eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 1 (WST01)

Statistics 1 Practice Paper 2

Worked Solutions

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WST01/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	The larger of two spins	10	2
2	Combining and correcting two classes	11	3
3	Regression with coded data	11	5
4	Normal mean and standard deviation	10	7
5	A Venn diagram of a survey	11	8
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7	Two observations of a random variable	11	12

Question paper & mark scheme | Exam questions by topic | More practice papers

1.

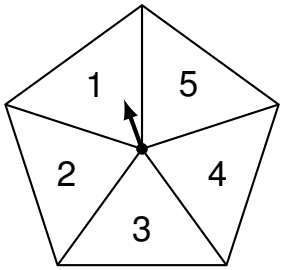

Figure 1

Figure 1 shows a fair five-sided spinner with sides numbered 1, 2, 3, 4 and 5

Rosa spins the spinner twice. The random variable X is the larger of the two scores. If the two scores are equal, X is that score.

(a) Find the probability distribution of X

(3)

(b) Find E(X)

(2)

(c) Find Var(X)

(3)

In a game, Rosa wins W points, where $W = 3X - 8$

(d) Find the probability that Rosa wins more than E(W) points.

(2)

(a) 25 equally likely pairs. $X \leqslant x$ when both scores $\leqslant x$: $P(X \leqslant x) = x^2/25$ M1

$P(X = x) = (x^2 - (x-1)^2)/25 = (2x-1)/25$ (or count: $X = 3$ from 13, 23, 33, 32, 31) A1

x: 1, 2, 3, 4, 5 P(X = x): 1/25, 3/25, 5/25, 7/25, 9/25 A1

(b) $E(X) = (1\times1 + 2\times3 + 3\times5 + 4\times7 + 5\times9)/25 = 95/25$ $E(X) = 3.8$ M1 A1

(c) $E(X^2) = (1\times1 + 4\times3 + 9\times5 + 16\times7 + 25\times9)/25 = 395/25$ M1

$= 15.8$ A1

$Var(X) = 15.8 - 3.8^2 = 15.8 - 14.44$ $Var(X) = 1.36$ A1

(d) $E(W) = 3E(X) - 8 = 3.4$. $W > 3.4 \Rightarrow 3X - 8 > 3.4 \Rightarrow X > 3.8$, so $X = 4$ or 5 M1

$P(X = 4) + P(X = 5) = 7/25 + 9/25$ $16/25 = 0.64$ A1

(Total for Question 1 is 10 marks)

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2. The marks, x, of the 20 students in class A and the marks, y, of the 30 students in class B in a test are summarised below.

Class A : $\sum x = 1240$ $\sum x^2 = 78\,160$ Class B : $\sum y = 1680$ $\sum y^2 = 97\,080$

(a) Find the mean and the standard deviation of the marks of all 50 students.

(4)

It was later discovered that one mark in class B had been recorded as 48 when it should have been 78

(b) Find the corrected mean and standard deviation of the marks of all 50 students.

(3)

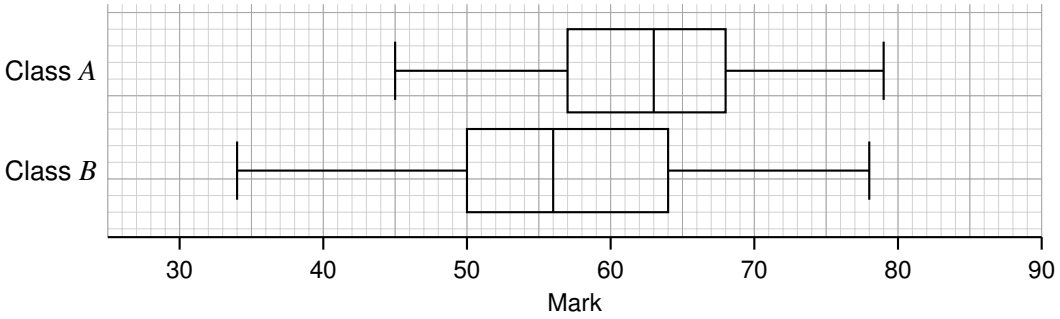

Figure 2

Figure 2 shows box plots of the corrected marks for the two classes.

(c) Compare the marks of the two classes, giving two comparisons.

(2)

The teacher scales every corrected mark using $s = 1.5x - 10$, where x is the corrected mark.

(d) Find the mean and the standard deviation of the scaled marks of all 50 students.

