eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 3 (WMA13)

Pure 3 Practice Paper 2

Edexcel IAL Pure Mathematics 3

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA13/01) • Calculator allowed

A full 75-mark paper, harder than a typical real series, with fewer steps given and topics combined in every question

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 9 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	Inverse function, sketch and a root	10	
2	Double angle identity and equation	8	
3	Logarithmic graph of a model	8	
4	Differentiating arctan using $\frac{dx}{dy}$	8	
5	Area using a trigonometric identity	8	
6	Exponential model, maximum and rate	8	
7	Modulus of an exponential function	9	
8	Differentiating and integrating $\sec x$	8	
9	Rational function and its inverse	8	
	Total	**75**	

Rough grade guide (out of 75): A* **60** • A **53** • B **47** • C **41**

Based on recent WMA13 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

1. The function f is defined by

$$f(x) = e^{2x} - 4 \qquad x \in \mathbb{R}$$

(a) Find $f^{-1}(x)$, stating its domain.

(3)

(b) On the same axes, sketch the curves with equations $y = f(x)$ and $y = f^{-1}(x)$

On your sketch, show the coordinates of each point where each curve meets the coordinate axes, and the equation of each asymptote.

(3)

The curves $y = f(x)$ and $y = f^{-1}(x)$ meet at a point with positive x coordinate α.

(c) Explain why α is a root of the equation

$$e^{2x} - x - 4 = 0$$

(2)

(d) Show that $\alpha = 0.7825$ to 4 decimal places.

(2)

DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 1 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Express $\cot\theta - \tan\theta$ in the form $k\cot 2\theta$, where k is a constant to be found.

(3)

(b) Hence solve, for $0 < \theta < \pi$, the equation

$$\cot\theta - \tan\theta = 4$$

giving your answers to 3 decimal places.

(3)

(c) Use the result of part (a) to find the exact value of $\tan\left(\frac{\pi}{12}\right)$, giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(2)

DO NOT WRITE IN THIS AREA

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 2 is 8 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3. The resting energy use, R kJ per hour, of a mammal of mass m kg is modelled by the equation

$$R = a m^n$$

where a and n are constants.

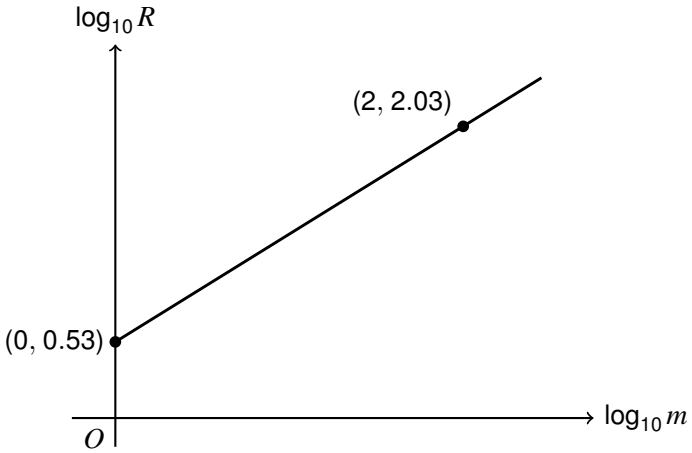

Figure 1

Figure 1 shows the linear relationship between $\log_{10} R$ and $\log_{10} m$

The line passes through the points (0, 0.53) and (2, 2.03)

(a) Find, according to the model, the value of n and the value of a, giving a to 3 significant figures.

(4)

(b) Use the model to estimate the resting energy use of a mammal of mass 70 kg.

(1)

(c) Find, according to the model, the mass of a mammal for which $\frac{dR}{dm} = 1$

(3)

DO NOT WRITE IN THIS AREA

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 3 is 8 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4.

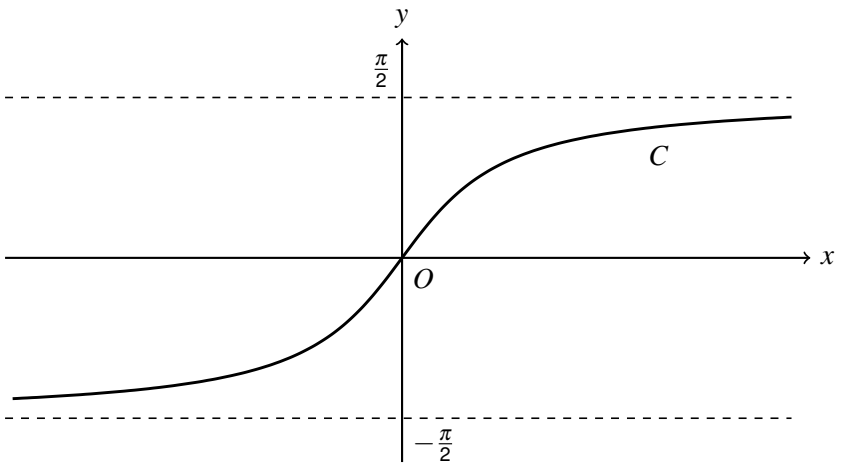

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = \arctan\left(\frac{x}{2}\right) \qquad x \in \mathbb{R}$$

(a) Use $x = 2\tan y$ to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x, giving your answer in its simplest form.

