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GCSE MATHEMATICS • ALGEBRA

Proof

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Full worked solutions for the free Proof worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Proof worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution. A different correct counterexample or method also earns the marks.

FOUNDATION

A1 **9** is odd and greater than 1, but $9 = 3 \times 3$, so it is not prime.

A2 $n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5(n + 2)$, which is 5 times a whole number: a multiple of 5

A3 $6n + 4 = 2(3n + 2)$, which is 2 times a whole number, so it is even.

HIGHER

A4 $(2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$, which is 2 times a whole number, so it is **even**

A5 $4n^2 + 12n + 9 - (4n^2 - 4n + 1) = 16n + 8 = \mathbf{8(2n + 1)}$, a multiple of 8

A6 $x^2 + 3x - 10 - (x^2 + 2x - 3) = \mathbf{x - 7}$ **as required**

A7 $2n(2n + 2) = 4n(n + 1)$. One of n and $n + 1$ is even, so $n(n + 1)$ is a multiple of 2 and $4n(n + 1)$ is a multiple of 8

B**Fluency solutions**

One mark for each correct part. Other correct counterexamples are fine. The notes in italics explain why each sequence was chosen.

FOUNDATION

B1 (a) **2** (b) **6** (c) **3**: half of 3 is 1.5 (d) $\mathbf{1 \times 1 = 1}$, but $1 + 1 = 2$ *One counterexample disproves a statement.*

B2 (a) $\mathbf{n + 1}$ (b) $\mathbf{2n}$ (c) $\mathbf{2n + 2}$ (d) $\mathbf{2n + 1}$

B3 (a) **always** (b) **never**: it is always even (c) **sometimes**: 3 is odd, 6 is even (d) **always**



- B4** (a) $2(n+2)$ (b) $3(n+2)$ (c) $5n+10-10=5n$ (d) $2(2n+1)$ is even; $4n+2$ is 2 more than a multiple of 4, so it is never a multiple of 4

Factorise to show the multiple: write the expression as the number times a whole number.

- B5** (a) $n+(n+1)+(n+2)=3n+3=3(n+1)$ (b) $n+(n+1)=2n+1$, which is odd
(c) $2n+(2n+2)=4n+2$, which is 2 more than a multiple of 4 Stopping at $3n+3$ is not enough: factorise.

- B6** (a) **yes**: $3x-6+6=3x$ (b) **no**: when $x=1$, $16 \neq 10$ (c) **yes**: $x^2-2x+1=(x-1)^2$ (d) **yes**

HIGHER

- B7** (a) $2m \times n = 2(mn)$, even (b) $(2n+1)^2 = 4n^2+4n+1 = 2(2n^2+2n)+1$, odd
(c) $(2m+1)-2n = 2(m-n)+1$, odd (d) $(2n)^2 = 4n^2$, a multiple of 4
- B8** (a) $4n+4 = 4(n+1)$ (b) $4n+12 = 4(n+3)$ (c) $(9n^2+6n+1)-(9n^2-6n+1) = 12n$
(d) $n^3+3n^2+3n+1-n^3-1 = 3n^2+3n = 3n(n+1)$, and $n(n+1)$ is even, so it is a multiple of 6
- B9** (a) $x^2+4x+4-4 = x^2+4x = x(x+4)$ (b) $(x+1)(x^2+2x+1) = x^3+3x^2+3x+1$
(c) $a^2+2ab+b^2-(a^2-2ab+b^2) = 4ab$ (d) $x^4+2x^2+1-(x^4-2x^2+1) = 4x^2$
- B10** (a) **true**: $n^2+n = n(n+1)$ is even, so n^2+n+1 is odd (b) **false**: $n=3$ gives $2^3+1=9$
(c) **true**: $(2n+1)+(2n+3)+(2n+5) = 6n+9 = 3(2n+3)$ (d) **false**: $n=2$ gives 6

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

- C1** One example only shows it works once. A proof must show it works for every pair of odd numbers, using algebra.
- C2** n and m could be any two integers. Consecutive integers must be written as n and $n+1$
- C3** Of any two consecutive whole numbers, one is even. Any whole number multiplied by an even number is even.



C4 $\frac{n + (n+1) + (n+2)}{3} = \frac{3n+3}{3} = n+1$, which is the middle number

HIGHER

C5 Using $2n+1$ twice means both odd numbers are the same. He must use different letters, $2m+1$ and $2n+1$

C6 $n^2 - 1 = (n-1)(n+1)$. For $n > 2$, both $n-1$ and $n+1$ are greater than 1, so $n^2 - 1$ has two factors greater than 1 and is not prime.

