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A-LEVEL MATHEMATICS • FURTHER PURE 1

Series

Worked Solutions

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Full worked solutions for the free Series worksheet, A-Level and International A-Level Maths

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 1 Series worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $4 \times \frac{1}{2}n(n+1) + 3n = 2n^2 + 5n$: $\boxed{n(2n+5)}$

A2 $\frac{1}{4}(81)(100)$: $\boxed{2025}$

A3 $f(15) - f(5) = 1240 - 55$: $\boxed{1185}$

STRETCH

A4 $\frac{1}{3}n(n+1)(2n+1) - \frac{1}{2}n(n+1) = \frac{1}{6}n(n+1)\big(2(2n+1) - 3\big)$: $\boxed{\frac{1}{6}n(n+1)(4n-1)}$

A5 $\frac{1}{4}n^2(n+1)^2 - n(n+1) = \frac{1}{4}n(n+1)\big(n(n+1) - 4\big)$: $\boxed{\frac{1}{4}n(n+1)(n^2+n-4)}$

A6 $\frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}n(2n+1)\big(2(4n+1) - (n+1)\big)$: $\boxed{\frac{1}{6}n(2n+1)(7n+1)}$

A7 $n(n+1) + n = n^2 + 2n = 168$, $(n+14)(n-12) = 0$: $\boxed{n = 12}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{210}$ **(b)** $\boxed{385}$ **(c)** $\boxed{441}$ **(d)** 50×3: $\boxed{150}$

A constant summed n times gives n times the constant.

B2 **(a)** $\boxed{n(n+6)}$ **(b)** $\boxed{\frac{1}{2}n(2n^2+3n+3)}$ **(c)** $\boxed{\frac{1}{6}n(2n^2+9n+13)}$ **(d)** $\boxed{\frac{1}{3}n(n+1)(n-4)}$

Expand, split, then use the standard results.

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B3 **(a)** $820-210$: **610** **(b)** $650-30$: **620** **(c)** $1296-36$: **1260** **(d)** $2(465-45)$: **840**

Subtract the sum up to one less than the lower limit.

B4 **(a)** $\frac{1}{4}n(n+1)\big(n(n+1)+2(2n+1)\big)$, giving the result **(b)** $\frac{1}{12}n(n+1)\big(3n(n+1)+2(2n+1)\big)=\frac{1}{12}n(n+1)(3n^2+7n+2)$, and $3n^2+7n+2=(n+2)(3n+1)$

(c) $\frac{2}{3}n(n+1)(2n+1)+2n(n+1)+n=\frac{1}{3}n\big(2(n+1)(2n+1)+6(n+1)+3\big)$, giving the result **(d)** $\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)=\frac{1}{6}n(n+1)(2n-2)$, giving the result

Take out the common factor before expanding.

B5 **(a)** $2n-n$: n **(b)** $2n(2n+1)-2n(n+1)$: $2n(3n+1)$ **(c)** $n^2(2n+1)^2-\frac{1}{4}n^2(n+1)^2$: $\frac{1}{4}n^2(3n+1)(5n+3)$

Always $f(2n)-f(n)$

B6 **(a)** $n^2+n-650=0$: **25** **(b)** $n^2=400$: **20** **(c)** $\frac{1}{2}n(n+1)=55$: **10** **(d)** $3n^2+n-310=0$: **10**

n must be a positive integer: reject the other root.

STRETCH

B7 **(a)** $\frac{a}{2}=\frac{3}{2}$: $a=3$ **(b)** $\frac{a}{2}+b=\frac{7}{2}$: $b=2$ **(c)** $670-185$: **485** **(d)** $3n^2+7n>2000$: $n=25$

Compare coefficients of n^2 and n

B8 **(a)** $n(2n+1)-\frac{1}{2}(n-1)n$: $\frac{3}{2}n(n+1)$ **(b)** $n+1$ **(c)** $2n(2n+1)$ **(d)** $n(2n+1)-n(n+1)$: n^2

A lower limit n subtracts $f(n-1)$

B9 **(a)** $\sum_{r=1}^{20} r(r+1)$: **3080** **(b)** $4\sum_{r=1}^{20} r^2$: **11480** **(c)** $\sum_{r=1}^{30}(2r-1)$: **900** **(d)** $\sum_{r=1}^{20}(3r+2)$: **670**

Find the general term and the number of terms first.

B10 **(a)** $\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)=\frac{1}{12}n(n+1)(3n^2-n-2)$, and $3n^2-n-2=(n-1)(3n+2)$

(b) **2640** **(c)** $41230-2640$: **38590** **(d)** $0+4+18=22=\frac{1}{12}(3)(2)(4)(11)$

One formula gives values, sums between limits and a check.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $\sum_{r=1}^{n} 5 = 5n$, not 5: $\boxed{n(n+1)+5n = n(n+6)}$

C2 Subtracting up to 10 removes the term $r = 10$; subtract up to 9: $210 - 45$: $\boxed{\mathbf{165}}$

C3 $n = 4$: $1+4+9+16 = 30$ but $10^2 = 100$: $\boxed{\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)}$

C4 $n = 2$: $1\times 2 + 2\times 3 = 8$ but $3\times 5 = 15$. Expand first: $\boxed{\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r}$