(3)

The point A on C has x coordinate 2

The normal to C at A meets the x-axis at the point B.

(b) Find the exact x coordinate of B.

(3)

(c) Find the exact coordinates of each point on C at which the gradient of C is $\dfrac{1}{5}$

(2)

DO NOT WRITE IN THIS AREA

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 4 is 8 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

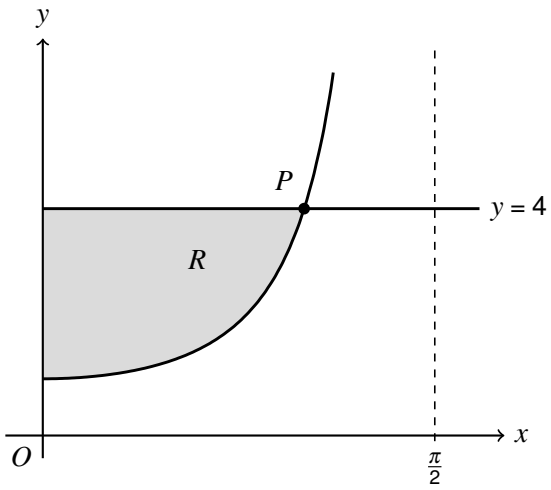

Figure 3

Figure 3 shows part of the curve C with equation

$$y = \frac{2}{1+\cos 2x} \qquad 0 \leqslant x < \frac{\pi}{2}$$

and the line with equation $y = 4$

The line meets C at the point P.

The region R, shown shaded in Figure 3, is bounded by C, the line $y = 4$ and the y-axis.

(a) Find the exact x coordinate of P.

(2)

(b) Find the exact area of R, giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational constants.

(6)

DO NOT WRITE IN THIS AREA

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 5 is 8 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6. A drug is given to a patient. The concentration of the drug in the patient's blood, C mg per litre, t hours after it is given, is modelled by the equation

$$C = 12\left(e^{-0.2t} - e^{-0.8t}\right) \qquad t \geqslant 0$$

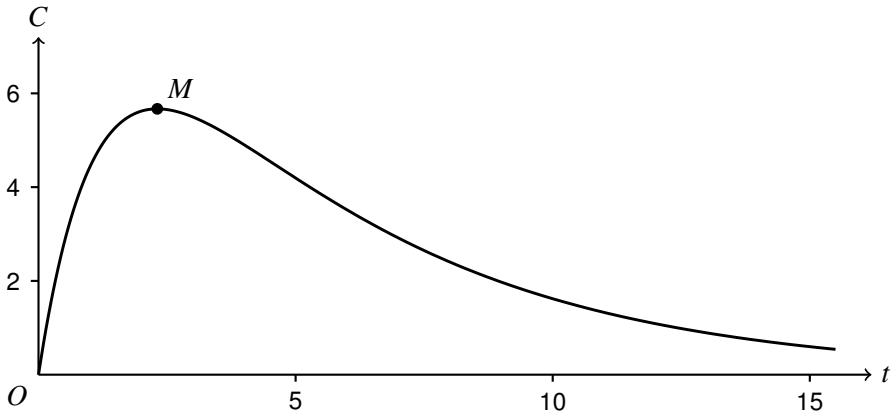

Figure 4

Figure 4 shows the graph of C against t. The concentration is greatest at the point M.

(a) Find the exact value of t at M, giving your answer in the form $p \ln q$, where p and q are rational.

(3)

(b) Find the greatest concentration of the drug, according to the model.

(2)

The drug is effective only while $C \geqslant 2$

(c) Show that, after the greatest concentration is reached, the drug stops being effective between $t = 8$ and $t = 9$

(2)

(d) Find the rate of change of the concentration when $t = 10$, stating the units of your answer.

(1)

DO NOT WRITE IN THIS AREA

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 6 is 8 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Sketch the graph with equation

$$y = |e^x - 4|$$

showing the coordinates of each point where the graph meets the coordinate axes, and the equation of the asymptote.

(3)

(b) Solve the equation $|e^x - 4| = 3$

(2)

(c) Find the exact solutions of the equation

$$|e^x - 4| = 2e^{-x}$$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 9 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$y = \ln(\sec x + \tan x) \qquad 0 \leqslant x < \frac{\pi}{2}$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in its simplest form.

(3)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{3}} \sec x \, \mathrm{d}x$$

(2)

(c) Find the exact value of

$$\int_0^{\frac{\pi}{3}} (\sec x + \tan x)^2 \, \mathrm{d}x$$

(3)

DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 8 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9. The function f is defined by

$$f(x) = \frac{2x^2 + 3x - 2}{x^2 - 4} \qquad x \in \mathbb{R},\ x \neq -2,\ x \neq 2$$

(a) Write $f(x)$ as a fraction in its simplest form.

(2)

(b) Find $f^{-1}(x)$, stating its domain.

(3)

(c) Find the coordinates of each point on the curve $y = f(x)$ at which the gradient is -3

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk