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INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2 (WST02)

Statistics 2 Practice Paper 2

Worked Solutions

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WST02/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	A binomial probability and an approximation	9	2
2	A two-tailed Poisson test	10	3
3	The sampling distribution of a range	10	4
4	A uniform distribution with unknown limits	11	5
5	A normal approximation in a Poisson test	11	6
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Question paper & mark scheme | Exam questions by topic | More practice papers

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1. Each email that arrives at an office is spam with probability p, independently of all other emails. The probability that neither of two emails is spam is 0.9025

(a) Find the value of p

(2)

(b) Find the probability that, of 10 emails, at least 2 are spam.

(3)

On one day, 120 emails arrive at the office.

(c) Using a suitable approximation, find the probability that more than 8 of these emails are spam.

(3)

(d) Give a reason why the approximation in part (c) is suitable.

(1)

(a) $P(\text{neither is spam}) = (1-p)^2 = 0.9025$ M1

$1 - p = 0.95$ ($1 - p$ cannot be -0.95) $\boxed{p = 0.05}$ A1

(b) $X \sim B(10, 0.05)$ B1

$P(X \geqslant 2) = 1 - P(X \leqslant 1) = 1 - 0.9139$ M1

$\boxed{0.0861}$ A1

(c) $Y \sim B(120, 0.05)$: n large, p small, so $Y \approx Po(120 \times 0.05) = Po(6)$ B1

$P(Y > 8) = 1 - P(Y \leqslant 8) = 1 - 0.8472$ M1

$\boxed{0.1528}$ A1

(d) $n = 120$ is large and $p = 0.05$ is small. B1

(Total for Question 1 is 9 marks)

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2. In the past, accidents at a road junction have occurred at a mean rate of 4 per month. The junction is redesigned, and the council wishes to test whether the mean rate of accidents has changed. The number of accidents in the two months after the redesign is recorded.

(a) Give two conditions, in context, under which a Poisson distribution is a suitable model for the number of accidents.

(2)

(b) Write down suitable hypotheses for the test.

(1)

(c) Using a 10% level of significance, find the critical region for the test. The probability in each tail should be less than 0.05

(4)

(d) Write down the actual significance level of the test.

(1)

In the two months after the redesign, 3 accidents occur at the junction.

(e) Comment on whether the mean rate of accidents has changed, giving a reason for your answer.

(2)

(a) Accidents happen independently of each other, one at a time. B1

Accidents happen at a constant average rate. B1

(b) Two months: $\lambda = 2 \times 4 = 8$. $H_0: \lambda = 8$, $H_1: \lambda \neq 8$ B1

(c) Under H_0, $X \sim Po(8)$; two-tailed 10%, so each tail under 0.05: M1

Lower: $P(X \leqslant 3) = 0.0424$, $P(X \leqslant 4) = 0.0996 \Rightarrow X \leqslant 3$ A1

Upper: $P(X \geqslant 14) = 1 - 0.9658 = 0.0342$, $P(X \geqslant 13) = 1 - 0.9362 = 0.0638$ M1

$X \leqslant 3$ or $X \geqslant 14$ A1

(d) $0.0424 + 0.0342$ 0.0766 B1

(e) 3 is in the critical region, so reject H_0. M1

There is evidence that the mean rate of accidents has changed (it has gone down). A1

(Total for Question 2 is 10 marks)

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3. A large bag contains counters, each with a number written on it. Of the counters, 40% show the number 1, 40% show the number 2 and 20% show the number 5. A random sample of three counters is taken from the bag, and R is the range of the three numbers.

(a) Find the sampling distribution of R

(6)

(b) Find E(R)

(2)

Eight such samples are taken.

