



eClassroom

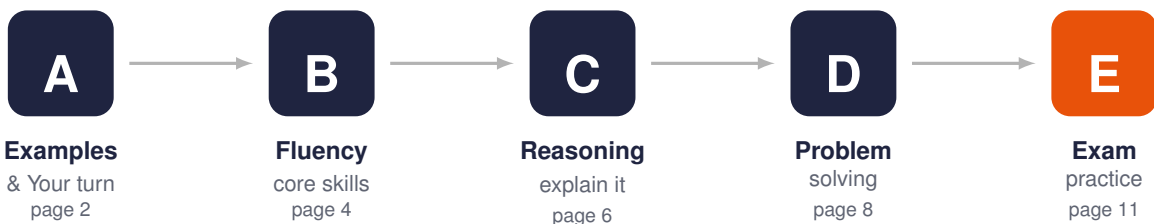
A-LEVEL MATHEMATICS • PURE 1

Differentiation

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Differentiation worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: differentiate powers of x , including negative and fractional powers • rewrite roots, brackets and fractions before differentiating • find the gradient of a curve at a point • find the equations of tangents and normals • *Stretch:* use second derivatives, find points with a given gradient, find unknown constants and prove results about tangents.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Points with a given gradient, unknown constants from tangents, first principles and proofs about tangents.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Differentiate $y = 4x^3 - \frac{6}{x^2} + 5\sqrt{x}$

Write every term as a power of x first.

$$y = 4x^3 - 6x^{-2} + 5x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 12x^2 + 12x^{-3} + \frac{5}{2}x^{-\frac{1}{2}}$$

Your turn A1

Differentiate $y = 2x^5 + \frac{3}{x} - 8\sqrt{x}$

Worked example A2

Find the gradient of the curve $y = x^3 - 4x + 1$ at the point where $x = 2$

$$\frac{dy}{dx} = 3x^2 - 4$$

Substitute $x = 2$ into $\frac{dy}{dx}$, not into y .

$$3 \times 4 - 4 = 8$$

Your turn A2

Find the gradient of the curve $y = 2x^2 + \frac{8}{x}$ at the point where $x = 2$

Worked example A3

Find the equation of the tangent to $y = x^2 - 3x + 5$ at the point where $x = 4$

Point: $y = 16 - 12 + 5 = 9$. Gradient: $2x - 3 = 5$

$$y - 9 = 5(x - 4)$$

$$y = 5x - 11$$

Your turn A3

Find the equation of the tangent to $y = x^3 - 2x$ at the point where $x = -1$



STRETCH

Worked example A4: rewrite first

Differentiate $y = \frac{(2x+1)^2}{x}$

Expand and split: $\frac{4x^2 + 4x + 1}{x} = 4x + 4 + x^{-1}$

$$\frac{dy}{dx} = 4 - x^{-2}$$

Your turn A4

Differentiate $y = \frac{x^2 - 5x}{2\sqrt{x}}$

Worked example A5: a normal

Find the equation of the normal to $y = \frac{4}{x}$ at the point (2, 2)

$\frac{dy}{dx} = -4x^{-2} = -1$ at $x = 2$, so the normal has gradient 1

$$y = x$$

Your turn A5

Find the equation of the normal to $y = 2\sqrt{x}$ at the point (4, 4), in the form $ax + by + c = 0$

Worked example A6: second derivatives

$f(x) = x^4 - 6\sqrt{x}$. Find $f''(1)$

$f'(x) = 4x^3 - 3x^{-\frac{1}{2}}$, $f''(x) = 12x^2 + \frac{3}{2}x^{-\frac{3}{2}}$

$$f''(1) = \frac{27}{2}$$

Your turn A6

$f(x) = 2x^3 + \frac{8}{x}$. Find $f''(2)$

Worked example A7: a given gradient

Find the points on $y = x^3 - 6x^2 + 2$ where the gradient is 15

$3x^2 - 12x = 15$, so $x^2 - 4x - 5 = 0$, $(x-5)(x+1) = 0$

Find y for each x from the curve's equation.

$$(5, -23), (-1, -5)$$

Your turn A7

Find the points on $y = x^3 + 3x^2 - 7x$ where the gradient is 2



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Differentiate with respect to x

(a) x^6

(b) $5x^{-2}$

(c) $4x^{\frac{3}{2}}$

(d) $6x^{-\frac{1}{3}}$

[4 marks]**B2** Write each expression as a power of x , then differentiate.

