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A-LEVEL MATHEMATICS • PURE 2

Algebraic Methods

Worked Solutions

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Algebraic Methods worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 2 Algebraic Methods worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

$$\text{A1} \quad \frac{(2x+1)(x+3)}{(x-3)(x+3)} = \frac{2x+1}{x-3}$$

$$\text{A2} \quad 2x^3 - 3x^2 - 4x + 5 = (x+1)(2x^2 - 5x + 1) + 4: \text{quotient } 2x^2 - 5x + 1, \text{remainder } 4$$

$$\text{A3} \quad f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{8}\right) + 5\left(\frac{1}{4}\right) + 2 - 3 = -\frac{1}{4} + \frac{5}{4} - 1 = 0$$

$$f\left(-\frac{1}{2}\right) = 0, \text{ so } (2x+1) \text{ is a factor}$$

STRETCH

$$\text{A4} \quad f(-2) = -16 - 4 - 2k - 6 = -26 - 2k = -2, \text{ so } -2k = 24: k = -12$$

$$\text{A5} \quad f(2) = 16 + 4 - 26 + 6 = 0, \text{ so } (x-2) \text{ is a factor: } (x-2)(2x^2 + 5x - 3)$$

$$(x-2)(2x-1)(x+3)$$

$$\text{A6} \quad f(-1) = 0: -2 + a - b + 3 = 0, \text{ so } a - b = -1; \quad f(2) = 15: 16 + 4a + 2b + 3 = 15, \text{ so } 2a + b = -2$$

$$\text{Adding: } 3a = -3: a = -1, b = 0$$

$$\text{A7} \quad \frac{(x+3)(x-1)}{(x-1)(x+1)} = \frac{x+3}{x+1} = 2, \text{ so } x+3 = 2x+2, x = 1$$

But the original fraction is undefined at $x = 1$: no solution

B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\frac{x}{x-3}$ (b) $\frac{x-3}{x+2}$ (c) $\frac{(2x+1)(x-1)}{(2x+1)(2x-1)} : \frac{x-1}{2x-1}$ (d) $\frac{-(x-3)}{(x-3)(x+3)} : \frac{1}{x+3}$

Only whole factors cancel; $3-x$ is $-(x-3)$

B2 (a) x^2+3x+2 , rem 5 (b) x^2+x-2 , rem 5 (c) x^2+4x+7 , rem 17 (d) x^2-x+2 , rem -3

Each remainder equals f at the root of the divisor: a check for every division.

B3 (a) $f(1) = 0$ (b) $f(-1) = -10$ (c) $f(2) = 14$ (d) $f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{4} + \frac{3}{2} - 4 = -\frac{5}{2}$

For $(2x-1)$, substitute the value that makes it zero: $x = \frac{1}{2}$

B4 (a) $f(1) = 0$: a factor (b) $f(-1) = 8 \neq 0$: not a factor (c) $f(3) = 0$: a factor (d) $f(-2) = 0$: a factor

Three factors found, so $f(x) = (x-1)(x-3)(x+2)$

B5 (a) $f(1) = 0$: $(x-1)(x-2)(x-3)$ (b) $f(3) = 0$: $(x-3)(x+2)^2$ (c) $f(1) = 0$: $(x-1)(2x+1)(x-3)$

Find one factor, divide, then factorise the quadratic; a repeated factor is written squared.

B6 (a) $8+2k-4=0$: $k=-2$ (b) $-8-2k-4=0$: $k=-6$ (c) $8+2k-4=6$: $k=1$ (d) $\frac{1}{2} + \frac{k}{2} - 4 = 0$: $k=7$

A factor sets f equal to 0; a remainder sets it equal to the remainder.

STRETCH

B7 (a) $x+1$ (b) $\frac{(x-2)(x+2)}{x(x+3)} \times \frac{x}{x+2} = \frac{x-2}{x+3}$ (c) $\frac{(2x-1)(x+2)}{(x-2)(x+2)} = \frac{2x-1}{x-2}$

(d) $x^3 - 8 = (x-2)(x^2+2x+4) : \frac{x^2+2x+4}{x+2}$

Dividing by a fraction means multiplying by its reciprocal.

B8 (a) x^2+2x+4 (b) x^2-3x+9 (c) $2x^2+2x-3$ (d) x^3-x^2+x-1

Write the missing powers with zero coefficients before dividing, e.g. x^3+0x^2+0x-8



- B9** (a) $f(1) = 1 + a + b + 6 = 0$, so $a + b = -7$ (b) $f(2) = 8 + 4a + 2b + 6 = 0$, so $2a + b = -7$
 (c) $a = 0$, $b = -7$ (d) $(x-1)(x-2)(x+3)$

Two conditions give two equations; the third factor comes from the constant term.

