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INTERNATIONAL A LEVEL MATHEMATICS • DECISION MATHEMATICS 1

Decision 1

Chapter 5: The Travelling Salesman Problem

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WDM11/01) • Calculator allowed • Answer book pages included

Multi-step exam questions at the difficulty of the real WDM11 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Algorithms** Flow charts, bubble sort, quick sort, bin packing, binary search	10	80	separate file	
2	**Graphs and Networks** Vocabulary, types of graph, degrees, adjacency and distance matrices	10	80	separate file	
3	**Algorithms on Graphs** Kruskal, Prim (networks and matrices), Dijkstra	10	80	separate file	
4	**Route Inspection** Eulerian graphs, pairing odd vertices, start and finish points	10	80	separate file	
5	**The Travelling Salesman Problem** Classical and practical problems, upper and lower bounds	10	80	2	
6	**Critical Path Analysis** Activity networks, event times, floats, Gantt charts, scheduling	10	80	separate file	
7	**Linear Programming** Formulation, feasible regions, objective lines, integer solutions	10	80	separate file	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 5 • KEY SKILLS

5 The Travelling Salesman Problem

Specification D1: The travelling salesman problem • 10 questions • 80 marks • about 96 minutes

1 Two problems

Classical: visit every vertex exactly once and return to the start. Practical: visit every vertex at least once and return to the start.

2 Least distances

Replace each entry by the shortest distance between the vertices (maybe via others); the table then satisfies the triangle inequality.

3 Upper bound from a tree

Twice the weight of a minimum spanning tree is an upper bound.

4 Shortcuts

Replace a repeated section of the doubled-tree walk by a direct arc; the saving lowers the upper bound. State each shortcut.

5 Nearest neighbour

From the start, go to the nearest unvisited vertex each time, then return to the start. Give the route and its length.

6 Lower bound

Delete a vertex, find a minimum spanning tree of the rest, then add the two least arcs from the deleted vertex.

7 Best bounds

The best upper bound is the least; the best lower bound is the greatest.

8 The interval

Lower bound $\leqslant$ optimal $\leqslant$ upper bound, with non-strict inequalities.

9 Optimal?

A tour is known to be optimal only if its length equals a lower bound.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Two problems	•					•	•			
2 Least distances	•						•			
3 Upper bound from a tree			•			•				•
4 Shortcuts			•							•
5 Nearest neighbour		•				•	•	•	•	•
6 Lower bound				•		•	•	•	•	•
7 Best bounds		•		•	•	•		•		
8 The interval					•	•	•	•	•	•
9 Optimal?					•			•		
Interleaved: Prim's algorithm on a matrix (Chapter 3)			•	•		•	•	•	•	•
Interleaved: Dijkstra and shortest paths (Chapter 3)	•						•			
Interleaved: inequalities in an unknown (P1)									•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: the table (with a spare copy), the tree for the shortcut method, and answer lines.

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1.

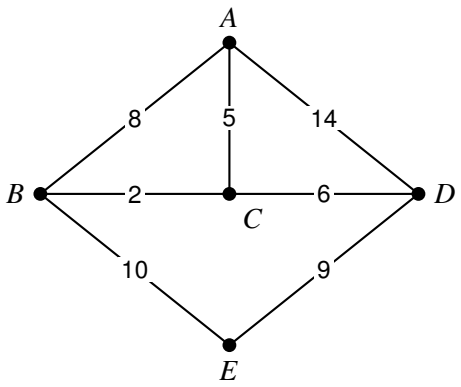


Figure 1

Figure 1 shows the roads between five towns, with lengths in km.

(a) Complete the table of least distances between the towns in the answer book.

(4)

(b) Explain the difference between the classical and the practical travelling salesman problems.

(2)

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Question 1 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E
A	–	7	5	11	17
B	7	–			
C	5		–		
D	11			–	
E	17				–

DO NOT WRITE IN THIS AREA

(Total for Question 1 is 6 marks)

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2. The table shows the travelling times, in minutes, between six offices.

	A	B	C	D	E	F
A	–	40	18	35	41	31
B	40	–	24	23	67	16
C	18	24	–	29	55	22
D	35	23	29	–	48	9
E	41	67	55	48	–	52
F	31	16	22	9	52	–

(a) Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of a tour. You must state your route.

(3)

(b) Repeat part (a) starting at D.

(2)

(c) State which of your upper bounds is better, giving a reason.

(1)

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Question 2 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	40	18	35	41	31
B	40	–	24	23	67	16
C	18	24	–	29	55	22
D	35	23	29	–	48	9
E	41	67	55	48	–	52
F	31	16	22	9	52	–

	A	B	C	D	E	F
A	–	40	18	35	41	31
B	40	–	24	23	67	16
C	18	24	–	29	55	22
D	35	23	29	–	48	9
E	41	67	55	48	–	52
F	31	16	22	9	52	–

Route from A and its length: _____

Route from D and its length: _____

(Total for Question 2 is 6 marks)

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3. The table shows the distances, in km, between six depots.