(a) $\sqrt{x}$

(b) $\frac{3}{x}$

(c) $\frac{2}{\sqrt{x}}$

(d) $x^2\sqrt{x}$

[4 marks]**B3** Find the gradient of the curve $y = x^3 - 12x$ at the point where

(a) $x = 0$

(b) $x = 2$

(c) $x = -2$

(d) $x = 3$

[4 marks]**B4** Find the equation of the tangent at the point where $x = 1$ to the curve

(a) $y = x^2$

(b) $y = x^2 + 3$

(c) $y = 3x^2$

(d) $y = x^3$

[4 marks]

CORE*continued: normals and second derivatives*

B5 Find the equation of the normal at the point where $x = 1$, in the form $ax + by + c = 0$, to the curve

(a) $y = x^2$

(b) $y = x^3$

(c) $y = \frac{2}{x}$

[3 marks]

B6 Find $\frac{d^2y}{dx^2}$ when y is

(a) x^4

(b) x^{-1}

(c) $\sqrt{x}$

(d) $4x^{\frac{3}{2}}$

[4 marks]

STRETCH

B7 $f(x) = x^3 - 3x$. Find the values of x for which

(a) $f'(x) = 0$

(b) $f'(x) = 9$

(c) $f'(x) = -3$

(d) $f'(x) = -6$

[4 marks]

B8 The points $P(2, 8)$ and $Q(2+h, (2+h)^3)$ lie on the curve $y = x^3$

(a) Find the gradient of the chord PQ when $h = 1$

(b) Find the gradient of the chord PQ when $h = 0.1$

(c) Show that the gradient of the chord PQ is $12 + 6h + h^2$

(d) Hence write down the gradient of the curve at P

[4 marks]

B9 Find the value of the constant k in each case.

(a) $y = kx^3$ has gradient 12 at $x = 2$

(b) $y = x^2 + kx$ has gradient 0 at $x = 3$

(c) $y = \frac{k}{x}$ has gradient -2 at $x = 2$

(d) $y = k\sqrt{x}$ has gradient 3 at $x = 4$

[4 marks]

B10 $f(x) = x^3 - 6x^2 + 4$

(a) Find $f''(1)$

(b) Find the value of x for which $f''(x) = 0$

(c) Find the gradient of the curve $y = f(x)$ at the point where $f''(x) = 0$

(d) Find the exact values of x for which $f'(x) = f''(x)$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam differentiates $y = \frac{3}{x^2}$ and writes $\frac{dy}{dx} = \frac{3}{2x}$
Explain Sam's mistake, and find the correct derivative. [2 marks]
- C2** Lily differentiates $y = \frac{x^2 + 1}{x}$ by differentiating the top and the bottom separately, and writes $\frac{dy}{dx} = \frac{2x}{1} = 2x$
Explain why this is wrong, and find the correct derivative. [2 marks]
- C3** Show that the curve $y = x^3 - 3x^2 + 5$ has gradient -3 at exactly one point, and find the equation of the tangent at that point. [3 marks]
- C4** Kai finds the normal to the curve $y = x^3 - 2x$ at the point $(2, 4)$. He finds that the gradient of the curve is 10 there, and writes the normal as $y - 4 = 10(x - 2)$
Explain Kai's mistake, and find the correct equation of the normal in the form $ax + by + c = 0$ [3 marks]



STRETCH

- C5** Jo wants the other point on $y = x^3 - 3x$ where the tangent is parallel to the tangent at $x = 2$. She solves the equation of the tangent at $x = 2$ simultaneously with the equation of the curve.
Explain why Jo's method is wrong, and find the coordinates of the correct point. [3 marks]
- C6** Prove, from first principles, that the derivative of x^3 is $3x^2$ [3 marks]
- C7** Show that no tangent to the curve $y = x^3 + 3x$ has gradient 2, and state the least possible gradient of a tangent to this curve. [3 marks]
- C8** The curve C has equation $y = x^3$, and a is a positive constant.
- (a) Show that the tangents to C at the points where $x = a$ and $x = -a$ are parallel. (2)
- (b) Show that the tangent at the point where $x = a$ has equation $y = 3a^2x - 2a^3$ (1)
- (c) The two tangents cross the y -axis 4 units apart. Find the value of a , and the value of $\frac{d^2y}{dx^2}$ at each point. (2)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

D1 The curve C has equation

$$y = x^3 - 5x^2 + 4x + 7$$

The point P on C has x coordinate 3

(a) Find the equation of the tangent to C at P , giving your answer in the form $y = mx + c$

(2)

(b) Find the coordinates of the point where this tangent crosses the x -axis.

(1)

[3 marks]

D2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The curve C has equation

$$y = x\sqrt{x} - 6\sqrt{x} \quad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(2)

The point P on C has x coordinate 4

(b) Find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(2)

[4 marks]

D3 The curve C has equation

$$y = kx^2 - \frac{6}{x} \quad x \neq 0$$

where k is a constant. The gradient of C at the point where $x = 1$ is 10

(a) Find the value of k .