C7 The number is $10a+b$ and its reverse is $10b+a$ $(10a+b) + (10b+a) = 11a + 11b = 11(a+b)$, a multiple of 11

C8 Let the numbers be $2n, 2n+2, 2n+4$ $2n(2n+2)(2n+4) = 2n \times 2(n+1) \times 2(n+2) = 8n(n+1)(n+2)$
 $n(n+1)(n+2)$ is the product of three consecutive integers, so it is $6k$. The product is $8 \times 6k = 48k$, a multiple of 48

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

D1 $n \rightarrow 2n \rightarrow 2n+10 \rightarrow n+5 \rightarrow n+5-n=5$ The answer is always **5**

D2 (a) $44 \times 55 = 2420$ and $45 \times 54 = 2430$ Difference **10**

(b) Top left n : the box is $n, n+1, n+10, n+11$

$$(n+1)(n+10) - n(n+11) = n^2 + 11n + 10 - n^2 - 11n = 10 \text{ for every } n$$

D3 $5n+5(n+1) = 10n+5 = 5(2n+1)$. $2n+1$ is odd, so the sum is an odd multiple of 5

D4 (a) $2a+2b = 2(a+b)$, which is even (b) For example, $3 \times 5 = 15$, which is odd

HIGHER

D5 Let the integers be $n-1, n, n+1$ $n^2 - (n-1)(n+1) = n^2 - (n^2 - 1) = 1$



- D6** $(n-1)^2 + n^2 + (n+1)^2 = n^2 - 2n + 1 + n^2 + n^2 + 2n + 1 = 3n^2 + 2$, which is 2 more than the multiple of 3, $3n^2$
-
- D7** $2^{n+2} - 2^n = 2^n \times 2^2 - 2^n = 2^n(4 - 1) = 3 \times 2^n$, a multiple of 3
-
- D8** (a) $(n+1)^2 - n^2 = 2n + 1$. Every odd number has the form $2n + 1$, so it is the difference of the squares of $n + 1$ and n
- (b) $99 = 2 \times 49 + 1$, so $n = 49$: $50^2 - 49^2$
-

E

Exam practice solutions

Marks are shown on the right of each line.

FOUNDATION

Foundation and Higher crossover

- E1** $2 + 3 = 5$, and 5 is odd B2
-
- E2** (a) $n + 1$ B1
- (b) $n + (n + 1) + (n + 2) = 3n + 3$ M1
- $= 3(n + 1)$, a multiple of 3 C1
-
- E3** **No**: checking some values does not show it is true for every odd number C1
- A proof must use algebra for a general odd number, $2k + 1$ C1
-
- E4** (a) $73 - 37 = 36 = 9 \times 4$ B1
- (b) $(10a + b) - (10b + a) = 9a - 9b$ M1
- $= 9(a - b)$, a multiple of 9 C1
-
- E5** $2m + (2n + 1)$ M1
- $= 2m + 2n + 1 = 2(m + n) + 1$ M1
- This is 1 more than an even number, so it is odd C1
-
- E6** Try $n = 1$: $(1 + 1)^2 = 4$ but $1^2 + 1 = 2$ M1
- So it is **not** true for every n C1
-



HIGHER

E7	$(2n+1)(2n+3)$ $= 4n^2 + 8n + 3 = 2(2n^2 + 4n + 1) + 1$ 1 more than an even number, so odd	M1 M1 C1
E8	$n^2 + 6n + 9 - (n^2 - 6n + 9)$ $= 12n$ 12 times an integer, so a multiple of 12	M1 A1 C1
E9	(a) $n^2 + (n+1)^2 = 2n^2 + 2n + 1$ $= 2(n^2 + n) + 1$, odd (b) The mean is $\frac{2(n^2 + n) + 1}{2} = n^2 + n + \frac{1}{2}$ $n^2 + n$ is a whole number, so the mean is a whole number plus $\frac{1}{2}$	M1 C1 M1 C1
E10	An odd number is $2n + 1$ $(2n + 1)^2 = 4n^2 + 4n + 1$ $= 4n(n + 1) + 1$ One of n and $n + 1$ is even, so $n(n + 1)$ is a multiple of 2 and $4n(n + 1)$ is a multiple of 8 So $(2n + 1)^2$ is 1 more than a multiple of 8	B1 M1 M1 M1 C1