(c) Find the probability that exactly two of the samples have a range of 0

(2)

(a) Ranges: all the same 0; 1s and 2s only 1; 2s and 5s only 3; a 1 and a 5 gives 4. B1

$P(R = 0) = 0.4^3 + 0.4^3 + 0.2^3 = 0.064 + 0.064 + 0.008$ M1

$P(R = 0) = 0.136$ A1

$P(R = 1)$: only 1s and 2s, not all equal: $0.8^3 - 0.4^3 - 0.4^3 = 0.512 - 0.128$ M1

$P(R = 1) = 0.384$; $P(R = 3) = 0.6^3 - 0.4^3 - 0.2^3 = 0.216 - 0.072 = 0.144$ A1

$P(R = 4) = 1 - 0.136 - 0.384 - 0.144 = 0.336$ A1

(b) $E(R) = 0 \times 0.136 + 1 \times 0.384 + 3 \times 0.144 + 4 \times 0.336 = 0.384 + 0.432 + 1.344$ M1

$\boxed{2.16}$ A1

(c) $Y \sim B(8, 0.136)$: $P(Y = 2) = 28 \times 0.136^2 \times 0.864^6$ M1

$= 0.2146\ldots$ $\boxed{0.215}$ A1

(Total for Question 3 is 10 marks)

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4. The continuous random variable X is uniformly distributed over the interval $[a, b]$. Given that $\mathrm{E}(X) = 7$ and $\mathrm{Var}(X) = 12$

(a) find the value of a and the value of b

(4)

(b) find $\mathrm{E}\big(X(X-1)\big)$

(3)

(c) Find $\mathrm{P}(X > 10)$

(1)

Twenty independent observations of X are taken.

(d) Find the probability that at least 8 of the observations are greater than 10

(3)

(a) Mean: $(a + b)/2 = 7 \Rightarrow a + b = 14$ B1

Variance: $(b - a)^2/12 = 12 \Rightarrow (b - a)^2 = 144$ B1

$b > a$, so $b - a = 12$; with $a + b = 14$: M1

$a = 1,\ b = 13$ A1

(b) $E(X^2) = Var(X) + (E(X))^2$ M1

$= 12 + 7^2 = 61$ A1

$E(X(X-1)) = E(X^2) - E(X) = 61 - 7$ 54 A1

(c) $P(X > 10) = (13 - 10) \div 12$ 0.25 B1

(d) Y = number of observations above 10: $Y \sim B(20, 0.25)$ M1

$P(Y \geqslant 8) = 1 - P(Y \leqslant 7) = 1 - 0.8982$ M1

0.1018 A1

(Total for Question 4 is 11 marks)

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5. A shop sells umbrellas at a mean rate of 30 per week, and the number sold in a week is modelled by a Poisson distribution.

(a) Using a suitable approximation, find the probability that more than 36 umbrellas are sold in a week.

(4)

(b) Explain why the approximation used in part (a) is suitable.

(1)

After an advertising campaign, the manager believes that the mean rate of sales has increased. In the week after the campaign, 40 umbrellas are sold.

(c) Using a suitable approximation, test the manager's belief at the 5% level of significance. State your hypotheses clearly.

(6)

(a) $X \sim Po(30)$; λ large, so $X \approx Y \sim N(30, 30)$ B1

$P(X > 36) = P(X \geqslant 37) \approx P(Y > 36.5)$ (continuity correction) M1

$z = (36.5 - 30) \div \sqrt{30} = 1.186... = 1.19$: $P(Z > 1.19) = 1 - 0.8830$ M1

$\boxed{0.1170}$ A1

(b) $\lambda = 30$ is large. B1

(c) $\boxed{H_0: \lambda = 30, \quad H_1: \lambda > 30}$ B1

$P(X \geqslant 40) \approx P(Y > 39.5)$ ($X \geqslant 40$ starts at 39.5) M1

$z = (39.5 - 30) \div \sqrt{30} = 1.734... = 1.73$ M1

$P(Z > 1.73) = 1 - 0.9582 = 0.0418$ A1

$0.0418 < 0.05$: significant, so reject H_0. M1

There is evidence that the mean rate of sales has increased. A1

(Total for Question 5 is 11 marks)

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6.

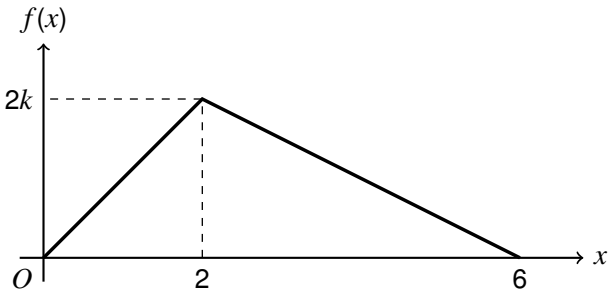

Figure 1

The continuous random variable X has probability density function

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant 2 \\ \frac{1}{2}k(6-x) & 2 < x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant, as shown in Figure 1.

