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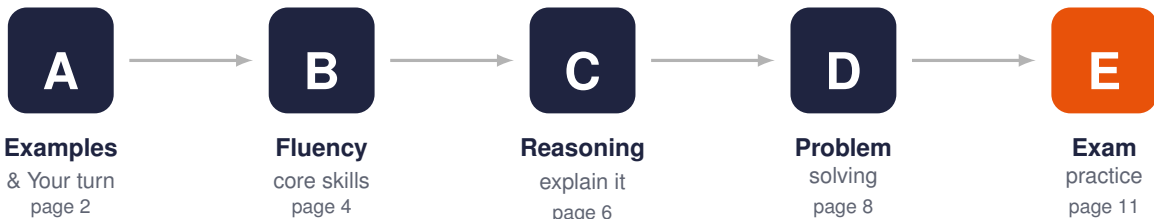
A-LEVEL MATHEMATICS • PURE 2

Trigonometric Identities and Equations

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free Trigonometric Identities and Equations worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: identities and equations • *Stretch*: radians, models, proofs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Quadratics, transformed angles, radians, models and proofs.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find the exact values of $\sin 150^\circ$, $\cos 240^\circ$ and $\tan 315^\circ$

Find the related acute angle, then the sign from the quadrant.

$\sin 30^\circ$, $-\cos 60^\circ$, $-\tan 45^\circ$

$$\frac{1}{2}, -\frac{1}{2}, -1$$

Your turn A1

Find the exact values of $\sin 225^\circ$, $\cos 120^\circ$ and $\tan 210^\circ$

Worked example A2

$\sin \theta = \frac{3}{5}$ and θ is obtuse. Find the exact values of $\cos \theta$ and $\tan \theta$

$$\cos^2 \theta = 1 - \sin^2 \theta = \frac{16}{25}; \text{ obtuse, so } \cos \theta < 0$$

$$\cos \theta = -\frac{4}{5}, \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$-\frac{4}{5}, -\frac{3}{4}$$

Your turn A2

$\cos \theta = -\frac{12}{13}$ and $180^\circ < \theta < 270^\circ$. Find the exact values of $\sin \theta$ and $\tan \theta$

Worked example A3

Solve $2\sin x = 1$ for $0 \leq x < 360^\circ$

Principal value 30° ; sine is also positive in the second quadrant: $180^\circ - 30^\circ$

$$x = 30^\circ, 150^\circ$$

Your turn A3

Solve $4\cos x + 3 = 0$ for $0 \leq x < 360^\circ$, giving answers to 1 decimal place.



STRETCH

Worked example A4: the tan identity

Solve $3\sin x = 2\cos x$ for $0 \leq x < 360^\circ$, to 1 decimal place.

Divide by $\cos x$ (it cannot be 0 here): $\tan x = \frac{2}{3}$

33.7° , and $+180^\circ$

$$x = 33.7^\circ, 213.7^\circ$$

Your turn A4

Solve $\sin x + 4\cos x = 0$ for $0 \leq x < 360^\circ$, to 1 decimal place.

Worked example A5: a multiple angle

Solve $\sin 2x = \frac{1}{2}$ for $0 \leq x < 360^\circ$

$0 \leq 2x < 720^\circ$: find every value of $2x$ first.

$2x = 30^\circ, 150^\circ, 390^\circ, 510^\circ$

$$x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$$

Your turn A5

Solve $\cos(x - 40^\circ) = \frac{1}{2}$ for $0 \leq x < 360^\circ$

Worked example A6: a quadratic in one function

Solve $2\cos^2 x + 3\sin x = 3$ for $0 \leq x < 360^\circ$

Use $\cos^2 x = 1 - \sin^2 x$: $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$$x = 30^\circ, 90^\circ, 150^\circ$$

Your turn A6

Solve $2\sin^2 x = 1 + \cos x$ for $0 \leq x < 360^\circ$

Worked example A7: radians

Solve $\tan\left(x + \frac{\pi}{6}\right) = 1$ for $0 \leq x < 2\pi$

$\frac{\pi}{6} \leq x + \frac{\pi}{6} < \frac{13\pi}{6}$: $x + \frac{\pi}{6} = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

$$x = \frac{\pi}{12}, \frac{13\pi}{12}$$

Your turn A7

Solve $\sin\left(x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ for $0 \leq x < 2\pi$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Find the exact value of

(a) $\sin 210^\circ$

(b) $\cos 135^\circ$

(c) $\tan 120^\circ$

(d) $\sin 330^\circ$

[4 marks]**B2** $\sin \theta = 0.6$. Find the exact value of

(a) $\cos \theta$, when θ is acute

(b) $\tan \theta$, when θ is acute

(c) $\cos \theta$, when θ is obtuse

(d) $\tan \theta$, when θ is obtuse

[4 marks]**B3** Simplify fully

(a) $\sin^2 x + \cos^2 x + 2$

(b) $\frac{1 - \cos^2 x}{\sin x}$

(c) $\frac{\sin x}{\tan x}$

(d) $\tan x \cos x$

[4 marks]**B4** Solve for $0 \leq x < 360^\circ$, giving answers to 1 decimal place where necessary.

