Trigonometric Functions

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 3

Question 1 — Worked Solution

(a) $\sec 60° = \dfrac{1}{\cos 60°} = \dfrac{1}{\frac{1}{2}} = 2$

(b) $\operatorname{cosec} 270° = \dfrac{1}{\sin 270°} = \dfrac{1}{-1} = -1$

(c) $\cot 135° = \dfrac{\cos 135°}{\sin 135°} = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

(d) $\sec 150° = \dfrac{1}{\cos 150°} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$

Question 2 — Worked Solution

(a) $\sin x \cot x = \sin x \cdot \dfrac{\cos x}{\sin x} = \cos x$

(b) $\tan x \operatorname{cosec} x = \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} = \dfrac{1}{\cos x} = \sec x$

(c) $\cos^2 x(\sec^2 x - 1) = \cos^2 x \cdot \tan^2 x = \cos^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x} = \sin^2 x$

Question 3 — Worked Solution

Prove cosecx − sinx ≡ cosxcotx

Start with the left-hand side:

$$\text{LHS} = \operatorname{cosec} x - \sin x = \frac{1}{\sin x} - \sin x$$

$$= \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$$= \cos x \cdot \frac{\cos x}{\sin x} = \cos x \cot x = \text{RHS} \quad \checkmark$$

Question 4 — Worked Solution

Prove $(\sec x + 1)(\sec x - 1) \equiv \tan^2 x$

Expand the left-hand side (difference of two squares):

$$\text{LHS} = \sec^2 x - 1$$

Use the identity $\sec^2 x \equiv 1 + \tan^2 x$:

$$= (1 + \tan^2 x) - 1 = \tan^2 x = \text{RHS} \quad ✓$$

Question 5 — Worked Solution

Prove (1+sinx)/cosx + cosx/(1+sinx) ≡ 2secx

Combine over common denominator cosx(1+sinx):

$$\text{LHS} = \frac{(1+\sin x)^2 + \cos^2 x}{\cos x(1+\sin x)}$$

Expand the numerator:

$$= \frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1+\sin x)}$$

Use $\sin^2 x + \cos^2 x = 1$:

$$= \frac{2+2\sin x}{\cos x(1+\sin x)} = \frac{2(1+\sin x)}{\cos x(1+\sin x)} = \frac{2}{\cos x} = 2\sec x = \text{RHS} \quad \checkmark$$

Question 6 — Worked Solution

Prove cosecx / (cosecx – sinx) ≡ sec²x

Rewrite the denominator (from Q3 we know cosecx – sinx = cosxcotx):

$$\text{LHS} = \frac{\operatorname{cosec} x}{\cos x \cot x}$$

$$= \frac{\frac{1}{\sin x}}{\cos x \cdot \frac{\cos x}{\sin x}} = \frac{\frac{1}{\sin x}}{\frac{\cos^2 x}{\sin x}}$$

$$= \frac{1}{\sin x} \times \frac{\sin x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS} \quad ✓$$

Question 7 — Worked Solution

Solve $\sec^2 x + \tan x = 3$, $0° \le x \le 360°$

Replace $\sec^2 x$ with $1 + \tan^2 x$:

$$1 + \tan^2 x + \tan x = 3 \;\Rightarrow\; \tan^2 x + \tan x - 2 = 0$$

$$(\tan x - 1)(\tan x + 2) = 0$$

$\tan x = 1$: $x = 45°, 225°$

$\tan x = -2$: $x = 180° - \arctan(2) = 116.57°$, $x = 360° - \arctan(2) = 296.57°$

$$\therefore x = 45°,\ 116.57°,\ 225°,\ 296.57°$$

Question 8 — Worked Solution

Solve $2\text{cosec}^2x - 3\cot x - 1 = 0,\ 0 \le x \le 2\pi$

Replace cosec^2x with $1 + \cot^2x$:

$$2(1+\cot^2 x) - 3\cot x - 1 = 0 \Rightarrow 2\cot^2 x - 3\cot x + 1 = 0$$

$$(2\cot x - 1)(\cot x - 1) = 0$$

$\cot x = 1 \to \tan x = 1 \to x = \pi/4,\ 5\pi/4$

$\cot x = 1/2 \to \tan x = 2 \to x = \arctan(2) = 1.11$ rad, $x = \pi + \arctan(2) = 4.25$ rad

$$\therefore x = \frac{\pi}{4},\ 1.11,\ \frac{5\pi}{4},\ 4.25\ (3\text{ s.f.})$$

Question 9 — Worked Solution

Solve $3\cot^2 x + 2\cot x - 1 = 0$, $0 \le x \le 2\pi$

Factorise as a quadratic in cotx:

