



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Quadratics

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Quadratics worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Quadratics worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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Page 1 of 9

A

Your turn answers

Compare each line of your working with the solution.

CORE

A1 $a = 2, b = 6, c = -1$: $x = \frac{-6 \pm \sqrt{36+8}}{4} = \frac{-6 \pm \sqrt{44}}{4}$ $\sqrt{44} = 2\sqrt{11}$, so $x = \frac{-3 \pm \sqrt{11}}{2}$

A2 $3(x^2 - 6x) + 20 = 3[(x-3)^2 - 9] + 20 = 3(x-3)^2 - 7$ minimum point $(3, -7)$

A3 $a = k, b = -6, c = k - 8$: $36 - 4k(k-8) = 0$, so $k^2 - 8k - 9 = 0$, $(k-9)(k+1) = 0$ $k = 9$, $k = -1$

STRETCH

A4 $-2\left(x^2 - \frac{5}{2}x\right) + 3 = -2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 3 = \frac{49}{8} - 2\left(x - \frac{5}{4}\right)^2$

The square is never negative, so the maximum is $\frac{49}{8}$, when $x = \frac{5}{4}$

A5 $y = a(x+2)^2 - 9$ through $(1, 9)$: $9 = 9a - 9$, so $a = 2$ $2(x+2)^2 - 9 = 2x^2 + 8x - 1$, so $y = 2x^2 + 8x - 1$

A6 Two distinct real roots: $b^2 - 4ac > 0$ $(k+3)^2 - 16k > 0$, so $k^2 - 10k + 9 > 0$, so $(k-1)(k-9) > 0$

A U-shaped graph is above the axis *outside* its roots: $k < 1$ or $k > 9$

A7 Let $u = x^{\frac{1}{3}}$: $u^2 + 2u - 8 = 0$, $(u+4)(u-2) = 0$, so $u = 2$ or $u = -4$ $x = 2^3$ or $x = (-4)^3$: $x = 8$, $x = -64$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $(x-7)(x+4) = 0$: $x = 7, x = -4$ (b) $(2x+5)(2x-3) = 0$: $x = \frac{3}{2}, x = -\frac{5}{2}$

(c) $x^2 + 2x - 15 = 0$, $(x+5)(x-3) = 0$: $x = 3, x = -5$ (d) $(3x-4)(3x+4) = 0$: $x = \frac{4}{3}, x = -\frac{4}{3}$

Always rearrange to $= 0$ before you factorise. In (c), $x = 15$ or $x+2 = 15$ is wrong: a product equal to 15 tells you nothing.

B2 (a) $x = \frac{-6 \pm \sqrt{36-16}}{2} = \frac{-6 \pm \sqrt{20}}{2}$: $x = -3 \pm \sqrt{5}$ (b) $36 - 36 = 0$, $(x+3)^2 = 0$: $x = -3$ (repeated)

(c) $36 - 44 = -8$, which is negative: **no real roots** (d) $x = \frac{-6 \pm \sqrt{36-24}}{4} = \frac{-6 \pm \sqrt{12}}{4}$: $x = \frac{-3 \pm \sqrt{3}}{2}$

The number under the square root, $b^2 - 4ac$, decides everything: positive, zero, negative.

B3 (a) $(x+5)^2 - 25 + 3 = (x+5)^2 - 22$ (b) $\left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + 2 = \left(x - \frac{7}{2}\right)^2 - \frac{41}{4}$

(c) $5(x^2+4x)+12 = 5(x+2)^2 - 20 + 12 = 5(x+2)^2 - 8$ (d) $-(x^2+4x)+10 = -(x+2)^2 + 4 + 10 = 14 - (x+2)^2$

Take the x^2 coefficient out of the x terms only, including a negative one.

B4 (a) $25 - 28 = -3$: **no real roots** (b) $144 - 144 = 0$: **one repeated root**

(c) $4 + 60 = 64$: **two distinct real roots** (d) $5x^2 - 2x + 1 = 0$: $4 - 20 = -16$: **no real roots**

In (d), $a = 5$, $b = 2$, $c = -1$ would be wrong: rearrange to $= 0$ first.

