+2) = (3,\ 2,\ 1)$$

Intersection at (3, 2, 1)

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Question 11 — Worked Solution

$(1+2s, 1+s, 2-s) = (3+t, 4-t, 1+2t)$

i: $2s-t=2$ ...(1) **j:** $s+t=3$ ...(2) **k:** $s+2t=1$ ...(3)

(2)–(3): $-t=2 \Rightarrow t=-2, s=5$

Check **i**: $1+10=11$, $3+(-2)=1 \Rightarrow 11 \neq 1$ — CONTRADICTION

Lines are skew.

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Question 12 — Worked Solution

$$\mathbf{a} \cdot \mathbf{b} = (3)(1) + (-2)(4) + (1)(-3) = 3 - 8 - 3$$

$$\therefore = -8$$

Question 13 — Worked Solution

$$\mathbf{a} \cdot \mathbf{b} = (3)(2) + (-1)(4) + (2)(1) = 6 - 4 + 2 = 0$$

Since a·b = 0, the vectors are perpendicular. ✓

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Question 14 — Worked Solution

$$\mathbf{a} \cdot \mathbf{b} = 2\lambda + 3(-1) + (-2)(1) = 2\lambda - 5 = 0$$

$$\therefore \lambda = \frac{5}{2}$$

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Question 15 — Worked Solution

$$\mathbf{a} \cdot \mathbf{b} = 2 - 2 - 4 = -4$$

$$|\mathbf{a}| = 3, \quad |\mathbf{b}| = 3$$

$$\cos\theta = \frac{-4}{9} \Rightarrow \theta = \arccos\left(-\frac{4}{9}\right)$$

$$\therefore \approx 116.4°$$

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Question 16 — Worked Solution

$\mathbf{d}_1$=(1,2,–1), $\mathbf{d}_2$=(2,–1,2)

$$\mathbf{d}_1 \cdot \mathbf{d}_2 = 2 - 2 - 2 = -2$$

$$|\mathbf{d}_1| = \sqrt{6}, \quad |\mathbf{d}_2| = 3$$

$$\cos\theta = \frac{|-2|}{3\sqrt{6}} = \frac{2}{3\sqrt{6}}$$

$$\therefore \theta \approx 74.21°$$

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Question 17 — Worked Solution

$$\mathbf{d}_1 \cdot \mathbf{d}_2 = 2 - 1 + 2 = 3$$

$$|\mathbf{d}_1| = \sqrt{6}, \quad |\mathbf{d}_2| = \sqrt{6}$$

$$\cos\theta = \frac{3}{\sqrt{6} \times \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \theta = 60°$$

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Question 18 — Worked Solution

$$\overrightarrow{BA} = (-2, -2, 2), \quad \overrightarrow{BC} = (2, -2, 2)$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = -4 + 4 + 4 = 4$$

$$|\overrightarrow{BA}| = |\overrightarrow{BC}| = 2\sqrt{3}$$

$$\cos(\angle ABC) = \frac{4}{12} = \frac{1}{3}$$

$$\therefore \angle ABC = \arccos\left(\frac{1}{3}\right) \approx 70.53°$$

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Question 19 — Worked Solution

(a) Intersection

$(1+s, 2+s, -s) = (2+2t, 3-t, -1+t)$

i: $s-2t=1$ ...(1) **j:** $s+t=1$ ...(2) **k:** $s+t=1$ (same as (2))

(1)–(2): $-3t=0 \Rightarrow t=0, s=1$

l_1 at $s=1$: $(2,3,-1)$. l_2 at $t=0$: $(2,3,-1)$ ✓

Intersection at (2, 3, –1)

(b) Angle

$$\mathbf{d}_1 \cdot \mathbf{d}_2 = (1)(2) + (1)(-1) + (-1)(1) = 2 - 1 - 1 = 0$$

$$\therefore \theta = 90° \quad \text{(lines are perpendicular)}$$