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INTERNATIONAL A LEVEL MATHEMATICS • PURE 1 (WMA11)

Pure 1 Practice Paper 2

Worked Solutions

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA11/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	Surds in a right-angled triangle	7	2
2	Inequalities in context	8	4
3	Trigonometric graph	8	5
4	Parallel and perpendicular lines	8	7
5	Gradient conditions, $f''(x)$	9	9
6	Tangents from a point	8	10
7	Radians: segment and triangle	8	11
8	Integration, parallel lines	9	13
9	Reciprocal graph, intersections	10	14

Question paper & mark scheme | Exam questions by topic | More practice papers

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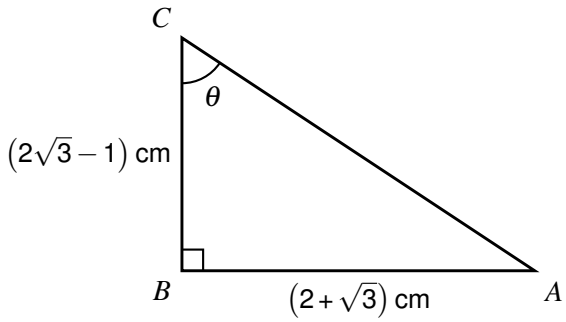

Diagram not drawn to scale

Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows a triangle ABC in which angle $ABC = 90°$.

$AB = (2+\sqrt{3})$ cm and $BC = (2\sqrt{3}-1)$ cm.

Angle $ACB = \theta$.

(a) Find the exact length of AC, giving your answer as a simplified surd.

(2)

(b) Find the exact area of triangle ABC, giving your answer in the form $p+q\sqrt{3}$, where p and q are rational numbers.

(2)

(c) Show that $\tan\theta = \dfrac{a+b\sqrt{3}}{c}$, where a, b and c are integers to be found.

(3)

(a) Pythagoras: $AC^2 = (2+\sqrt{3})^2 + (2\sqrt{3}-1)^2$ M1

$(2+\sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$ $\quad (2\sqrt{3}-1)^2 = 12 - 4\sqrt{3} + 1 = 13 - 4\sqrt{3}$

$AC^2 = 7 + 4\sqrt{3} + 13 - 4\sqrt{3} = 20$, so $AC = \sqrt{20} = \boxed{2\sqrt{5}\text{ cm}}$ A1

(b) Area $= \frac{1}{2} \times AB \times BC = \frac{1}{2}(2+\sqrt{3})(2\sqrt{3}-1)$ M1

$(2+\sqrt{3})(2\sqrt{3}-1) = 4\sqrt{3} - 2 + 6 - \sqrt{3} = 4 + 3\sqrt{3}$

Area $= \frac{1}{2}(4+3\sqrt{3}) = \boxed{2 + \frac{3}{2}\sqrt{3}\ \text{cm}^2}$ A1

(c) θ is the angle at C, so the opposite side is AB and the adjacent side is BC:

$\tan\theta = \frac{AB}{BC} = \frac{2+\sqrt{3}}{2\sqrt{3}-1}$ B1

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Question 1 continued

Multiply the top and bottom by $(2\sqrt{3} + 1)$: the bottom becomes $12 - 1 = 11$ M1

Top: $(2 + \sqrt{3})(2\sqrt{3} + 1) = 4\sqrt{3} + 2 + 6 + \sqrt{3} = 8 + 5\sqrt{3}$

$\tan\theta = \frac{8 + 5\sqrt{3}}{11}$ so $a = 8$, $b = 5$, $c = 11$ A1

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(Total for Question 1 is 7 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A rectangular animal enclosure has length $(2x+1)$ metres and width $(x-2)$ metres.

The perimeter of the enclosure must be less than 56 metres.

(a) Show that $x < \frac{29}{3}$

(2)

The area of the enclosure must be greater than $52\,m^2$.

(b) Find the set of possible values of x.

(6)

(a) Perimeter = $2(2x + 1) + 2(x - 2) = 4x + 2 + 2x - 4 = 6x - 2$ M1

$6x - 2 < 56 \Rightarrow 6x < 58 \Rightarrow x < \frac{29}{3}$ as required A1*

(b) Area condition: $(2x + 1)(x - 2) > 52$ M1

$2x^2 - 4x + x - 2 > 52 \Rightarrow 2x^2 - 3x - 54 > 0$ A1

Critical values: $(2x + 9)(x - 6) = 0$, so $x = 6$ or $x = -\frac{9}{2}$ M1

A positive quadratic is above zero outside its roots: $x < -\frac{9}{2}$ or $x > 6$ A1

Now combine every condition. The perimeter needs $x < \frac{29}{3}$. M1

The width $x - 2$ must be positive, so $x > 2$, which rules out $x < -\frac{9}{2}$.