(2)

(a) All 50: $\sum x = 1240 + 1680 = 2920$, $\sum x^2 = 78160 + 97080 = 175240$ M1

mean $= 2920/50$ $\boxed{58.4}$ A1

$\sigma = \sqrt{175240/50 - 58.4^2} = \sqrt{3504.8 - 3410.56} = \sqrt{94.24}$ M1

$\boxed{\text{standard deviation} = 9.71}$ A1

(b) 48 becomes 78: $\sum x = 2920 + 30 = 2950$, $\sum x^2 = 175240 - 48^2 + 78^2 = 179020$ M1

mean $= 2950/50$ $\boxed{59}$ A1

$\sigma = \sqrt{179020/50 - 59^2} = \sqrt{3580.4 - 3481} = \sqrt{99.4}$ $\boxed{9.97}$ A1

(c) Median A = 63 > median B = 56, so class A generally scored higher. B1

IQR B = 64 − 50 = 14 > IQR A = 68 − 57 = 11, so class B's marks are more spread out. B1

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Question 2 continued

(d) $s = 1.5x - 10$: mean $= 1.5 \times 59 - 10$, s.d. $= 1.5 \times 9.97...$ (the -10 has no effect) M1

mean = 78.5 s.d. = 15.0 A1

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(Total for Question 2 is 11 marks)

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3. A biologist measures the length, x mm, and the mass, y g, of 8 fish of the same species. The data are coded using

$$u = \frac{x - 200}{10} \quad \text{and} \quad v = \frac{y - 100}{5}$$

and the coded results are summarised below.

$$\sum u = 24 \quad \sum v = 40 \quad \sum u^2 = 314 \quad \sum v^2 = 404 \quad \sum uv = 331$$

(a) Find S_{uu}, S_{vv} and S_{uv}

(3)

(b) Find the product moment correlation coefficient between x and y, explaining why the coding does not affect your answer.

(2)

(c) Find the equation of the regression line of y on x, in the form $y = a + bx$, giving a and b to 3 significant figures.

(4)

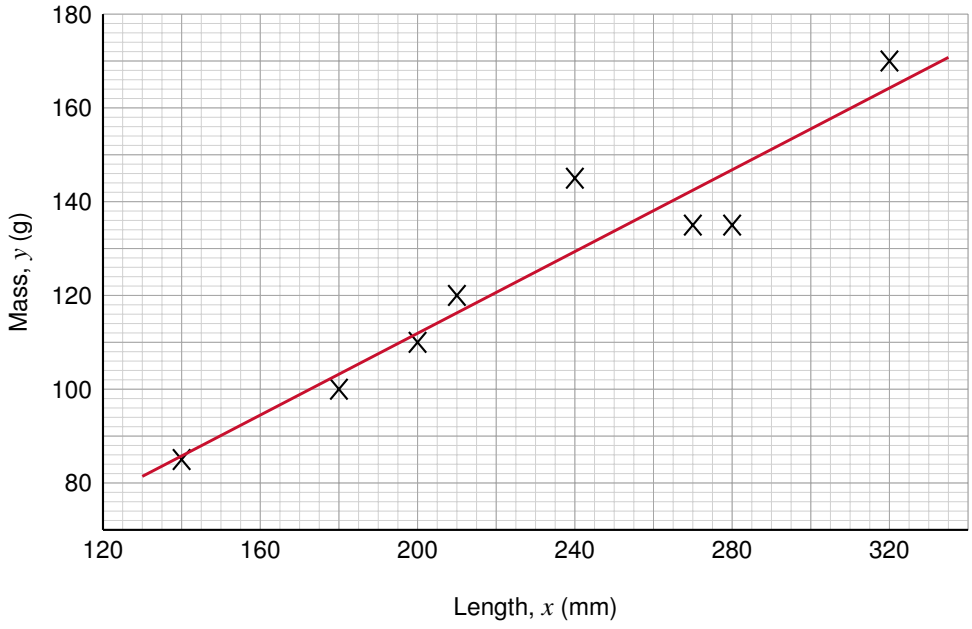

Figure 3

Figure 3 shows a scatter diagram of the original data.

(d) Draw your regression line on Figure 3.

(1)

(e) Interpret the value of b in the context of the question.

(1)

(a) $S_{uu} = 314 - 24^2/8 = 314 - 72$, $S_{vv} = 404 - 40^2/8 = 404 - 200$ M1

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Question 3 continued

$S_{uv} = 331 - (24 \times 40)/8 = 331 - 120$ $\boxed{S_{uu} = 242}$ A1

$\boxed{S_{vv} = 204}$ $\boxed{S_{uv} = 211}$ A1

(b) $r = 211 / \sqrt{242 \times 204} = 0.9496...$ $\boxed{r = 0.950}$ M1

Coding only subtracts and divides by positive numbers, so r is unchanged. A1

(c) $b_v = S_{uv}/S_{uu} = 211/242 = 0.87190...$ M1

$\bar{u} = 24/8 = 3$, $\bar{v} = 40/8 = 5$: $v = 5 - 0.8719 \times 3 + 0.8719u = 2.3843 + 0.8719u$ M1

$(y - 100)/5 = 2.3843 + 0.8719(x - 200)/10 \Rightarrow y - 100 = 11.921 + 0.43595x - 87.19$ M1

$\boxed{y = 24.7 + 0.436x}$ A1

(d) Drawn on Figure 3, through $(\bar{x}, \bar{y}) = (230, 125)$ and $(150, 90.1)$. B1

(e) Each extra 1 mm of length adds about 0.436 g to the mass of a fish. B1

(Total for Question 3 is 11 marks)

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4. The lengths, L cm, of the leaves on a plant are modelled by a normal distribution with mean μ and standard deviation σ

Of these leaves, 10% are shorter than 48.8 cm and 20% are longer than 54.1 cm.

