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A-LEVEL MATHEMATICS • STATISTICS 2

Binomial Distributions

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Binomial Distributions worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 2 Binomial Distributions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Statistics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\binom{12}{3}(0.25)^3(0.75)^9 = 220(0.015625)(0.075085)$: **0.2581**

A2 $1 - \mathrm{P}(X \leqslant 6) = 1 - 0.7800$: **0.2200**; $\mathrm{P}(X \leqslant 7) - \mathrm{P}(X \leqslant 3) = 0.8909 - 0.2340$: **0.6569**

A3 $np = 12$, $np(1-p) = 7.2$; $\mathrm{P}(X \leqslant 11)$: **12, 7.2, 0.4311**

STRETCH

A4 Failures $Y \sim \mathrm{B}(15, 0.1)$: $\mathrm{P}(X \leqslant 12) = \mathrm{P}(Y \geqslant 3) = 1 - 0.8159$: **0.1841**

A5 $1 - p = \dfrac{2.88}{4.8} = 0.6$, $p = 0.4$, $n = \dfrac{4.8}{0.4}$: $n = 12,\ p = 0.4$

A6 $1 - 0.85^n > 0.99$, $n > \dfrac{\ln 0.01}{\ln 0.85} = 28.3$: $n = 29$

A7 $\dfrac{\mathrm{P}(X \leqslant 2)}{\mathrm{P}(X \leqslant 5)} = \dfrac{0.1268}{0.7216}$: **0.1757**

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** **0.4032** **(b)** $\mathrm{P}(X \leqslant 4)$: **0.2173** **(c)** $1 - \mathrm{P}(X \leqslant 4)$: **0.7827** **(d)** $1 - \mathrm{P}(X \leqslant 5)$: **0.5968**

Rewrite every inequality as "$\leqslant$" before using the tables.

B2 **(a)** **0.1488** **(b)** **0.3115** **(c)** **0.1094** **(d)** **0.0038**

$28p^2(1-p)^6$ *in each case; (d) equals* $\mathrm{P}(X = 6)$ *for* $p = 0.25$

B3 **(a)** $0.7507 - 0.0553$: **0.6954** **(b)** $0.7507 - 0.1299$: **0.6208**

(c) $0.5914 - 0.0553$: **0.5361** **(d)** $0.5914 - 0.1299$: **0.4615**

Upper end: $P(X \leqslant 10)$ *or* $P(X \leqslant 9)$. *Lower end: subtract* $P(X \leqslant 5)$ *or* $P(X \leqslant 6)$

B4 **(a)** **10 and 8** **(b)** $0.5n = 12$: $n = 24$ **(c)** $30p(1-p) = 6.3$, $p^2 - p + 0.21 = 0$: $p = 0.3$ **or** 0.7

(d) $36p(1-p) = 9$, $(2p-1)^2 = 0$: $p = 0.5$

The variance alone gives two values of p *unless one is ruled out.*

B5 **(a)** Fixed number of independent tosses, constant probability: **yes,** B(10, 0.5)

(b) **No**: dealt without replacement, so the trials are not independent and the probability changes

(c) **No**: the number of trials is not fixed

A binomial model needs a fixed n*, independent trials, two outcomes and a constant* p

B6 **(a)** $P(X \leqslant 4) = 0.0905$, $P(X \leqslant 5) = 0.1935$: $a = 4$ **(b)** $1 - 0.9558 = 0.0442$: $b = 12$

(c) $P(X \leqslant 10) = 0.9022$: $c = 10$ **(d)** $1 - 0.8106 = 0.1894$: $d = 9$

Read along the column until the probability crosses the required value.

STRETCH

B7 Late trains $Y \sim B(20, 0.2)$

(a) 0.8^{20}: **0.0115** **(b)** $P(Y \leqslant 2)$: **0.2061** **(c)** $P(Y \geqslant 7) = 1 - 0.9133$: **0.0867** **(d)** $P(3 \leqslant Y \leqslant 6)$: **0.7072**

"At least 18 on time" is "at most 2 late".

B8 **(a)** $0.8^n < 0.01$, $n > 20.6$: **21** **(b)** $(1-p)^2 = 0.36$: $p = 0.4$ **(c)** $1 - p = 0.8$: $n = 16$**,** $p = 0.2$

(d) $p^5 = 5p^4(1-p)$, $p = 5 - 5p$: $p = \frac{5}{6}$

Write each condition as an equation in the unknown, then solve.

B9 **(a)** $1 - 0.4925$: **0.5075** **(b)** $\frac{0.2311}{0.5075}$: **0.455** **(c)** $2(0.5075)(0.4925)$: **0.500** **(d)** $10(0.50748)$: **5.07**

A binomial probability becomes the p *of a new binomial model.*

B10 **(a)** $3(7) - 5$: **16** **(b)** $9(4.55)$: **40.95** **(c)** $P(X > 7) = 1 - 0.6010$: **0.3990**

(d) $\sigma_Y = 6.399$; $3X - 5 \leqslant 9.60$ gives $X \leqslant 4$: **0.1182**

Translate a statement about Y *into one about* X *before using the tables.*

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $X \geqslant 8$ is the complement of $X \leqslant 7$, not $X \leqslant 8$: $1 - 0.7723$: **0.2277**

C2 A distribution must be stated with its parameters: B(30, 0.15)

C3 Count the failures: $Y \sim$ B(15, 0.3), and at least 12 pass means at most 3 fail

P($Y \leqslant 3$): **0.2969**

C4 The variance is the square of the standard deviation: $np(1-p) = 4$

$1 - p = \frac{4}{8}$: $p = 0.5, n = 16$

STRETCH

C5 A conditional probability divides by the probability of the condition

$\frac{\text{P}(X = 3)}{\text{P}(X \geqslant 2)} = \frac{0.2503}{1 - 0.2440}$: **0.3311**

C6 The equation has two roots, 0.3 and 0.7, and the question says $p > 0.5$: $p = 0.7$

C7 The cards are dealt without replacement, so the trials are not independent and the probability of red changes from card to card.