$6 < x < \frac{29}{3}$ A1

(Total for Question 2 is 8 marks)

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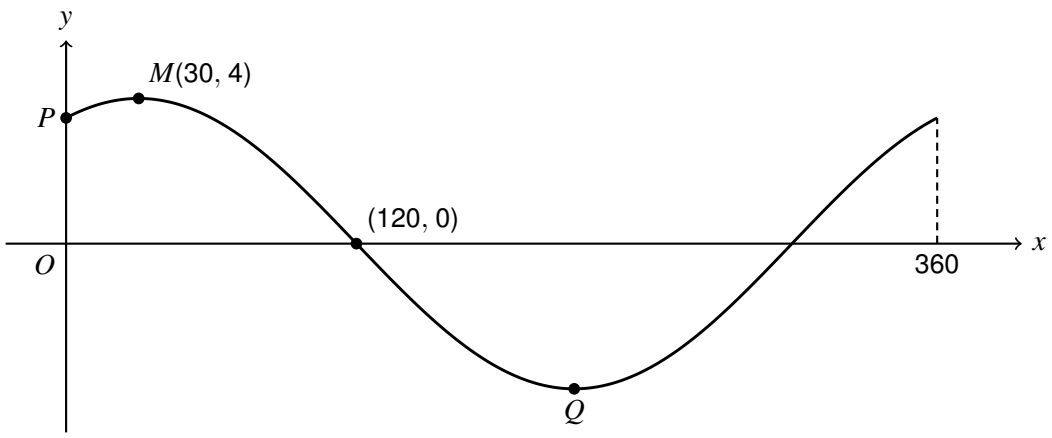

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = a\sin(x+b)^\circ \qquad 0 \leqslant x \leqslant 360$$

where a and b are constants, $a > 0$ and $0 < b < 360$

The curve has a maximum point at $M(30, 4)$ and crosses the x-axis at $(120, 0)$, as shown.

(a) Find the value of a and the value of b.

(2)

The curve meets the y-axis at P and has a minimum point at Q.

(b) Find the exact coordinates of P and the coordinates of Q.

(2)

The line with equation $y = k$, where k is a constant, meets the curve at exactly two points.

(c) Find the set of possible values of k.

(4)

(a) The largest value of a sin(...) is a, and the maximum is 4, so $a = 4$ B1

At the maximum, $\sin(30 + b)^\circ = 1$, so $30 + b = 90 \Rightarrow b = 60$ B1

(Check: at x = 120, $4 \sin 180^\circ = 0$ ✓)

(b) At P, x = 0: $y = 4 \sin 60^\circ = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$, so $P(0, 2\sqrt{3})$ B1

The minimum is 180° after the maximum: $x = 30 + 180 = 210$, so $Q(210, -4)$ B1

(c) The curve rises from $2\sqrt{3}$ to 4, falls to −4, then rises back to $2\sqrt{3}$ at x = 360. M1

For $2\sqrt{3} < k < 4$ the line meets it twice, either side of M.

For $-4 < k < 2\sqrt{3}$ the line meets it twice, either side of Q.

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Question 3 continued

But $y = 2\sqrt{3}$ meets it THREE times: $x = 0$, $x = 60$ and $x = 360$. ($k = \pm 4$ only touch.) B1

$-4 < k < 2\sqrt{3}$ or $2\sqrt{3} < k < 4$ A1 A1

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(Total for Question 3 is 8 marks)

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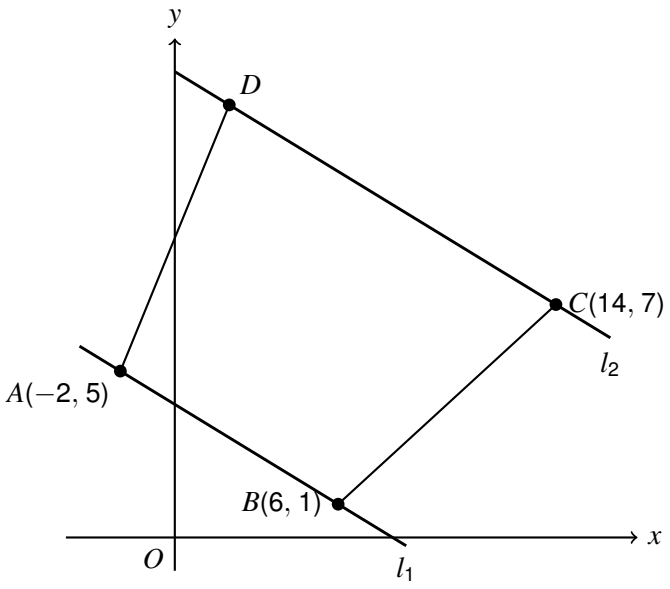