$$(3\cot x - 1)(\cot x + 1) = 0$$

$\cot x = -1 \rightarrow \tan x = -1 \rightarrow x = 3\pi/4, 7\pi/4$

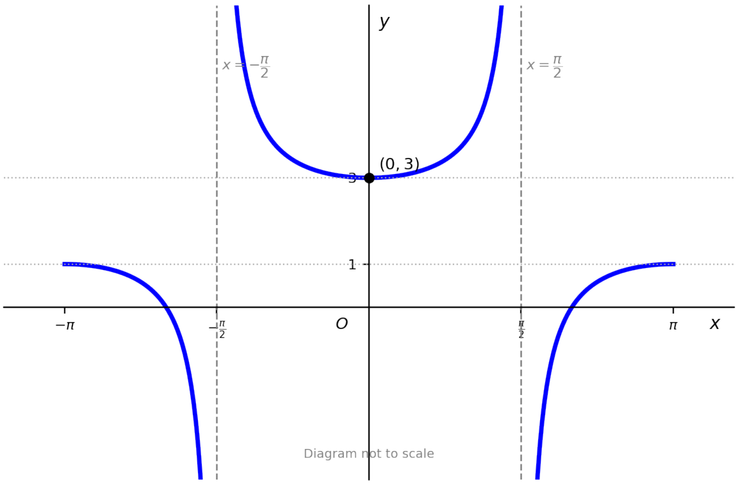
$\cot x = 1/3 \rightarrow \tan x = 3 \rightarrow x = \arctan(3) = 1.25$ rad, $x = \pi + \arctan(3) = 4.39$ rad

$$\therefore x = \frac{3\pi}{4},\ 1.25,\ \frac{7\pi}{4},\ 4.39\ (3\text{ s.f.})$$

Question 10 — Worked Solution

(a) Sketch y = secx + 2

y = secx translated 2 units upward. The minimum value of secx is 1 (at x=0), so the minimum of y = secx + 2 is 3. y-intercept: (0, 3).



(b) Asymptotes

$x = -\pi/2$ and $x = \pi/2$

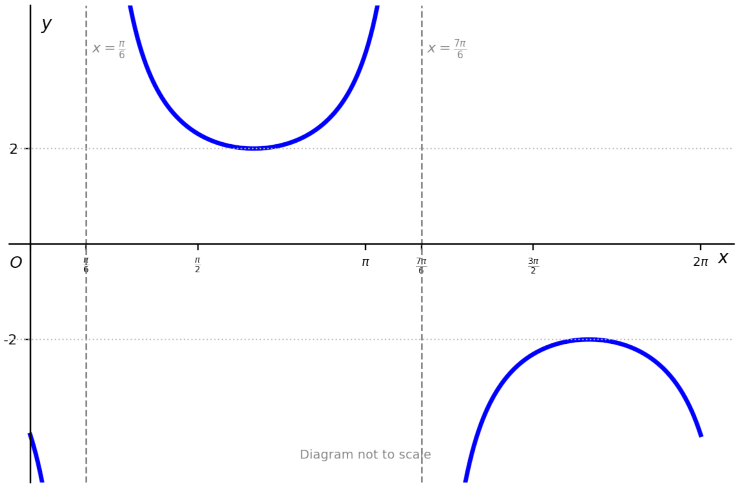
(c) Range

$y \le 1$ or $y \ge 3$

Question 11 — Worked Solution

(a) Sketch y = 2cosec(x – π/6)

y = cosec(x – π/6) is y = cosecx translated π/6 to the right. Then scaled by 2 vertically. Minima at y = 2, maxima at y = –2.



(b) Asymptotes

Asymptotes occur where sin(x – π/6) = 0, i.e. x – π/6 = 0, π, 2π, ... In [0, 2π]: x = π/6 and x = π + π/6 = 7π/6.

x = π/6   and   x = 7π/6

(c) Range

y ≤ –2   or   y ≥ 2

Question 12 — Worked Solution

(a) Domains and ranges

(i) arcsinx: Domain $-1 \le x \le 1$, Range $-\pi/2 \le y \le \pi/2$

(ii) arccosx: Domain $-1 \le x \le 1$, Range $0 \le y \le \pi$

(iii) arctanx: Domain $x \in$ ■, Range $-\pi/2 < y < \pi/2$

(b) Exact values

(i) $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ (since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$)

(ii) $\arccos(-1) = \pi$ (since $\cos\pi = -1$)

(iii) $\arctan(-1) = -\frac{\pi}{4}$ (since $\tan(-\frac{\pi}{4}) = -1$)

Question 13 — Worked Solution

(a) Solve arctan(2x – 1) = π/4

Apply tan to both sides:

$$2x - 1 = \tan\left(\frac{\pi}{4}\right) = 1$$

$$2x = 2$$

$$\therefore x = 1$$

(b) Solve arcsin(3x + 1) = –π/6

Apply sin to both sides:

$$3x + 1 = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$3x = -\frac{3}{2}$$

$$\therefore x = -\frac{1}{2}$$

Question 14 — Worked Solution

Prove $\cot^2 x - \cos^2 x \equiv \cos^2 x \cot^2 x$

Start with the left-hand side:

$$\text{LHS} = \cot^2 x - \cos^2 x = \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$$

$$= \cos^2 x\left(\frac{1}{\sin^2 x} - 1\right) = \cos^2 x \cdot \frac{1 - \sin^2 x}{\sin^2 x}$$

$$= \cos^2 x \cdot \frac{\cos^2 x}{\sin^2 x} = \cos^2 x \cot^2 x = \text{RHS} \quad ✓$$