B5 (a) **$(3, 5)$, minimum** (b) **$(-1, -7)$, minimum** (c) $2x - 1 = 0$ when $x = \frac{1}{2}$: **$\left(\frac{1}{2}, 4\right)$, maximum**

The 2 in (b) stretches the curve but does not move the turning point. In (c), the bracket is zero at $x = \frac{1}{2}$, not $x = 1$

B6 (a) $(x^2-1)(x^2-16) = 0$: $x = 1, x = -1, x = 4, x = -4$ (b) $(\sqrt{x}-2)(\sqrt{x}-4) = 0$: $x = 4, x = 16$

(c) $(x^3+8)(x^3-1) = 0$: $x = -2, x = 1$ (d) $u = x+1$: $(u-2)(u-3) = 0$, so $x+1 = 2$ or $x+1 = 3$: $x = 1, x = 2$

Find the block that is squared ($x^2, \sqrt{x}, x^3, x+1$), call it u , and solve for u first.



STRETCH

B7 (a) $3(x^2 - 3x) + 2 = 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 2 = \boxed{3\left(x - \frac{3}{2}\right)^2 - \frac{19}{4}}$

(b) $-2\left(x^2 - \frac{5}{2}x\right) - 1 = -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8} - 1 = \boxed{-2\left(x - \frac{5}{4}\right)^2 + \frac{17}{8}}$

(c) $(x+k)^2 - k^2 + 5 = \boxed{(x+k)^2 + 5 - k^2}$ (d) $2x^2 + 5x - 3 = 2\left(x + \frac{5}{4}\right)^2 - \frac{25}{8} - 3 = \boxed{2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}}$

Fractions, a jumbled order, a letter, then brackets to expand first. The method never changes.

B8 (a) $k^2 - 36 = 0$: $\boxed{k = 6, k = -6}$ (b) $16 - 4k^2 = 0$: $\boxed{k = 2, k = -2}$

(c) $(k-1)^2 - 4(k+2) = 0$, so $k^2 - 6k - 7 = 0$, $(k-7)(k+1) = 0$: $\boxed{k = 7, k = -1}$

(d) $4k^2 - 4(k-2)(k+3) = 0$, so $4k^2 - 4k^2 - 4k + 24 = 0$: $\boxed{k = 6}$

Write down a , b and c before you substitute. In (d) the k^2 terms cancel, so there is only one value.

B9 (a) $(2^x)^2 - 6(2^x) + 8 = 0$, $(2^x - 2)(2^x - 4) = 0$: $\boxed{x = 1, x = 2}$

(b) $u = \sqrt{x}$: $u^2 - 4u - 1 = 0$, $u = 2 \pm \sqrt{5}$. Since $\sqrt{x}$ cannot be negative, reject $2 - \sqrt{5}$, so $x = (2 + \sqrt{5})^2 = \boxed{9 + 4\sqrt{5}}$

(c) $u = \frac{1}{x}$: $2u^2 + 3u - 2 = 0$, $(2u-1)(u+2) = 0$, so $\frac{1}{x} = \frac{1}{2}$ or $\frac{1}{x} = -2$: $\boxed{x = 2, x = -\frac{1}{2}}$

(d) $(x^2 + 5)(x^2 - 2) = 0$. The factor $x^2 + 5$ is never zero: $\boxed{x = \sqrt{2}, x = -\sqrt{2}}$

B10 (a) $y = a(x-1)(x-5)$; at $(0, 10)$, $5a = 10$, $a = 2$: $\boxed{y = 2x^2 - 12x + 10}$

(b) $y = a(x-2)^2 - 3$; at $(0, 5)$, $4a - 3 = 5$, $a = 2$: $\boxed{y = 2x^2 - 8x + 5}$

(c) $y = a(x+3)^2$; at $(0, -18)$, $9a = -18$, $a = -2$: $\boxed{y = -2x^2 - 12x - 18}$

(d) The roots solve $(x-1)^2 = 2$, that is $x^2 - 2x - 1 = 0$, so $y = a(x^2 - 2x - 1)$; at $(0, -3)$, $-a = -3$, $a = 3$: $\boxed{y = 3x^2 - 6x - 3}$

Never forget the constant a : a curve through the roots alone is not enough.



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** Dividing by x loses the solution $x = 0$, because you cannot divide by x if x might be 0

$$x^2 - 6x = 0, \text{ so } x(x - 6) = 0: \boxed{x = 0, x = 6}$$

- C2** $2(x+4)^2 - 29 = 2x^2 + 16x + 3$, which is not $2x^2 + 8x + 3$. Ben did not take the 2 out of the $8x$ term.

$$2(x^2 + 4x) + 3 = 2(x+2)^2 - 8 + 3 = \boxed{2(x+2)^2 - 5}$$

- C3** $4(x^2 - 3x) + 10 = 4\left(x - \frac{3}{2}\right)^2 - 9 + 10 = 4\left(x - \frac{3}{2}\right)^2 + 1$

A square is never negative, so $4\left(x - \frac{3}{2}\right)^2 + 1 \geq 1$, which is positive for every x . Minimum value **1**

- C4** Ali's factors do not multiply back to the equation: $(x-5)(x+3) = x^2 - 2x - 15$, not $3x^2 - 6x - 45$. Examiners do not give the method mark, because the roots could have come from a calculator.

