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A-LEVEL MATHEMATICS • FURTHER PURE 1

Coordinate Systems

Worksheet

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Free Coordinate Systems worksheet for A-Level and International A-Level Maths, with full worked solutions

A — **Examples** & Your turn — page 2

B — **Fluency** core skills — page 4

C — **Reasoning** explain it — page 6

D — **Problem** solving — page 7

E — **Exam** practice — page 9

Tap a section to jump straight to it

By the end of this worksheet you can: use the focus and directrix of a parabola • work with the rectangular hyperbola • convert parametric forms • find tangents and normals • *Stretch:* general tangents and normals, where lines meet the curves again, and loci, lengths and areas.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH General normals, normals that meet the curve again, and focus–directrix problems with areas and lengths.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: focus and directrix

State the focus and the directrix of $y^2 = 20x$

Compare with $y^2 = 4ax$: $4a = 20$

$S(5, 0),\ x = -5$

Your turn A1

State the focus and the directrix of $y^2 = 6x$

Worked example A2: parametric to Cartesian

Find the Cartesian equation of the curve $x = 2t^2$, $y = 4t$

$t = \frac{y}{4}$, so $x = 2\left(\frac{y}{4}\right)^2 = \frac{y^2}{8}$

$y^2 = 8x$

Your turn A2

Find the Cartesian equation of the curve $x = 5t$, $y = \frac{5}{t}$

Worked example A3: the focal distance

P lies on $y^2 = 16x$ with x-coordinate 6. Find PS, where S is the focus.

PS equals the distance from P to the directrix: $x + a$

$6 + 4 = 10$

Your turn A3

$P(10, 10)$ lies on $y^2 = 10x$. Find PS, where S is the focus.

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STRETCH

Worked example A4: a tangent to a parabola

Find the tangent to $y^2 = 8x$ at $(2, 4)$

$y = \sqrt{8}x^{\frac{1}{2}}$, $\frac{dy}{dx} = \sqrt{2}x^{-\frac{1}{2}} = 1$ at $x = 2$

$y = x + 2$

Your turn A4

Find the tangent to $y^2 = 12x$ at $(3, -6)$

Worked example A5: a normal to a hyperbola

Find the normal to $xy = 16$ at $(2, 8)$

$y = 16x^{-1}$, $\frac{dy}{dx} = -\frac{16}{x^2} = -4$; normal gradient $\frac{1}{4}$

$4y = x + 30$

Your turn A5

Find the normal to $xy = 12$ at $(6, 2)$

Worked example A6: a general normal

Show that the normal to $y^2 = 4ax$ at $(at^2, 2at)$ is $y + tx = 2at + at^3$

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$; normal gradient $-t$: $y - 2at = -t(x - at^2)$

$y + tx = 2at + at^3$

Your turn A6

Show that the tangent to $xy = 25$ at $\left(5t, \frac{5}{t}\right)$ is $x + t^2y = 10t$

Worked example A7: where a line meets a curve

Find where $y = x - 1$ meets $y^2 = 8x$

$(x - 1)^2 = 8x$, $x^2 - 10x + 1 = 0$, $x = 5 \pm 2\sqrt{6}$

$(5 \pm 2\sqrt{6},\ 4 \pm 2\sqrt{6})$

Your turn A7

Find where $y = x + 1$ meets $xy = 6$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 State the focus and the directrix of

(a) $y^2 = 4x$

(b) $y^2 = 24x$

(c) $y^2 = 3x$

(d) Find the equation of the parabola with focus $(7, 0)$ and directrix $x = -7$

[4 marks]

B2 Find the Cartesian equation of each curve.

(a) $x = 4t^2,\ y = 8t$

(b) $x = 3t,\ y = \dfrac{3}{t}$

(c) $x = \dfrac{1}{4}t^2,\ y = \dfrac{1}{2}t$

(d) $x = 2t,\ y = \dfrac{8}{t}$

[4 marks]

B3 S is the focus of $y^2 = 12x$

(a) Find PS for $P(3, 6)$

(b) Find PS for $P(12, 12)$

(c) Find the points P with $PS = 10$

(d) Find PS for $P(3t^2, 6t)$

[4 marks]

B4 Find the equation of the tangent to $y^2 = 16x$

(a) at $(4, 8)$

(b) at $(1, -4)$

(c) at $(4t^2, 8t)$

(d) Where does the tangent at $t = 2$ meet the x-axis?

