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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 3 (WFM03)

Further Pure 3 Practice Paper 1

Edexcel IAL Further Pure Mathematics 3

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM03/01) • Calculator allowed

A full 75-mark paper of multi-step exam questions covering every Further Pure 3 chapter, with a separate mark scheme

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 9 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A hyperbolic equation	6	
2	A derivative and an exact integral	7	
3	Integrals by completing the square	7	
4	A 3×3 matrix with a parameter	8	
5	The foci and directrices of an ellipse	8	
6	A plane, a line and the angle between them	9	
7	A reduction formula	9	
8	Eigenvalues and diagonalisation	10	
9	A tangent to a hyperbola and its asymptotes	11	
	Total	**75**	

Rough grade guide (out of 75): A* **68** • A **62** • B **55** • C **48**

Based on recent WFM03 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$2\cosh 2x - 9\cosh x + 7 = 0$$

giving your answers as exact logarithms where appropriate.

(6)

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(Total for Question 1 is 6 marks)

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2.

$$y = x\,\text{arsinh}\,x - \sqrt{1+x^2}$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \text{arsinh}\,x$

(3)

(b) Hence find the exact value of

$$\int_0^{\frac{3}{4}} \text{arsinh}\,x\,\mathrm{d}x$$

giving your answer in the form $a\ln 2 + b$, where a and b are rational numbers.

(4)

(Total for Question 2 is 7 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that

$$\int_{-2}^{2} \frac{1}{\sqrt{x^2+4x+13}}\,\mathrm{d}x = \ln 3$$

(4)

(b) Find the exact value of

$$\int_{0}^{1} \frac{1}{4x^2+4x+5}\,\mathrm{d}x$$

(3)

(Total for Question 3 is 7 marks)

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4.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & k \\ 0 & k & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

where k is a constant.

(a) Find $\det \mathbf{M}$ in terms of k

(2)

(b) Find the values of k for which $\mathbf{M}$ is singular.

(2)

Given that $k = 2$

(c) find $\mathbf{M}^{-1}$

(4)

(Total for Question 4 is 8 marks)

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5. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

(a) Find the eccentricity of E

(2)

(b) Find the coordinates of the foci of E and the equations of its directrices.

(3)

The point $P(4, p)$, where $p > 0$, lies on E. The foci of E are S and S'

(c) Find the lengths PS and PS', and verify that $PS + PS' = 10$

(3)

(Total for Question 5 is 8 marks)

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6. The plane Π contains the points $A(1, 0, 2)$, $B(3, 1, 1)$ and $C(2, -1, 4)$

(a) Find a Cartesian equation of Π

(4)

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

(b) Find the coordinates of the point where l meets Π

(3)

(c) Find the acute angle between l and Π, giving your answer in degrees to one decimal place.

(2)

(Total for Question 6 is 9 marks)

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7.

$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, \mathrm{d}x, \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$$

(5)

(b) Hence find the exact value of I_4

(4)

(Total for Question 7 is 9 marks)

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8.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & -2 \end{pmatrix}$$

(a) Show that 3 is an eigenvalue of $\mathbf{A}$, and find the other two eigenvalues.

(4)

(b) Find an eigenvector corresponding to each eigenvalue.

(3)

(c) Find an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$

(3)

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9.

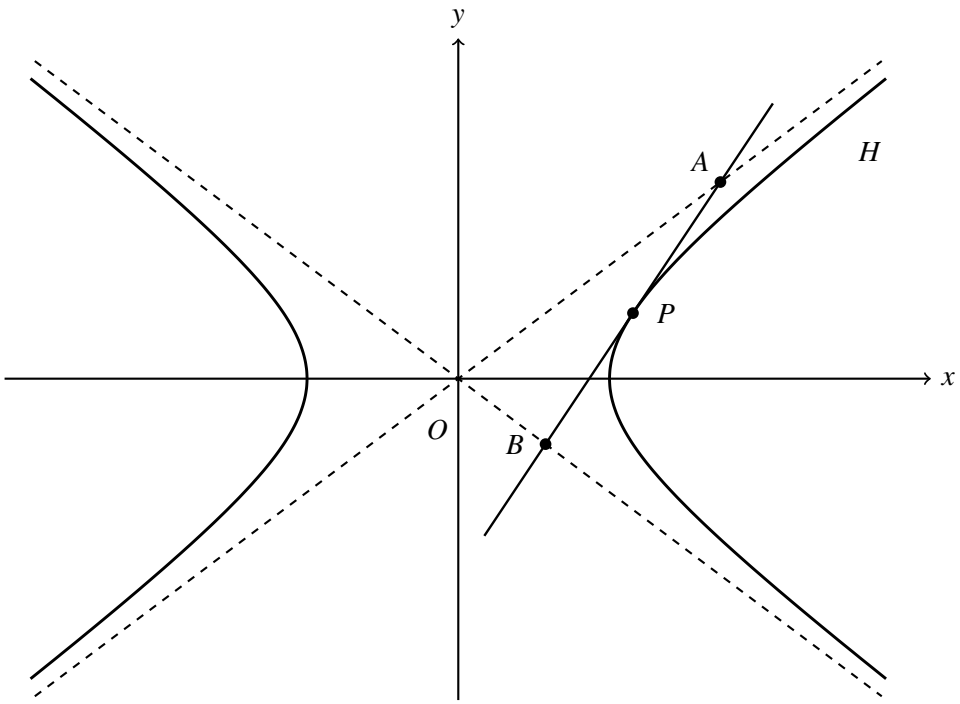

Figure 1

Figure 1 shows the hyperbola H with equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and its asymptotes. The point $P(4\cosh t, 3\sinh t)$ lies on H. The tangent to H at P meets the asymptotes at A and B

(a) Show that an equation of the tangent to H at P is

$$3x\cosh t - 4y\sinh t = 12$$

(4)

(b) Find, in terms of t, the coordinates of A and B, giving your answers in terms of e^t

(4)

(c) Show that the area of the triangle OAB, where O is the origin, does not depend on t

(3)

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Question 9 continued

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(Total for Question 9 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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