STRETCH

C5 The terms up to n must be removed: $\sum_{r=1}^{2n} r - \sum_{r=1}^{n} r = n(2n+1) - \frac{1}{2}n(n+1)$: $\boxed{\frac{1}{2}n(3n+1)}$

C6 n must be a positive integer, and $\sum_{r=1}^{13} r = 91 < 100 < 105 = \sum_{r=1}^{14} r$, so $\boxed{\textbf{no } n \textbf{ gives exactly 100}}$

C7 The form asked for is factorised: $\frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) = \frac{1}{6}n(n+1)(2n+10)$: $\boxed{\frac{1}{3}n(n+1)(n+5)}$

C8 **(a)** $n\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2 = \frac{1}{2}n^2(n+1) - \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}n(n+1)\big(3n-(2n+1)\big) = \frac{1}{6}n(n+1)(n-1)$, as required

(b) $n = 10$, the terms $r(10-r)$ for $r = 1$ to 9 (the $r = 10$ term is 0): $\frac{1}{6}(10)(11)(9)$: $\boxed{\mathbf{165}}$

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $3\times\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)=\frac{1}{2}n(n+1)\big((2n+1)-1\big)=\frac{1}{2}n(n+1)(2n)=n^2(n+1)$, as required

D2 **(a)** $\sum_{r=1}^{n}(r^3+4r^2+3r)=\frac{1}{4}n^2(n+1)^2+\frac{2}{3}n(n+1)(2n+1)+\frac{3}{2}n(n+1)$

$=\frac{1}{12}n(n+1)\big(3n(n+1)+8(2n+1)+18\big)=\frac{1}{12}n(n+1)(3n^2+19n+26)$, as required

(b) $\frac{1}{12}(10)(11)(516)$: **4730**

D3 $\sum_{r=3}^{30}r(r+2)=\text{f}(30)-\text{f}(2)=\frac{1}{6}(30)(31)(67)-\frac{1}{6}(2)(3)(11)=10385-11$: **10374**

D4 $\sum_{r=1}^{n}(25r^2-20r+4)=\frac{25}{6}n(n+1)(2n+1)-10n(n+1)+4n=\frac{n}{6}(50n^2+75n+25-60n-60+24)$

$a=50,\ b=15,\ c=-11$

STRETCH

D5 **(a)** $n(n+1)(2n+1)+n(n+1)=n(n+1)(2n+2)=2n(n+1)^2$, as required

(b) $2n(n+1)^2=10n(n+1)$, so $n+1=5$: $n=4$

D6 **(a)** $\frac{1}{6}n(n+1)(2n+1)+\frac{5}{2}n(n+1)=\frac{1}{6}n(n+1)(2n+16)=\frac{1}{3}n(n+1)(n+8)$, as required

(b) $\frac{1}{3}n(n+1)(n+8)=6n(n+1)$, so $n+8=18$ ($n=0$ and $n=-1$ are not positive): $n=10$

D7 **(a)** $n(n+1)(2n+1)-n=n(2n^2+3n)=n^2(2n+3)$, as required

(b) $\text{f}(2n)-\text{f}(n)=4n^2(4n+3)-n^2(2n+3)=n^2(14n+9)$, as required

D8 **(a)** $\sum_{r=1}^{n}(r^2+r-2)=\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)-2n=\frac{1}{6}n(2n^2+6n-8)=\frac{1}{3}n(n-1)(n+4)$, as required

(b) $\sum_{r=19}^{39}(r+2)(r-1)=\text{f}(39)-\text{f}(18)=21242-2244$: **18998**

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E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\sum_{r=1}^{n}(2r^2+3r-2)$ *B1*

$=\frac{1}{3}n(n+1)(2n+1)+\frac{3}{2}n(n+1)-2n$ *M1*

$=\frac{n}{6}\left(2(2n^2+3n+1)+9(n+1)-12\right)$ *M1*

$=\frac{n}{6}(4n^2+15n-1)$: $a=4,\ b=15,\ c=-1$ *A1*

(b) $\frac{20}{6}(1600+300-1)$: **6330** *M1 A1*

E2 **(a)** $\sum_{r=1}^{n}(6r^2-2r+1)=n(n+1)(2n+1)-n(n+1)+n$ *M1 A1*

$=n\left((n+1)(2n+1)-(n+1)+1\right)$ *M1*

$=n(2n^2+2n+1)$, as required *A1**

(b) $n(2n^2+2n+1)-\frac{1}{2}n(n+1)(2n+1)=237$, so $2n^3+n^2+n-474=0$ *M1*

$(n-6)(2n^2+13n+79)=0$, and $13^2-8\times79<0$, so the quadratic has no real roots *M1*

exactly one: $n=6$ *A1*

STRETCH

E3 **(a)** $\sum_{r=1}^{n}(r^3-4r)=\frac{1}{4}n^2(n+1)^2-2n(n+1)$ *M1*

$=\frac{1}{4}n(n+1)\left(n(n+1)-8\right)$ *M1*

$=\frac{1}{4}n(n+1)(n^2+n-8)$, as required *A1**

(b) $\frac{1}{4}n(n+1)(n^2+n-8)=3n(n+1)$ *M1*

$n^2+n-20=0$, $(n+5)(n-4)=0$ *M1*

$n=4$ *A1*

(c) f(20) − f(9) *M1*

$=43260-1845$ *M1*

41415 *A1*