(a) Find the value of μ and the value of σ

(5)

(b) A leaf is chosen at random. Given that it is longer than 52 cm, find the probability that it is longer than 55 cm.

(3)

Three leaves are chosen at random.

(c) Find the probability that exactly one of them is longer than 55 cm.

(2)

(a) $P(L < 48.8) = 0.1$: from the tables, $z = -1.2816$ (below the mean) B1

$P(L > 54.1) = 0.2$: $P(Z < z) = 0.8 \Rightarrow z = 0.8416$ B1

$(48.8 - \mu)/\sigma = -1.2816$ and $(54.1 - \mu)/\sigma = 0.8416$ M1

$\mu - 1.2816\sigma = 48.8$, $\mu + 0.8416\sigma = 54.1$: subtract: $2.1232\sigma = 5.3$ M1

$\sigma = 2.4962...$, $\mu = 48.8 + 1.2816 \times 2.4962... = 51.999...$ $\boxed{\mu = 52.0}$ $\boxed{\sigma = 2.50}$ A1

(b) $P(L > 55) = P(Z > (55 - 52)/2.5) = P(Z > 1.2) = 1 - 0.8849 = 0.1151$ M1

$P(L > 55 \mid L > 52) = P(L > 55)/P(L > 52) = 0.1151/0.5$ (52 is the mean) M1

$\boxed{0.230}$ A1

(c) 3 orders for exactly one: $3 \times 0.1151 \times 0.8849^2 = 0.2703...$ $\boxed{0.270}$ M1 A1

(Total for Question 4 is 10 marks)

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5.

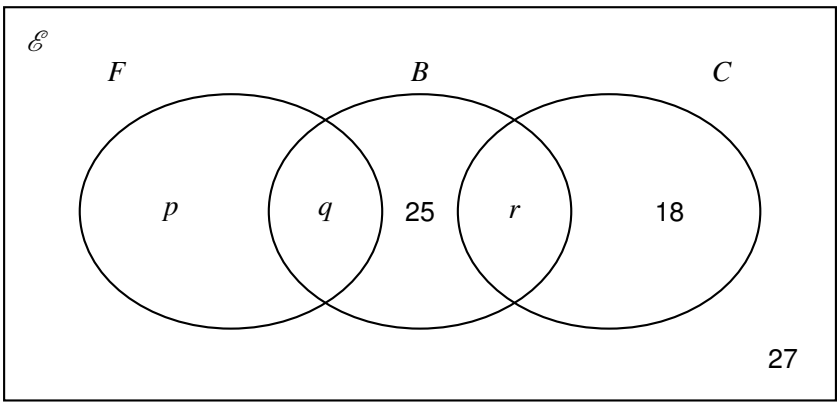

Figure 4

Each of the 120 students in a year group was asked which of three sports, football (F), basketball (B) and cricket (C), they play. The Venn diagram in Figure 4 shows the numbers of students, where p, q and r are integers.

No student plays both football and cricket. A total of 45 students play basketball. The events "a randomly chosen student plays football" and "a randomly chosen student plays basketball" are independent.

(a) Find the values of p, q and r

(5)

A student is chosen at random.

(b) Given that the student plays basketball, find the probability that the student plays cricket.

(2)

(c) Given that the student does not play cricket, find the probability that the student plays football.

(2)

(d) Show that the events B and C are not independent.

(2)

(a) Total: $p + q + 25 + r + 18 + 27 = 120 \Rightarrow p + q + r = 50$ M1

Basketball: $q + 25 + r = 45 \Rightarrow q + r = 20$ M1

Subtract: $\boxed{p = 30}$ A1

F and B independent: $q/120 = (p + q)/120 \times 45/120 \Rightarrow q = (3/8)(30 + q)$ M1

$8q = 90 + 3q \Rightarrow 5q = 90$ $\boxed{q = 18}$ $\boxed{r = 2}$ A1

(b) $P(C \mid B) = (\text{number in B and C})/(\text{number in B}) = r/45$ $\boxed{2/45}$ M1 A1

(c) Not cricket: $120 - (2 + 18) = 100$ students; football: $30 + 18 = 48$ (none in C) M1

$P(F \mid C') = 48/100$ $\boxed{0.48}$ A1

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Question 5 continued

(d) $P(B \cap C) = 2/120 = 1/60$; $P(B) \times P(C) = 45/120 \times 20/120 = 1/16$ M1

$1/60 \neq 1/16$, so B and C are not independent ✓ A1

(Total for Question 5 is 11 marks)

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6. The masses, m grams, of 200 tomatoes are summarised in the table below.