Diagram not drawn to scale

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The line l_1 passes through the points $A(-2, 5)$ and $B(6, 1)$.

The line l_2 is parallel to l_1 and passes through the point $C(14, 7)$.

(a) Find an equation of l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

The point D lies on l_2 such that AD is perpendicular to l_1, as shown in Figure 3.

(b) Find the coordinates of D.

(3)

(c) Find the exact area of the trapezium $ABCD$.

(3)

(a) Gradient of $l_1 = \frac{1-5}{6-(-2)} = \frac{-4}{8} = -\frac{1}{2}$. l_2 is parallel, so it has the same gradient.

l_2: $y - 7 = -\frac{1}{2}(x - 14) \Rightarrow 2y - 14 = -x + 14$ M1

$\boxed{x + 2y - 28 = 0}$ A1

(b) AD is perpendicular to l_1, so its gradient is 2: $y - 5 = 2(x + 2) \Rightarrow y = 2x + 9$ M1

D is also on l_2: $x + 2(2x + 9) - 28 = 0 \Rightarrow 5x - 10 = 0 \Rightarrow x = 2$ M1

$y = 2(2) + 9 = 13$, so $\boxed{D(2, 13)}$ A1

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Question 4 continued

(c) $AB = \sqrt{8^2 + 4^2} = 4\sqrt{5}$, $DC = \sqrt{12^2 + 6^2} = 6\sqrt{5}$, $AD = \sqrt{4^2 + 8^2} = 4\sqrt{5}$ M1

AD is perpendicular to both parallel sides, so it is the height of the trapezium.

Area $= \frac{1}{2}(AB + DC) \times AD = \frac{1}{2}(4\sqrt{5} + 6\sqrt{5}) \times 4\sqrt{5} = 5\sqrt{5} \times 4\sqrt{5}$ M1

$= 20 \times 5 = \boxed{100}$ A1

(Total for Question 4 is 8 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, where

$$f(x) = 2x^3 + ax^2 + bx + 3$$

and a and b are constants.

The tangent to C at the point where $x = 1$ is parallel to the line with equation $4x + y = 7$

Given also that $f''(2) = 18$

(a) find the value of a and the value of b.

(4)

(b) Find the coordinates of the other point on C at which the tangent is parallel to the line $4x + y = 7$

(2)

(c) Find the set of values of x for which the gradient of C is less than 8

(3)

(a) Differentiate twice: $f'(x) = 6x^2 + 2ax + b$ $f''(x) = 12x + 2a$ M1

$f''(2) = 18$: $24 + 2a = 18 \Rightarrow a = -3$ M1

The line $4x + y = 7$ is $y = -4x + 7$, with gradient -4. So $f'(1) = -4$:

$6 + 2a + b = -4 \Rightarrow 6 - 6 + b = -4 \Rightarrow b = -4$ M1

$\boxed{a = -3,\ b = -4}$ A1

(b) $f'(x) = 6x^2 - 6x - 4$. Tangent parallel to the line: $6x^2 - 6x - 4 = -4$ M1

$6x(x - 1) = 0$, so $x = 0$ ($x = 1$ is the point we started from). $f(0) = 3$, so $\boxed{(0, 3)}$ A1

(c) The gradient is $f'(x)$: $6x^2 - 6x - 4 < 8 \Rightarrow 6x^2 - 6x - 12 < 0 \Rightarrow x^2 - x - 2 < 0$ M1

Critical values: $(x - 2)(x + 1) = 0$, so $x = 2$ or $x = -1$ M1

A positive quadratic is below zero between its roots: $\boxed{-1 < x < 2}$ A1

(Total for Question 5 is 9 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = x^2 - 2x + 7$

The line with equation $y = kx - 2$, where k is a constant, is a tangent to C.

(a) Find the two possible values of k and, for each one, the coordinates of the point where the line touches C.

(6)

(b) The two tangents and the line joining their points of contact form a triangle.