	A	B	C	D	E	F
A	–	25	14	9	13	24
B	25	–	39	23	29	49
C	14	39	–	20	18	11
D	9	23	20	–	6	26
E	13	29	18	6	–	22
F	24	49	11	26	22	–

A minimum spanning tree for the depots is AD, DE, AC, CF, DB; it is drawn in the answer book.

(a) Use the minimum spanning tree to find an initial upper bound for the length of a tour.

(2)

(b) Starting with the minimum spanning tree in the answer book, use the shortcut method to find an upper bound below 120 km. You must state the shortcut(s) you use and the length of your upper bound.

(4)

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Question 3 continued D1 ANSWER BOOK

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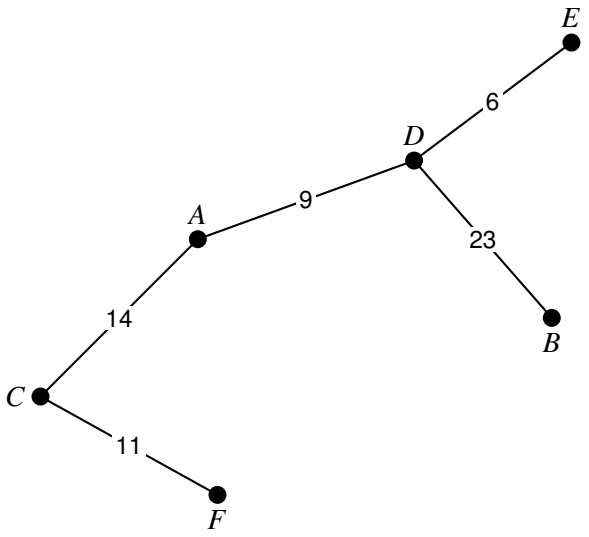


Diagram 1

	A	B	C	D	E	F
A	–	25	14	9	13	24
B	25	–	39	23	29	49
C	14	39	–	20	18	11
D	9	23	20	–	6	26
E	13	29	18	6	–	22
F	24	49	11	26	22	–

Upper bound: _____

(Total for Question 3 is 6 marks)

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4. The table shows the distances, in km, between six farms.

	A	B	C	D	E	F
A	–	11	16	44	40	13
B	11	–	27	45	43	21
C	16	27	–	38	32	20
D	44	45	38	–	7	55
E	40	43	32	7	–	50
F	13	21	20	55	50	–

(a) By deleting A, and all of its arcs, find a lower bound for the length of a tour. You must make your method clear.

(3)

(b) By deleting B instead, find a second lower bound.

(2)

(c) State which lower bound is better, giving a reason.

(1)

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Question 4 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	11	16	44	40	13
B	11	–	27	45	43	21
C	16	27	–	38	32	20
D	44	45	38	–	7	55
E	40	43	32	7	–	50
F	13	21	20	55	50	–

	A	B	C	D	E	F
A	–	11	16	44	40	13
B	11	–	27	45	43	21
C	16	27	–	38	32	20
D	44	45	38	–	7	55
E	40	43	32	7	–	50
F	13	21	20	55	50	–

Lower bound deleting A: _____

Lower bound deleting B: _____

(Total for Question 4 is 6 marks)

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5. For a travelling salesman problem on six cities, the nearest neighbour algorithm gave tours of lengths 135, 128 and 143 km, starting at A, B and C respectively. Deleting A, B and D in turn gave lower bounds of 104, 100 and 117 km.

(a) Write down the best interval within which the length, T km, of an optimal tour must lie.

(2)

(b) Explain why the lower bounds are not all the same, and why the greatest one is used.

(2)

(c) A tour of length 120 km is suggested. State, with a reason, whether it could be optimal, and whether the information shows that it is.

(2)

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Question 5 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Best interval: _____

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(Total for Question 5 is 6 marks)

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6. The table shows the distances, in km, between six warehouses. A driver must visit each warehouse, starting and finishing at A, and wishes to minimise the distance travelled.

	A	B	C	D	E	F
A	–	45	47	38	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	38	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

(a) Explain why the classical and practical problems are the same here.

(1)

(b) Use Prim's algorithm, starting at A, to find a minimum spanning tree, listing arcs in the order selected, and hence find an initial upper bound.

(4)

(c) Use the nearest neighbour algorithm, starting at A, to find a second upper bound. You must state your route.

(2)

(d) By deleting A, find a lower bound.

(2)

(e) Write down the best interval for the length, T km, of an optimal route.