Mass (m grams)	Frequency
$60 \leqslant m < 70$	6
$70 \leqslant m < 80$	28
$80 \leqslant m < 85$	40
$85 \leqslant m < 90$	52
$90 \leqslant m < 100$	60
$100 \leqslant m < 120$	14

A histogram is drawn to represent these data. The bar for the class $70 \leqslant m < 80$ is 3 cm wide and 1.4 cm high.

(a) Find the width and the height of the bar for the class $85 \leqslant m < 90$

(3)

(b) Use linear interpolation to estimate the lower quartile and the upper quartile of the masses.

(3)

The median mass is 87.5 grams. The masses are to be modelled by a normal distribution with mean equal to this median and with the same interquartile range as the data.

(c) Find the standard deviation of the normal model.

(3)

(d) Use the model to estimate the number of these tomatoes with a mass of more than 100 grams, and comment on the suitability of the model.

(2)

(a) 85 to 90 is 5 wide, half of 70 to 80 (10 wide) $\Rightarrow$ width 1.5 cm B1

70–80 bar: $3 \times 1.4 = 4.2$ cm² for 28 tomatoes $\Rightarrow 0.15$ cm² each; $52 \times 0.15 = 7.8$ cm² M1

height $= 7.8 \div 1.5$ height 5.2 cm A1

(b) Cumulative frequencies: 6, 34, 74, 126, 186, 200

Q_1: 50th value, in 80 to 85: $80 + (50 - 34)/40 \times 5 = 80 + 2$ M1

$Q_1 = 82$ g A1

Q_3: 150th value, in 90 to 100: $90 + (150 - 126)/60 \times 10 = 90 + 4$ $Q_3 = 94$ g A1

(c) Quartiles of a normal: $\mu \pm 0.6745\sigma$ (from the tables, $P(Z < z) = 0.75$) B1

IQR: $2 \times 0.6745\sigma = 94 - 82 = 12 \Rightarrow 1.349\sigma = 12$ M1

$\sigma = 8.895\ldots$ $\sigma = 8.90$ g A1

(d) $P(M > 100) = P(Z > 12.5/8.90) = P(Z > 1.41) = 1 - 0.9207 = 0.0793$; $\times 200 = 15.9$ M1

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Question 6 continued

About 16 predicted against 14 in the table: close, so the model is reasonable. A1

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(Total for Question 6 is 11 marks)

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7. The discrete random variable X has the probability distribution below, where a and b are constants.

x	1	2	3	4	5
$P(X = x)$	0.2	a	0.25	b	0.1

Given that $E(X) = 2.85$

(a) find the value of a and the value of b

(3)

(b) Find $\text{Var}(5 - 3X)$

(3)

Two independent observations, X_1 and X_2, are taken of X

(c) Find $P(X_1 + X_2 = 4)$

(3)

(d) Find $P(X_1 > X_2)$

(2)

(a) Sum to 1: $0.2 + a + 0.25 + b + 0.1 = 1 \Rightarrow a + b = 0.45$ M1

$E(X)$: $0.2 + 2a + 0.75 + 4b + 0.5 = 2.85 \Rightarrow 2a + 4b = 1.4 \Rightarrow a + 2b = 0.7$ M1

Subtract: $b = 0.25$, $a = 0.2$ $a = 0.2$ $b = 0.25$ A1

(b) $E(X^2) = 1(0.2) + 4(0.2) + 9(0.25) + 16(0.25) + 25(0.1) = 9.75$ M1

$\text{Var}(X) = 9.75 - 2.85^2 = 1.6275$; $\text{Var}(5 - 3X) = (-3)^2 \times 1.6275 = 9 \times 1.6275$ M1

$= 14.6475$ $\text{Var}(5 - 3X) = 14.6$ A1

(c) $X_1 + X_2 = 4$ from (1, 3), (3, 1) or (2, 2) M1

$0.2 \times 0.25 + 0.25 \times 0.2 + 0.2 \times 0.2 = 0.05 + 0.05 + 0.04$ M1

0.14 A1

(d) $P(X_1 = X_2) = 0.2^2 + 0.2^2 + 0.25^2 + 0.25^2 + 0.1^2 = 0.215$

By symmetry, $P(X_1 > X_2) = P(X_1 < X_2)$, and the three cases add to 1 M1

$P(X_1 > X_2) = (1 - 0.215)/2$ 0.3925 A1

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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