Find the area of this triangle.

(2)

(a) The line meets the curve where $x^2 - 2x + 7 = kx - 2$ M1

$x^2 - 2x - kx + 9 = 0 \Rightarrow x^2 - (2 + k)x + 9 = 0$ A1

A tangent touches at one point, so the discriminant is zero: $(2 + k)^2 - 4(1)(9) = 0$ M1

$(2 + k)^2 = 36 \Rightarrow 2 + k = \pm 6 \Rightarrow$ $\boxed{k = 4 \text{ or } k = -8}$ A1

The repeated root is $x = \frac{2 + k}{2}$: M1

$k = 4$: $x = 3$, $y = 4(3) - 2 = 10$ $\quad$ $k = -8$: $x = -3$, $y = -8(-3) - 2 = 22$

$\boxed{(3, 10)}$ and $\boxed{(-3, 22)}$ A1

(b) Both tangents have y-intercept -2, so they meet at $(0, -2)$.

The line joining $(3, 10)$ and $(-3, 22)$ crosses the y-axis at its midpoint, $(0, 16)$.

Split the triangle along the y-axis: base $16 - (-2) = 18$, heights 3 and 3 M1

Area $= \frac{1}{2} \times 18 \times 3 + \frac{1}{2} \times 18 \times 3 = \boxed{54}$ A1

(Total for Question 6 is 8 marks)

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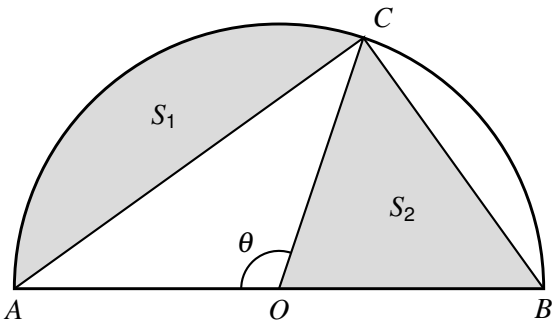

Figure 4

Figure 4 shows a semicircle with centre O and diameter $AB = 12$ cm.

The point C lies on the semicircle and angle $AOC = \theta$ radians, where $\frac{\pi}{2} < \theta < \pi$

The region S_1, shown shaded, is bounded by the chord AC and the arc AC.

The region S_2, shown shaded, is the triangle OBC.

The area of S_1 is equal to the area of S_2.

(a) Show that $\theta = 2\sin\theta$

(3)

Given that $\theta = 1.895$, correct to 3 decimal places,

(b) find the perimeter of S_1, in cm to 3 significant figures.

(3)

(c) Find the total area of the semicircle that is **not** shaded, in cm^2 to 3 significant figures.

(2)

(a) S_1 is a segment: S_1 = sector AOC − triangle AOC

$S_1 = \frac{1}{2} \times 6^2 \times \theta - \frac{1}{2} \times 6^2 \times \sin\theta = 18\theta - 18\sin\theta$ M1

Angle BOC $= \pi - \theta$, so $S_2 = \frac{1}{2} \times 6 \times 6 \times \sin(\pi - \theta)$ M1

$\sin(\pi - \theta) = \sin\theta$, so $S_2 = 18\sin\theta$

Equal areas: $18\theta - 18\sin\theta = 18\sin\theta \Rightarrow \theta = 2\sin\theta$ as required A1*

(b) Arc AC $= r\theta = 6 \times 1.895 = 11.37$ cm M1

Chord AC: $AC^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos 1.895 = 94.95\ldots$, so $AC = 9.744\ldots$ cm M1

Perimeter of $S_1 = 11.37 + 9.744\ldots = 21.11\ldots$, so **21.1 cm** (3 s.f.) A1

(c) Semicircle $= \frac{1}{2}\pi \times 6^2 = 18\pi$. Shaded $= S_1 + S_2 = 2 \times 18\sin\theta$ M1

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Question 7 continued

Not shaded = $18\pi - 36 \sin 1.895 = 56.55... - 34.12... =$ $\boxed{22.4 \text{ cm}^2}$ (3 s.f.) A1

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(Total for Question 7 is 8 marks)

8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 3\sqrt{x} + \frac{k}{x^2}$$

and k is a constant.

The point P on C has x coordinate 4. The tangent to C at P is parallel to the line l with equation $y = 5x + 6$

(a) Find the value of k.

(2)

Given that the point $(1, -2)$ lies on C,

(b) find $f(x)$, giving each term in its simplest form.