Correct: $3x^2 - 6x - 45 = 3(x-5)(x+3) = 0$ (or divide by 3 first): $\boxed{x = 5, x = -3}$

STRETCH

- C5** $16 - 4(k-2) = 0$ gives $\boxed{k = 6}$. Jo's condition only applies to a quadratic. When $\boxed{k = 2}$ the x^2 term disappears, and the equation becomes the linear equation $4x + 1 = 0$, which has exactly one real root,
- $$x = -\frac{1}{4}$$

- C6** The discriminant is $k^2 - 4(k-2) = k^2 - 4k + 8 = (k-2)^2 + 4$
 $(k-2)^2$ is never negative, so the discriminant is at least 4. It is positive for every k , so there are always two distinct real roots.

- C7** $\sqrt{x}$ is never negative, so $\sqrt{x} = -2$ has no solution. Squaring -2 gave a false answer. Check: $4 - 2 \times 2 - 8 = -8$, which is not 0

Only $\boxed{x = 16}$ works: $16 - 2 \times 4 - 8 = 0$



C8 (a) Divide by a , then complete the square:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0, \text{ so } \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \text{ (the } \pm \text{ covers the sign of } a\text{), so } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

as required

(b) When $b^2 = 4ac$ the square root is 0, so both solutions are $x = -\frac{b}{2a}$: the roots are equal.

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) Roots 1 and 9: $f(x) = a(x-1)(x-9)$. At the minimum point, $-8 = a(4)(-4) = -16a$, so $a = \frac{1}{2}$

$$\frac{1}{2}(x^2 - 10x + 9) = \frac{1}{2}x^2 - 5x + \frac{9}{2}$$

(b) $f(0) = \frac{9}{2}$: $\left(0, \frac{9}{2}\right)$

D2 $2x^2 - 5x - 9 = x + 11$

$$2x^2 - 6x - 20 = 0. \text{ Divide by 2: } x^2 - 3x - 10 = 0, \text{ so } (x-5)(x+2) = 0$$

Using $y = x + 11$: $P(-2, 9)$, $Q(5, 16)$

Writing $(x-5)(x+2)$ straight after $2x^2 - 6x - 20 = 0$ loses marks: those factors multiply to $x^2 - 3x - 10$. Divide by 2 first, or write $(2x-10)(x+2)$.

D3 (a) With $u = x^2$: $3u + \frac{4}{u} = 13$. Multiply by u :

$$3u^2 + 4 = 13u, \text{ so } 3u^2 - 13u + 4 = 0 \quad \text{as required}$$

(b) $(3u-1)(u-4) = 0$, so $x^2 = \frac{1}{3}$ or $x^2 = 4$

$$x = 2, x = -2, x = \frac{\sqrt{3}}{3}, x = -\frac{\sqrt{3}}{3} \quad \left(\sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3} \text{ is the simplest exact form}\right)$$



D4 (a) $-40(x^2 - 9x) - 500 = -40 \left[\left(x - \frac{9}{2}\right)^2 - \frac{81}{4} \right] - 500 = 310 - 40 \left(x - \frac{9}{2}\right)^2$

$A = 310$, $B = 40$, $C = 4.5$

(b) Maximum daily profit $\pounds 310$, when each sandwich is sold for $\pounds 4.50$

STRETCH

D5 (a) $k = -1$: $-2x^2 + 6x + 6 = 0$, so $x^2 - 3x - 3 = 0$, and $x = \frac{3 \pm \sqrt{9+12}}{2}$: $x = \frac{3 \pm \sqrt{21}}{2}$

(b) No real roots: $36 - 4(k-1)(k+7) < 0$, so $36 - 4k^2 - 24k + 28 < 0$

$4k^2 + 24k - 64 > 0$, so $k^2 + 6k - 16 > 0$, $(k+8)(k-2) > 0$: $k < -8$ or $k > 2$

Check $k = 1$: the equation becomes $6x + 8 = 0$, which has a root, and 1 is not in the set.