(2)

(b) Find the gradient of C at the point where $x = -1$

(1)

[3 marks]



D4 $f(x) = 2x^3 - 9x^2 + 12x$

(a) Solve the equation $f'(x) = 0$

(2)

(b) Find the value of $f''(x)$ at each of your solutions to part (a).

(1)

[3 marks]

STRETCH

D5 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = x + \frac{9}{x}$, $x \neq 0$

The tangents to C at the points A and B each have gradient -8

(a) Show that the x coordinates of A and B satisfy $x^2 = 1$

(2)

(b) Find the equations of the two tangents.

(2)

[4 marks]

D6 Figure 1 shows part of the curve C with equation

$$y = \frac{4}{x}, x > 0, \text{ and the tangent } l \text{ to } C \text{ at the point}$$

$P(1, 4)$. The tangent meets the x -axis at A and the y -axis at B .

(a) Find the equation of l , and the area of triangle OAB .

(2)

(b) Prove that, for every point on C , the tangent at that point forms a triangle with the coordinate axes of the same area.

(2)

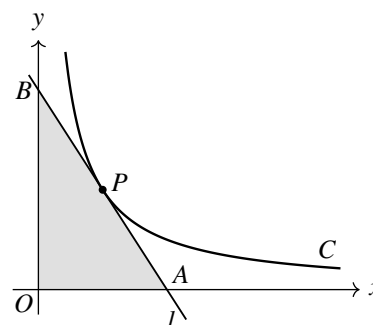


Diagram NOT accurately drawn

Figure 1

[4 marks]



D7 Figure 2 shows the curve C with equation $y = x^2 - 4x + 3$, and the line l , which is the normal to C at the point $P(3, 0)$. The line l meets C again at the point Q .

(a) Find an equation for l .

(2)

(b) Find the coordinates of Q .

(2)

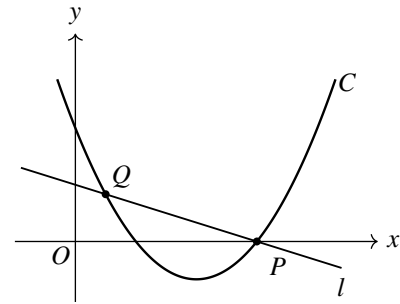


Diagram NOT accurately drawn

Figure 2

[4 marks]

D8 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^2 + \frac{b}{x} \quad x \neq 0$$

where a and b are constants. The curve $y = f(x)$ passes through the point $P(2, 10)$, and the tangent to the curve at P has gradient 7

(a) Show that $8a + b = 20$ and that $16a - b = 28$

(2)

(b) Find the value of a and the value of b .

(1)

(c) Find the value of $f''(2)$

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 2x^{\frac{3}{2}} - 3x + 5 \quad x > 0$$

- (a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

.....
(2)

The point P lies on C and has x coordinate 4

- (b) Find the equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

.....
(3)

(Total for Question E1 is 5 marks)



E2 The curve C has equation $y = f(x)$, where

$$f(x) = 2x^2 + \frac{8}{x} \quad x \neq 0$$

(a) Find, in simplest form,

(i) $f'(x)$

(ii) $f''(x)$

(2)

The point P lies on C and has x coordinate 2

(b) Find the equation of the normal to C at P , giving your answer in the form $px + qy + r = 0$, where p , q and r are integers.

.....
(3)

(c) Find the value of $f''(2)$

.....
(1)

(Total for Question E2 is 6 marks)



STRETCH

E3 Figure 3 shows part of the curve C with equation

$$y = \frac{1}{4}x^2$$

The point A on C has x coordinate $2p$, and the point B on C has x coordinate $2q$, where p and q are non-zero constants and $p \neq q$

The tangents to C at A and at B meet at the point T .

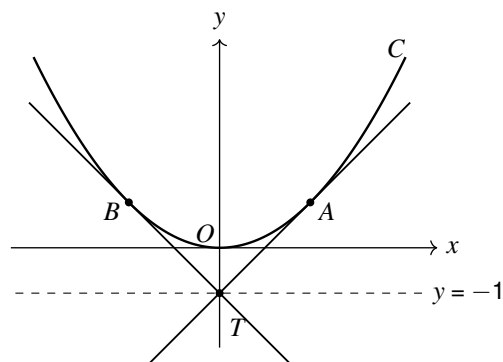


Figure 3

(a) Show that the tangent to C at A has equation $y = px - p^2$

(3)

(b) Show that the tangents at A and B are perpendicular when $pq = -1$

(2)

(c) Given that the tangents at A and B are perpendicular, show that T lies on the line $y = -1$

(3)

(Total for Question E3 is 8 marks)

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