[4 marks]

B5 Find the equation of the normal to $xy = 36$ at

(a) $(3, 12)$

(b) $(12, 3)$

(c) $(-6, -6)$

[3 marks]

B6 Find the points where

(a) $y = x$ meets $y^2 = 9x$

(b) $y = 2$ meets $y^2 = 8x$

(c) $y = x + 3$ meets $xy = 4$

(d) $xy = 2$ meets $y^2 = 4x$

[4 marks]

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STRETCH

B7 Show that

(a) the normal to $y^2 = 4ax$ at $(at^2, 2at)$ is $y + tx = 2at + at^3$

(b) the normal to $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$ is $t^3x - ty = c(t^4 - 1)$

(c) the tangent to $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$ is $x + t^2y = 2ct$

(d) the normal in (a) meets the x-axis at $(2a + at^2, 0)$

[4 marks]

B8 Find the Cartesian equation of the locus of

(a) the midpoint of OP, $P(at^2, 2at)$ on $y^2 = 4ax$

(b) the midpoint of PS, with S the focus of $y^2 = 4ax$

(c) the midpoint of OP, $P\left(ct, \frac{c}{t}\right)$ on $xy = c^2$

(d) Show that the point $N(2a + at^2, 0)$ in B7(d) satisfies $SN = SP$

[4 marks]

B9 $P(4, 8)$ lies on $y^2 = 16x$. The tangent and the normal at P meet the x-axis at T and N

(a) Find the tangent and T

(b) Find the normal and N

(c) Find the area of triangle PTN

(d) Show that the focus is the midpoint of TN

[4 marks]

B10 $P(1, 2)$ lies on $y^2 = 4x$

(a) Find the normal at P

(b) Show that it meets the curve where $x^2 - 10x + 9 = 0$

(c) Find the point Q where the normal meets the curve again.

(d) Find the exact length PQ

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Amir says that the focus of $y^2 = 12x$ is $(12, 0)$. Explain his error, and give the focus and the directrix.

[2 marks]

C2 Bea writes the curve $y^2 = 36x$, $y > 0$, as $y = 6\sqrt{x}$ and then $\frac{dy}{dx} = 6x^{-1}$. Explain her error, and find $\frac{dy}{dx}$

[2 marks]

C3 For the normal to $y^2 = 4ax$ at $(at^2, 2at)$, Cal writes $y - 2at = t(x - at^2)$, using t as the gradient.

Explain his error, and give the correct normal.

[3 marks]

C4 The normal to $xy = 4$ at $(2t, \frac{2}{t})$ with $t = 2$ meets the curve again at the point with parameter $-\frac{1}{8}$. Dev writes the point as $\left(-\frac{1}{8}, -32\right)$

Explain his error, and find the point.

[3 marks]

STRETCH

C5 $P(3, 6)$ and $Q(12, -12)$ lie on $y^2 = 12x$. Asked for the exact length PQ, Eve writes 20.12

Explain why this would not score full marks, and give the exact length.

[3 marks]

C6 Fay finds the equation of the normal to $xy = 9$ at $(3, 3)$ as $y = -x + 6$

Explain her error, and give the equation of the normal.

[3 marks]

C7 Gus says that the distance from $P(at^2, 2at)$ on $y^2 = 4ax$ to the focus is at^2

Explain his error, and give the correct distance.

[3 marks]

C8 The tangent to $xy = c^2$ at $P\left(ct, \frac{c}{t}\right)$ meets the x-axis at A and the y-axis at B

(a) Show that P is the midpoint of AB

(3)

(b) Show that the area of triangle OAB is $2c^2$, whatever the value of t

(2)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The parabola C has equation $y^2 = 20x$ and focus S. The point P, with $y > 0$, lies on C, and the shortest distance from P to the directrix of C is 15 units.

Determine the exact perimeter of the triangle OPS, where O is the origin, giving your answer in the form $p + q\sqrt{3}$, where p and q are integers to be found. You should show all your working clearly. **[3 marks]**

D2 The rectangular hyperbola H has equation $xy = 64$. The point $P\left(8t, \frac{8}{t}\right)$, $t \neq 0$, lies on H

(a) Use calculus to show that the normal to H at P has equation $t^3x - ty = 8(t^4 - 1)$

(2)

The normal to H at P meets the y-axis at the point A

(b) Determine, in simplest form, the coordinates of A in terms of t, and hence find the exact coordinates of A in the case when $t = \frac{1}{2}$

(2)

[4 marks]

D3 The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant, and the point $P\left(ct, \frac{c}{t}\right)$, $t > 0$, lies on H. The tangent to H at P is $x + t^2y = 2ct$, and it meets the x-axis at A and the y-axis at B

Determine the coordinates of A and B in terms of c and t. Given that the area of triangle AOB, where O is the origin, is 50 square units, determine the exact value of c **[3 marks]**

D4 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The rectangular hyperbola H has equation $xy = 25$, and the point $P(10, 2.5)$ lies on H

The normal to H at P has equation $y = 4x - 37.5$. Using algebra, determine the exact coordinates of the point where this normal meets H again, showing clearly how you obtain and solve the quadratic equation.