D6 (a) $x^2 - 2x + 4 = mx$

$x^2 - (2+m)x + 4 = 0$. Touching means equal roots: $(2+m)^2 - 16 = 0$

$m^2 + 4m + 4 - 16 = 0$, so $m^2 + 4m - 12 = 0$ **as required**

(b) $(m+6)(m-2) = 0$, so $m = 2$ or $m = -6$

$m = 2$: $x^2 - 4x + 4 = (x-2)^2 = 0$, so $x = 2$ $m = -6$: $x^2 + 4x + 4 = (x+2)^2 = 0$, so $x = -2$

Using $y = x^2 - 2x + 4$: $A(-2, 12)$, $B(2, 4)$ (l_1 is $y = -6x$, l_2 is $y = 2x$)

D7 (a) $2(x^2 - 6x) + 13 = 2(x-3)^2 - 18 + 13 = 2(x-3)^2 - 5$

(b) $2(x-3)^2 = 5$, so $(x-3)^2 = \frac{5}{2}$ and $x-3 = \pm \sqrt{\frac{5}{2}}$, where $\sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2}$

$x = 3 - \frac{1}{2}\sqrt{10}$, $x = 3 + \frac{1}{2}\sqrt{10}$

(c) $AB = \sqrt{10}$. The height is 5, the distance from $M(3, -5)$ to the x -axis. Area = $\frac{1}{2} \times \sqrt{10} \times 5 = \frac{5\sqrt{10}}{2}$

D8 (a) $u = \sqrt{x}$: $u^2 - 6u + 5 = 0$, $(u-1)(u-5) = 0$, so $\sqrt{x} = 1$ or $\sqrt{x} = 5$: $x = 1$, $x = 25$

(b) $(u-3)^2 = 9-k$, so $\sqrt{x} = 3 \pm \sqrt{9-k}$

Two different values need $9-k > 0$, so $k < 9$

Both values must be allowed, because $\sqrt{x} \geq 0$. The smaller needs $3 - \sqrt{9-k} \geq 0$, so $9-k \leq 9$, so $k \geq 0$

$0 \leq k < 9$ Check: $k = 0$ gives $x = 0$ and $x = 36$, two solutions; $k = -1$ gives only one.

The lower limit is the mark most often lost (Jun 2024 examiner report).



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

- E1** (a) $y = k(x - 1)$ B1: or $y = kx - k$
- (b) $x^2 + 2x + 6 = k(x - 1)$ M1: equates and collects to $= 0$
 $x^2 + (2 - k)x + 6 + k = 0$
 At least one root: $(2 - k)^2 - 4(6 + k) \geq 0$, so $k^2 - 8k - 20 \geq 0$ as required A1*: cso
- (c) $k^2 - 8k - 20 \geq 0$, $(k - 10)(k + 2) = 0$: critical values 10 and -2 M1
 $k \leq -2$ or $k \geq 10$ A1: outside region
-
- E2** (a) $-2(x^2 - 4x) - 5 = -2[(x - 2)^2 - 4] - 5 = 3 - 2(x - 2)^2$ M1: $\pm 2(x - 2)^2$ seen
- $a = 3$, $b = -2$, $c = -2$ A1: two correct; A1: all three
- (b) $(2, 3)$ B1ft
- (c) $2(x - 2)^2 = 3$, so $x = 2 \pm \sqrt{\frac{3}{2}}$ M1: solves their (a) = 0 exactly
- $AB = 2\sqrt{\frac{3}{2}} = \sqrt{4 \times \frac{3}{2}} = \sqrt{6}$ A1



STRETCH

E3 (a) $u^2 = x^2 + 2 + \frac{1}{x^2}$

M1: squares u to three terms

$$2u^2 - 9u + 10 = 2x^2 + 4 + \frac{2}{x^2} - 9x - \frac{9}{x} + 10$$

$$= 2x^2 - 9x + 14 - \frac{9}{x} + \frac{2}{x^2}, \text{ so the equation is } 2u^2 - 9u + 10 = 0$$

as required

A1*

(b) $(2u - 5)(u - 2) = 0$, so $u = \frac{5}{2}$ or $u = 2$

M1: solves the quadratic in u

$$x + \frac{1}{x} = 2: x^2 - 2x + 1 = 0, (x - 1)^2 = 0$$

$$x + \frac{1}{x} = \frac{5}{2}: 2x^2 - 5x + 2 = 0, (2x - 1)(x - 2) = 0$$

dM1: both quadratics in x

$$x = \frac{1}{2}, x = 1, x = 2$$

A1: all three, no extras

(c) $x + \frac{1}{x} = k$, so $x^2 - kx + 1 = 0$ ($x = 0$ is never a root)

M1

At least one real solution: $k^2 - 4 \geq 0$

dM1: discriminant ≥ 0

$$k \leq -2 \text{ or } k \geq 2$$

A1

