eClassroom

Radians

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 1

Question 1 — Worked Solution

(a) 135° to radians

$$135° \times \frac{\pi}{180} = \frac{135\pi}{180} = \frac{3\pi}{4}$$

$$\therefore 135° = \frac{3\pi}{4} \text{ rad}$$

(b) 240° to radians

$$240° \times \frac{\pi}{180} = \frac{240\pi}{180} = \frac{4\pi}{3}$$

$$\therefore 240° = \frac{4\pi}{3} \text{ rad}$$

(c) 7π/6 to degrees

$$\frac{7\pi}{6} \times \frac{180°}{\pi} = \frac{7 \times 180°}{6} = 7 \times 30° = 210°$$

$$\therefore \frac{7\pi}{6} = 210°$$

(d) 5π/4 to degrees

$$\frac{5\pi}{4} \times \frac{180°}{\pi} = \frac{5 \times 180°}{4} = 5 \times 45° = 225°$$

$$\therefore \frac{5\pi}{4} = 225°$$

Question 2 — Worked Solution

(a) Arc length

Using arc length = rθ:

$$\text{Arc } AB = r\theta = 8 \times \frac{\pi}{3} = \frac{8\pi}{3} \text{ cm}$$

$$\therefore \text{Arc } AB = \frac{8\pi}{3} \text{ cm}$$

(b) Area of sector

Using area = ½r²θ:

$$\text{Area} = \frac{1}{2} \times 8^2 \times \frac{\pi}{3} = \frac{1}{2} \times 64 \times \frac{\pi}{3} = \frac{32\pi}{3} \text{ cm}^2$$

$$\therefore \text{Area} = \frac{32\pi}{3} \text{ cm}^2$$

(c) Perimeter of sector

Perimeter = arc + 2 radii:

$$\text{Perimeter} = \frac{8\pi}{3} + 2(8) = \frac{8\pi}{3} + 16 \text{ cm}$$

$$\therefore \text{Perimeter} = \frac{8\pi}{3} + 16 \text{ cm}$$

Question 3 — Worked Solution

(a) Area of sector OAB

$$\text{Area}_{\text{sector}} = \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2 \times \frac{\pi}{4} = \frac{100\pi}{8} = \frac{25\pi}{2}\ \text{cm}^2$$

$$\therefore \text{Area of sector} = \frac{25\pi}{2}\ \text{cm}^2$$

(b) Area of minor segment

Area of segment = Area of sector – Area of triangle OAB

$$\text{Area}_{\triangle} = \frac{1}{2}r^2\sin\theta = \frac{1}{2}(100)\sin\frac{\pi}{4} = 50 \times \frac{\sqrt{2}}{2} = 25\sqrt{2}\ \text{cm}^2$$

$$\text{Area}_{\text{segment}} = \frac{25\pi}{2} - 25\sqrt{2}$$

$$\therefore \text{Area of segment} = \frac{25\pi}{2} - 25\sqrt{2}\ \text{cm}^2$$

(c) Perimeter of minor segment

Perimeter = arc AB + chord AB

$$\text{Arc } AB = r\theta = 10 \times \frac{\pi}{4} = \frac{5\pi}{2}\ \text{cm}$$

$$\text{Chord } AB = 2r\sin\frac{\theta}{2} = 2(10)\sin\frac{\pi}{8} = 20\sin\frac{\pi}{8}\ \text{cm}$$

$$\text{Perimeter} = \frac{5\pi}{2} + 20\sin\frac{\pi}{8}\ \text{cm}$$

$$\therefore \text{Perimeter} = \frac{5\pi}{2} + 20\sin\frac{\pi}{8} \approx 15.51\ \text{cm}$$

Question 4 — Worked Solution

Solve $2\sin^2 x + \sin x - 1 = 0$

Factorise as a quadratic in sin x:

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -1$$

For sin x = 1/2, in [0, 2π]:

$$x = \frac{\pi}{6} \quad \text{and} \quad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

For sin x = –1, in [0, 2π]:

$$x = \frac{3\pi}{2}$$

$$\therefore x = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{3\pi}{2}$$

Question 5 — Worked Solution

(a) Find θ

Using area of sector = ½r²θ:

$$\frac{1}{2} \times 8^2 \times \theta = 48$$

$$32\theta = 48 \Rightarrow \theta = \frac{48}{32} = \frac{3}{2} \text{ rad}$$

$$\therefore \theta = \frac{3}{2} \text{ rad}$$

(b) Arc length AB

$$\text{Arc } AB = r\theta = 8 \times \frac{3}{2} = 12 \text{ cm}$$

$$\therefore \text{Arc } AB = 12 \text{ cm}$$

(c) Perimeter of sector

$$\text{Perimeter} = \text{arc} + 2r = 12 + 2(8) = 12 + 16 = 28 \text{ cm}$$

$$\therefore \text{Perimeter} = 28 \text{ cm}$$

Question 6 — Worked Solution

(a) Area of shaded region

Shaded area = Area of sector – Area of triangle OAB

$$\text{Area}_{\text{sector}} = \frac{1}{2}r^2\theta = \frac{1}{2}(36)\cdot\frac{2\pi}{3} = 12\pi \text{ cm}^2$$

$$\text{Area}_{\triangle} = \frac{1}{2}r^2\sin\theta = \frac{1}{2}(36)\sin\frac{2\pi}{3} = 18\times\frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ cm}^2$$

$$\text{Shaded area} = 12\pi - 9\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Shaded area} = 12\pi - 9\sqrt{3} \text{ cm}^2$$

(b) Perimeter of shaded region

Perimeter = arc AB + chord AB

$$\text{Arc } AB = r\theta = 6\times\frac{2\pi}{3} = 4\pi \text{ cm}$$

$$\text{Chord } AB = 2r\sin\frac{\theta}{2} = 2(6)\sin\frac{\pi}{3} = 12\times\frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ cm}$$

$$\text{Perimeter} = 4\pi + 6\sqrt{3} \text{ cm}$$

$$\therefore \text{Perimeter} = 4\pi + 6\sqrt{3} \text{ cm}$$