[3 marks]

STRETCH

D5 The parabola C has equation $y^2 = 12x$, and the point $P(3t^2, 6t)$, $t \neq 0$, lies on C. The normal to C at P has equation $y + tx = 6t + 3t^3$

The normal to C at the point where $t = 2$ meets C again at the point Q. Using algebra, and without using a calculator to solve any equation, determine the exact coordinates of Q **[4 marks]**

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D6 The parabola C has equation $y^2 = 4x$, and the point $P(t^2, 2t)$ lies on C

(a) Use calculus to show that the normal to C at P has equation $y + tx = 2t + t^3$

(2)

(b) Hence find the equations of all the normals to C which pass through the point $(6, 0)$, giving your answers in the form $y = mx + k$, where m and k are constants.

(2)

[4 marks]

D7 The parabola C has equation $y^2 = 4ax$, where a is a positive constant, and S is the focus of C. The point P on C, with $y > 0$, is such that $PS = 5a$

(a) Find the coordinates of P in terms of a

(2)

(b) Given that the area of triangle OPS, where O is the origin, is 18 square units, find the value of a and hence write down the coordinates of P

(2)

[4 marks]

D8 Figure 1 shows a sketch of the parabola C with equation $y^2 = 16x$, its focus S and its directrix. The point P on C, with $y > 0$, is at a distance 20 units from S, and M is the point on the directrix such that PM is perpendicular to the directrix.

(a) Find the coordinates of P

(2)

(b) Find the exact area of the triangle PMS, showing your working clearly.

(3)

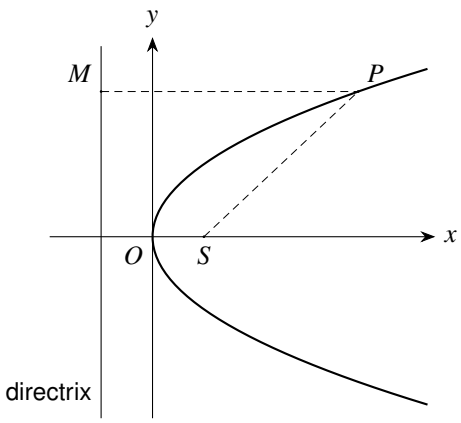

Figure 1

[5 marks]

E Exam practice

Exam-style questions in the style of WFM01. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The rectangular hyperbola H has equation $xy = 48$. The point P with coordinates $(6, 8)$ lies on H

(a) Show, using calculus, that the normal to H at P has equation $4y = 3x + 14$

(3)

The normal to H at P crosses H again at the point Q

(b) Determine the exact coordinates of Q, giving your answer in the form $\left(\frac{a}{b}, \frac{c}{d}\right)$, where a, b, c and d are integers.

(3)

(Total for Question E1 is 6 marks)

E2 The parabola C has equation $y^2 = 12x$. The points $P(3p^2, 6p)$ and $Q(3q^2, 6q)$, where $p \neq q$, lie on C

(a) Use calculus to show that the tangent to C at P has equation $py = x + 3p^2$

(2)

(b) Show that the tangents to C at P and at Q meet at the point $R\big(3pq, 3(p+q)\big)$

(3)

(c) Given that $p = 1$ and $q = 3$, determine the exact area of the triangle PQR, showing all your working.

(2)

(Total for Question E2 is 7 marks)

STRETCH

E3 The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant. The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H

(a) Use calculus to show that an equation of the normal to H at P is $t^3x - ty = c(t^4 - 1)$

(3)

The parabola C has equation $y^2 = 9x$. The normal to the hyperbola $xy = 16$ at the point $(4, 4)$ meets C at the origin and at the point Q

(b) Determine the coordinates of Q

(2)

Given that the point R is the focus of C, the line l is the directrix of C, and the line through Q and R meets l at the point S

(c) determine the exact length of QS

(4)

(Total for Question E3 is 9 marks)

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