- B10** (a) $16 - 20 < 0$: 1 (b) $(x+2)(x-1)^2$: 2 (c) $x = 3, \pm\sqrt{5}$: 3 (d) $1 - 4 < 0$: 1

The quadratic factor decides: negative discriminant, no extra roots; a square, one repeated root.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** $(x+2)$ is zero when $x = -2$, so Sam must find $f(-2)$, not $f(2)$

$$f(-2) = -8 + 12 - 4 = 0: \text{so } (x+2) \text{ is a factor}$$

- C2** Lily must evaluate the expression, show it equals 0, and conclude: $f(3) = 0$, so $(x-3)$ is a factor

- C3** $(2x-1) = 0$ when $x = \frac{1}{2}$, so Kai should use $f\left(\frac{1}{2}\right)$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = 0: \text{so } (2x-1) \text{ is a factor}$$

- C4** "Completely" means every factor must be linear if possible, and $2x^2 + 5x - 12$ factorises.

$$(x+1)(2x-3)(x+4)$$

STRETCH

- C5** A cubic has at most three real roots. Here $x^2 + 2x + 5$ has discriminant $4 - 20 = -16 < 0$, so it gives no real roots.

$$\text{one real root, } x = 2$$

- C6** Terms cannot be cancelled, only factors. Factorise first: $\frac{x(x+5)}{x(x+2)} = \frac{x+5}{x+2}$

- C7** $f(-1) = 0 \times (\dots) + 9$: remainder 9

$$f(2) = 3(4 - 6 + k) + 9 = 0, \text{ so } 3(k - 2) = -9: k = -1$$



C8 (a) Substituting $x = a$: $a^n - a^n = 0$, so by the factor theorem $(x - a)$ is a factor

(b) $a = 3$, $n = 3$: dividing gives $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$

The discriminant of $x^2 + 3x + 9$ is $9 - 36 = -27 < 0$, so the only real root is $x = 3$

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $f(2) = 8a + 12 - 10 + 4 = 22$, so $8a = 16$: $a = 2$

(b) $f(-1) = -2 + 3 + 5 + 4 = 10$

D2 (a) $f\left(\frac{3}{2}\right) = \frac{27}{4} + \frac{9}{4} + \frac{3k}{2} - 6 = 3 + \frac{3k}{2} = 0$, so $k = -2$

(b) $(2x - 3)(x^2 + 2x + 2)$; discriminant $4 - 8 = -4 < 0$, so **one real root, $x = \frac{3}{2}$**

D3 $3x^3 - 2x^2 + 4x - 7 = (x - 2)(3x^2 + 4x + 12) + 17$

quotient $3x^2 + 4x + 12$, remainder 17

D4 $x^2 + 3x - 1 = (x - 2)(x + 5) + 9$, so $\frac{x^2 + 3x - 1}{x - 2} = x + 5 + \frac{9}{x - 2}$: $A = 1, B = 5, C = 9$

STRETCH

D5 (a) $f(-2) = 0$: $-8 + 4a - 2b - 12 = 0$, so $2a - b = 10$; $f(1) = -18$: $1 + a + b - 12 = -18$, so $a + b = -7$

Adding: $3a = 3$: $a = 1$, $b = -8$

(b) $x^3 + x^2 - 8x - 12 = (x + 2)(x^2 - x - 6)$: $(x + 2)^2(x - 3)$

D6 (a) $f(3) = 27 - 27 - 12 + 12 = 0$, so $(x - 3)$ is a factor

(b) $x^2(x - 3) - 4(x - 3) = (x^2 - 4)(x - 3)$: $(x - 3)(x - 2)(x + 2)$

(c) $8y^3 - 12y^2 - 8y + 12 = f(2y)$, so $2y = 3$, 2 or -2 : $y = \frac{3}{2}, 1, -1$



D7 (a) $g(2) = 8 - 24 + 26 - 10 = 0$, so $(x - 2)$ is a factor

(b) $g(x) = (x - 2)(x^2 - 4x + 5)$, and $16 - 20 = -4 < 0$, so one real root, $x = 2$

(c) $x + 1 = 2$: $x = 1$

D8 (a) $f(2) = 16 - 12 + 2a + 6 = 0$: $a = -5$

(b) $(x - 2)(2x^2 + x - 3)$: $(x - 2)(x - 1)(2x + 3)$

(c) With $x = 2^y$ the equation is $f(x) = 0$, so $2^y = 2$, 1 or $-\frac{3}{2}$; $2^y > 0$, so reject the negative value:

$y = 1$ or $y = 0$

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $f(3) = 54 - 45 + 3a + b = 0$, so $3a + b = -9$

as required

B1: = 0 shown*

(b) $f(2) = 16 - 20 + 2a + b = -9$, so $2a + b = -5$

M1

$a = -4$, $b = 3$

A1

(c) $2x^3 - 5x^2 - 4x + 3 = (x - 3)(2x^2 + x - 1)$

M1: divides by $(x - 3)$

$(x - 3)(2x - 1)(x + 1)$

A1

E2 (a) $f(1) = 1 + p + 7 - 6 = 0$, so $p = -2$

as required

*B1**

(b) $x^3 - 2x^2 + 7x - 6 = (x - 3)(x^2 + x + 10) + 24$

M1 A1

quotient $x^2 + x + 10$, remainder 24

A1: both stated

(c) $f(x) = (x - 1)(x^2 - x + 6)$, and $1 - 24 = -23 < 0$

M1

the quadratic has no real roots, so only $x = 1$

A1



STRETCH

- E3** (a) $f(1) = 2$: $a + b + c = 1$; $f(2) = 5$: $4a + 2b + c = -3$; $f(3) = 14$: $9a + 3b + c = -13$ B1 B1 B1
- (b) Subtract: $3a + b = -4$ and $5a + b = -10$, so $2a = -6$ M1
- $a = -3$, $b = 5$, $c = -1$ A1
- (c) $f\left(\frac{1}{2}\right) = \frac{1}{8} - \frac{3}{4} + \frac{5}{2} - 1 = \frac{7}{8}$ B1
- (d) $f(x) - 2 = x^3 - 3x^2 + 5x - 3$, and $1 - 3 + 5 - 3 = 0$, so $(x - 1)$ is a factor M1
- $(x - 1)(x^2 - 2x + 3)$; $4 - 12 = -8 < 0$: one real root, $x = 1$ A1