(3)

(c) Find the exact shortest distance between the tangent to C at P and the line l.

(4)

(a) At $x = 4$: $f'(4) = 3\sqrt{4} + \frac{k}{16} = 6 + \frac{k}{16}$

The tangent is parallel to $y = 5x + 6$, so its gradient is 5: $6 + \frac{k}{16} = 5$ M1

$\frac{k}{16} = -1 \Rightarrow \boxed{k = -16}$ A1

(b) $f'(x) = 3x^{1/2} - 16x^{-2}$. Integrate: $f(x) = 3x^{3/2} \div \frac{3}{2} - 16x^{-1} \div (-1) + c$ M1

$f(x) = 2x^{3/2} + 16x^{-1} + c$ A1

$(1, -2)$ lies on C: $2 + 16 + c = -2 \Rightarrow c = -20$, so $\boxed{f(x) = 2x^{3/2} + \frac{16}{x} - 20}$ A1

(c) $f(4) = 2(8) + 4 - 20 = 0$, so $P(4, 0)$. The normal at P has gradient $-\frac{1}{5}$.

Normal: $y = -\frac{1}{5}(x - 4)$. It is perpendicular to BOTH parallel lines. M1

It meets l where $-\frac{1}{5}(x - 4) = 5x + 6 \Rightarrow -x + 4 = 25x + 30 \Rightarrow x = -1$, $y = 1$ M1

Shortest distance = distance from $P(4, 0)$ to $(-1, 1) = \sqrt{5^2 + 1^2}$ M1

$\boxed{\sqrt{26}}$ A1

(Total for Question 8 is 9 marks)

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9.

$$f(x) = \frac{4}{x+1} - 2 \qquad x \neq -1$$

(a) Sketch the curve with equation $y = f(x)$. On your sketch, show the equations of the asymptotes and the coordinates of any points where the curve meets the coordinate axes.

(4)

In this question you must show all stages of your working.

The line with equation $y = kx + 2$, where k is a constant, meets the curve $y = f(x)$ at the point $(0, 2)$.

(b) Find the values of k for which the line meets the curve at **exactly one** point.

(4)

(c) The curve with equation $y = f(x) + c$, where c is a constant, does not meet the x-axis.

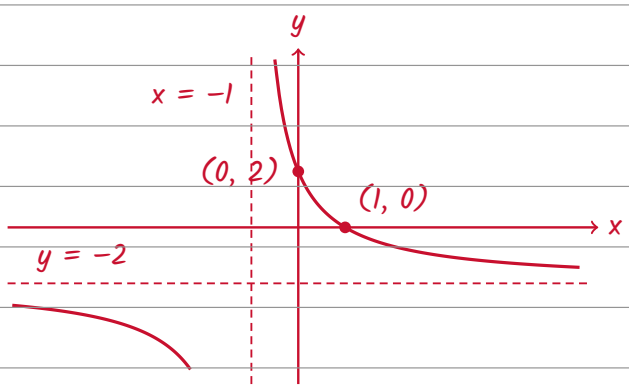
Find the value of c.

(2)

(a) Vertical asymptote $x = -1$, where $x + 1 = 0$.

Horizontal asymptote $y = -2$, because $\frac{4}{x+1} \to 0$ for large x.

$x = 0$: $y = 4 - 2 = 2$ $\quad$ $y = 0$: $\frac{4}{x+1} = 2 \Rightarrow x + 1 = 2 \Rightarrow x = 1$



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(b) $\frac{4}{x+1} - 2 = kx + 2$. Multiply by $(x + 1)$: $4 - 2(x + 1) = (kx + 2)(x + 1)$ M1

$2 - 2x = kx^2 + kx + 2x + 2 \Rightarrow kx^2 + (k + 4)x = 0$ A1

$x(kx + k + 4) = 0$: $x = 0$ is the point $(0, 2)$, and the other root is $x = -\frac{k+4}{k}$. M1

Exactly one point if that root is also 0: $k + 4 = 0$, $k = -4$ (the line is a tangent).

Or if there is no x^2 term at all: $k = 0$ gives $4x = 0$, so $x = 0$ only.

$\boxed{k = -4 \text{ or } k = 0}$ A1

(c) $y = f(x) + c$ moves the curve up by c. It always crosses the x-axis, unless M1

its horizontal asymptote becomes the x-axis itself: $-2 + c = 0 \Rightarrow \boxed{c = 2}$ A1

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Question 9 continued

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(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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