(1)

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Question 6 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	45	47	38	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	38	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

	A	B	C	D	E	F
A	–	45	47	38	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	38	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

Upper bound from the tree: _____

Nearest neighbour route and length: _____

Lower bound: _____

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Question 6 continued

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(Total for Question 6 is 10 marks)

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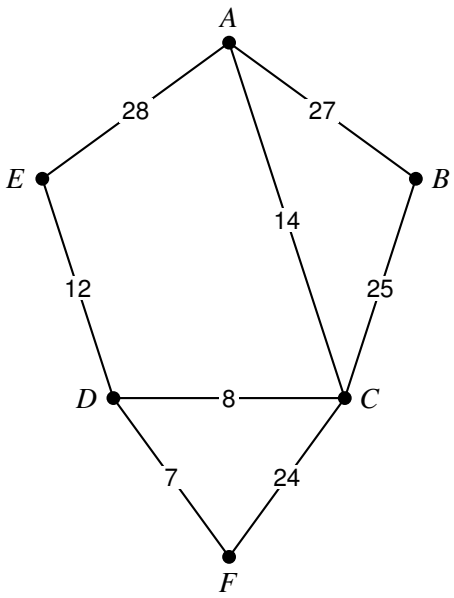


Figure 2

Figure 2 shows the roads between six villages, with lengths in km. A health visitor must visit every village, starting and finishing at A.

(a) Complete the table of least distances in the answer book.

(3)

(b) Use the nearest neighbour algorithm on your table, starting at A, to find an upper bound for the length of the route, and write down the corresponding route through the actual roads.

(4)

(c) By deleting A, find a lower bound for the length of the route.

(3)

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Question 7 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

Route through the roads: _____

Upper bound: _____

Lower bound: _____

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. The table shows the times, in minutes, to travel between six sites.

	A	B	C	D	E	F
A	–	35	46	43	57	61
B	35	–	37	11	52	49
C	46	37	–	48	15	16
D	43	11	48	–	62	58
E	57	52	15	62	–	12
F	61	49	16	58	12	–

(a) Use the nearest neighbour algorithm, starting at A and then starting at B, to find two upper bounds. State your routes.

(4)

(b) By deleting A and then deleting B, find two lower bounds.

(4)

(c) Hence write down the best interval for the time, T minutes, of an optimal tour, and state whether either nearest neighbour route is known to be optimal.

(2)

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Question 8 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	35	46	43	57	61
B	35	–	37	11	52	49
C	46	37	–	48	15	16
D	43	11	48	–	62	58
E	57	52	15	62	–	12
F	61	49	16	58	12	–

	A	B	C	D	E	F
A	–	35	46	43	57	61
B	35	–	37	11	52	49
C	46	37	–	48	15	16
D	43	11	48	–	62	58
E	57	52	15	62	–	12
F	61	49	16	58	12	–

Upper bounds: _____

Lower bounds: _____

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. The table shows the distances, in km, between six towns. The distance between A and D is x km, where $36 < x < 45$

	A	B	C	D	E	F
A	–	45	47	x	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	x	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

(a) By deleting A, show that a lower bound for the length of a tour is $89 + \min(x, 40)$ km, explaining each step.

(4)

(b) Explain why the nearest neighbour algorithm starting at A first travels to F, and show that it gives the route A–F–E–C–B–D–A with length $(119 + x)$ km.

(3)

(c) The upper bound in part (b) is 157 km. Find x, and hence the best interval for the length, T km, of an optimal tour that these two methods give.

(3)

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Question 9 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	45	47	x	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	x	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

	A	B	C	D	E	F
A	–	45	47	x	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	x	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

Lower bound in terms of x: _____

Interval: _____

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Question 9 continued

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(Total for Question 9 is 10 marks)

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10. The table shows the distances, in hundreds of metres, between seven stalls at a fair. An inspector must visit every stall, returning to the start.

	A	B	C	D	E	F	G
A	–	41	30	26	12	19	22
B	41	–	11	63	47	37	31
C	30	11	–	52	36	27	21
D	26	63	52	–	17	28	35
E	12	47	36	17	–	14	20
F	19	37	27	28	14	–	7
G	22	31	21	35	20	7	–

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree. List the arcs in the order selected, and state its weight.

(3)

(b) Starting from twice the weight of your tree, use the shortcut method to find an upper bound below 160. State the shortcut you use.

(3)

(c) Use the nearest neighbour algorithm, starting at D, to find another upper bound, stating the route.

(2)

(d) By deleting D, find a lower bound, and hence state the best interval for the length, T, of an optimal tour.

(2)

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Question 10 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F	G
A	–	41	30	26	12	19	22
B	41	–	11	63	47	37	31
C	30	11	–	52	36	27	21
D	26	63	52	–	17	28	35
E	12	47	36	17	–	14	20
F	19	37	27	28	14	–	7
G	22	31	21	35	20	7	–

	A	B	C	D	E	F	G
A	–	41	30	26	12	19	22
B	41	–	11	63	47	37	31
C	30	11	–	52	36	27	21
D	26	63	52	–	17	28	35
E	12	47	36	17	–	14	20
F	19	37	27	28	14	–	7
G	22	31	21	35	20	7	–

Upper bounds: _____

Lower bound and interval: _____

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 5 IS 80 MARKS

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