Replace and shuffle each card before the next is dealt: then B(5, 0.5) is valid.

C8 **(a)** $(1-p)^n = np(1-p)^{n-1}$; divide by $(1-p)^{n-1}$: $1 - p = np$: $p = \frac{1}{n+1}$

(b) $np(1-p) = \frac{n}{n+1} \times \frac{n}{n+1} = 0.81$, so $\frac{n}{n+1} = 0.9$: $n = 9$, $p = 0.1$

$1 - 0.9^9 - 9(0.1)(0.9)^8$: **0.2252**

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $X \sim \text{B}(25, 0.15)$: mean 3.75, $\sigma = \sqrt{3.1875} \approx 1.785$; $1.96 < X < 5.54$ gives $2 \leqslant X \leqslant 5$

$\text{P}(X \leqslant 5) - \text{P}(X \leqslant 1) = 0.8385 - 0.0931$: **0.7454**

D2 **(a)** $X \sim \text{B}(20, 0.25)$, score $S = 3X - (20 - X) = 4X - 20$; $\text{E}(S) = 4(5) - 20 = 0$, as required

(b) $4X - 20 \geqslant 10$ gives $X \geqslant 8$: $1 - 0.8982$: **0.1018**

D3 Failed seeds $Y \sim \text{B}(10, 0.15)$: a tray fails if $Y \geqslant 3$, $1 - 0.8202 = 0.1798$

Trays $T \sim \text{B}(6, 0.1798)$: $\binom{6}{2}(0.1798)^2(0.8202)^4$: **0.2195**

D4 **(a)** $1 - p = \dfrac{1.8^2}{3.6} = 0.9$: $p = 0.1, n = 36$

(b) 0.9^{36}: **0.0225**

STRETCH

D5 $\text{P}(X \geqslant 2) = 1 - 0.95^n - 0.05n(0.95)^{n-1} > 0.9$

$n = 76$: 0.8986; $n = 77$: 0.9027: $n = 77$

D6 **(a)** $\text{P}(X \leqslant k) = 0.9301$: from the tables $k = 18$

(b) $\dfrac{\text{P}(X > 18)}{\text{P}(X > 12)} = \dfrac{0.0699}{1 - 0.3143}$: **0.102**

D7 **(a)** $1 - (1 - p)^2 = 0.36$, $1 - p = 0.8$: $p = 0.2$

(b) $X \sim \text{B}(20, 0.2)$: $1 - \text{P}(X \leqslant 5) = 1 - 0.8042$: **0.1958**

D8 **(a)** 0.9^{10}: **0.3487**

(b) $\text{P}(X = 1) = 10(0.1)(0.9)^9 \approx 0.3874$; $0.3487 + 0.3874 \times 0.3487$: **0.4838**

(c) $10 + 10 \times 0.3874$: **13.9**

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E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $X \sim \text{B}(30, 0.35)$ *B1*

Customers pay by card independently of each other (with constant probability 0.35) *B1*

(b) $\binom{30}{10}(0.35)^{10}(0.65)^{20}$ *M1*

0.1502 *A1*

(c) $1 - \text{P}(X \leqslant 12) = 1 - 0.7802$ *M1*

0.2198 *A1*

(d) $\text{P}(X \leqslant 14) - \text{P}(X \leqslant 7) = 0.9348 - 0.1238$ *M1*

0.8110 *A1*

(e) $\text{P}(X < r) = \text{P}(X \leqslant r - 1)$ *M1*

$\text{P}(X \leqslant 5) = 0.0233$, $\text{P}(X \leqslant 6) = 0.0586$, so $r - 1 = 5$ *A1*

$r = 6$ *A1*

E2 **(a)** $np = 3$, $np(1-p) = 2.7$ *M1*

$1 - p = 0.9$, so $p = 0.1$ *A1*

$n = 30$, $p = 0.1$ *A1*

(b) **0.4114** *B1*

(c) $\dfrac{\text{P}(X = 4)}{1 - \text{P}(X \leqslant 1)}$ *M1*

$\dfrac{0.1771}{1 - 0.1837}$ *A1*

0.2170 *A1*

(d) At least 27 good means at most 3 faulty: $\text{P}(X \leqslant 3)$ *M1*

0.6474 *A1*

(e) $\text{P}(X > 5) = 1 - 0.9268$; $1 - 0.9268^5$ *M1*

0.3162 *A1*

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STRETCH

E3 **(a)** $(1-p)^4 = 0.0256$ *M1*

$1-p = 0.4$ *A1*

$\boxed{p = 0.6}$ *A1**

(b) Unopened $Y \sim B(25, 0.4)$; more than 18 opened means at most 6 unopened *M1 M1*

$P(Y \leqslant 6)$: $\boxed{\mathbf{0.0736}}$ *A1*

(c) $1 - 0.4^n > 0.999$ *M1*

$n > \dfrac{\ln 0.001}{\ln 0.4} = 7.54$ *dM1*

$\boxed{n = 8}$ *A1*

(d) P(at most 3 unopened in 10) = $P(Y \leqslant 3) = 0.3823$, as required *B1*

$\binom{12}{5}(0.3823)^5(0.6177)^7$ *M1*

$\boxed{\mathbf{0.2219}}$ *A1*

(e) $0.6177^m < 0.01$, $m > 9.56$ *M1*

$\boxed{m = 10}$ *A1*

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