(a) Find the value of k

(2)

(b) Find the cumulative distribution function F(x) for all values of x

(4)

(c) Find the lower quartile of X

(2)

(d) Find P$(X > 4)$

(2)

Five independent observations of X are taken.

(e) Find the probability that exactly two of the observations are greater than 4

(2)

(a) Total area under f is 1: a triangle, base 6, height 2k: ½ × 6 × 2k = 6k = 1 M1

k = 1/6 A1

(b) 0 ⩽ x ⩽ 2: F(x) = ∫ from 0 to x of t/6 dt = x²/12 M1

2 < x ⩽ 6: F(x) = F(2) + ∫ from 2 to x of (6 − t)/12 dt = 1/3 + [6t − t²/2]/12 M1

= 1/3 + (6x − x²/2 − 10)/12 = (12x − x² − 12)/24 (check F(6) = 24/24 = 1 ✓) A1

F = 0 (x < 0); x²/12 (0 ⩽ x ⩽ 2); (12x − x² − 12)/24 (2 < x ⩽ 6); 1 (x > 6) A1

(c) F(2) = 4/12 = 1/3 > 0.25, so Q_1 is in the first piece: x²/12 = 0.25 ⇒ x² = 3 M1

$Q_1 = \sqrt{3} = 1.73$ A1

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Question 6 continued

(d) $F(4) = (48 - 16 - 12)/24 = 20/24 = 5/6$; $P(X > 4) = 1 - 5/6$ M1

$1/6$ A1

(e) $W \sim B(5, 1/6)$: $P(W = 2) = 10 \times (1/6)^2 \times (5/6)^3 = 1250/7776$ M1

$= 0.1607...$ 0.161 A1

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(Total for Question 6 is 12 marks)

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7. A spinner is claimed to land on red with probability 0.25. Asha thinks that the probability is not 0.25, and she plans to spin the spinner 30 times and record the number of times, X, that it lands on red.

(a) Write down suitable hypotheses to test Asha's belief.

(1)

(b) Using a 5% level of significance, find the critical region for a two-tailed test. The probability in each tail should be as close as possible to 0.025 without exceeding it.

(4)

(c) Write down the actual significance level of the test.

(1)

The spinner lands on red 12 times.

(d) State whether or not there is evidence to support Asha's belief, giving a reason for your answer.

(2)

Ben carries out the same two-tailed test, at the 5% level of significance, but spins the spinner n times. He wants a result of no reds at all to be significant.

(e) Find the smallest possible value of n

(4)

(a) $H_0: p = 0.25$, $H_1: p \neq 0.25$ B1

(b) Under H_0, $X \sim B(30, 0.25)$; 2.5% in each tail. M1

Lower: $P(X \leqslant 2) = 0.0106 \leqslant 0.025$, $P(X \leqslant 3) = 0.0374 > 0.025 \Rightarrow X \leqslant 2$ A1

Upper: $P(X \geqslant 13) = 1 - 0.9784 = 0.0216$; $P(X \geqslant 12) = 1 - 0.9493 = 0.0507$ M1

$X \leqslant 2$ or $X \geqslant 13$ A1

(c) $0.0106 + 0.0216$ 0.0322 B1

(d) 12 is not in the critical region (just below 13), so do not reject H_0. M1

There is insufficient evidence to support Asha's belief that the probability is not 0.25. A1

(e) $P(\text{no reds in } n \text{ spins}) = 0.75^n$ B1

Two-tailed: the lower tail can be at most 0.025, so need $0.75^n \leqslant 0.025$ M1

$n \ln 0.75 \leqslant \ln 0.025 \Rightarrow n \geqslant \ln 0.025 \div \ln 0.75 = 12.8$ ($\ln 0.75 < 0$: sign flips) M1

Check: $0.75^{12} = 0.0317$, $0.75^{13} = 0.0238$ $n = 13$ A1

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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