(a) $\sin x = 0.4$

(b) $\cos x = -0.3$

(c) $\tan x = -2$

(d) $\sin x = -1$

[4 marks]

CORE

*continued: identities and counting solutions***B5** Prove that

(a) $(\sin x + \cos x)^2 \equiv 1 + 2 \sin x \cos x$

(b) $\cos^2 x - \sin^2 x \equiv 1 - 2 \sin^2 x$

(c) $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$

[3 marks]

B6 State the number of solutions for $0 \leq x < 360^\circ$ of

(a) $\sin x = 0.3$

(b) $\sin 2x = 0.3$

(c) $\sin 3x = 0.3$

(d) $\sin \frac{x}{2} = 0.3$

[4 marks]

STRETCH

B7 Solve for $0 \leq x < 180^\circ$

(a) $\cos 2x = \frac{1}{2}$

(b) $\tan 2x = 1$

(c) $\sin(x + 30^\circ) = 1$

(d) $\cos(2x - 30^\circ) = 0$

[4 marks]

B8 Solve for $0 \leq x < 360^\circ$

(a) $2 \sin^2 x = \sin x$

(b) $4 \cos^2 x = 3$

(c) $2 \cos^2 x - \cos x - 1 = 0$

(d) $\tan^2 x = 3$

[4 marks]

B9 Solve for $0 \leq x < 2\pi$, giving exact answers.

(a) $\sin x = \frac{\sqrt{2}}{2}$

(b) $\cos x = -\frac{1}{2}$

(c) $\tan x = \sqrt{3}$

(d) $\sin 2x = 0$

[4 marks]

B10 The temperature, $T^\circ\text{C}$, in a greenhouse t hours after midnight is modelled by $T = 18 - 6 \cos(15t)^\circ$, $0 \leq t < 24$

(a) State the maximum temperature.

(b) State the minimum temperature.

(c) Find the time of day at which the maximum occurs.

(d) Find the first time of day at which the temperature is 21°C

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam solves $\sin x \cos x = \sin x$ for $0 \leq x < 360^\circ$ by dividing by $\sin x$, and gives only $x = 0^\circ$
Explain Sam's mistake, and give all the solutions. [2 marks]
- C2** Lily is told that $\sin \theta = 0.6$ and θ is obtuse. She writes $\cos \theta = \sqrt{1 - 0.36} = 0.8$
Explain Lily's mistake. [2 marks]
- C3** To solve $\tan x + 2 \sin x = 0$ for $0 \leq x < 360^\circ$, Kai multiplies by $\cos x$ and writes $\sin x + 2 \sin x = 0$
Explain Kai's mistake, and solve the equation correctly. [3 marks]
- C4** Jo solves $\sin 2x = \frac{\sqrt{3}}{2}$ for $0 \leq x < 360^\circ$ and gives $x = 30^\circ$ and $x = 60^\circ$ only.
Explain what Jo has missed, and give all the solutions. [3 marks]



STRETCH

- C5** Max solves $\cos x = \frac{1}{2}$ for $0 \leq x < 2\pi$ and gives $x = 60^\circ$ and $x = 300^\circ$

Explain why Max would lose marks, and give the answers in the correct form. [3 marks]

- C6** Ali writes $\sin(x + 30^\circ) = \sin x + \sin 30^\circ$

Show, with a value of x , that this is false, and solve $\sin(x + 30^\circ) = \frac{1}{2}$ for $0 \leq x < 360^\circ$ [3 marks]

- C7** Solve $2\cos^2 x + 5\cos x - 3 = 0$ for $0 \leq x < 360^\circ$, explaining why one of the factors gives no solutions.

[3 marks]

- C8** (a) Prove that $\sin^4 x - \cos^4 x \equiv \sin^2 x - \cos^2 x$

(2)

- (b) Hence solve $\sin^4 x - \cos^4 x = \frac{1}{2}$ for $0 \leq x < 360^\circ$

(3)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The angle θ is an obtuse angle in a triangle, so that $90^\circ < \theta < 180^\circ$, and it is given that $\tan \theta = -\frac{\sqrt{5}}{2}$

Using the identities $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$, or a right-angled triangle, find the exact values of $\sin \theta$ and $\cos \theta$, explaining how the quadrant decides each sign. Give each answer in its simplest exact form. You should not use the inverse tangent function on your calculator.

[3 marks]

- D2** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Show that the equation $4 \sin^2 x + 4 \cos x = 5$ can be written as $(2 \cos x - 1)^2 = 0$, stating clearly the identity that you use.

(2)

(b) Hence solve the equation $4 \sin^2 x + 4 \cos x = 5$ for $0 \leq x < 360^\circ$, explaining why there are only two solutions even though the equation is a quadratic in $\cos x$

(2)

[4 marks]

- D3** A calculator gives only one solution of an equation such as $\sin u = 0.6$, its principal value. Solve, for $0 \leq x < 180^\circ$, the equation

$$5 \sin(2x + 10^\circ) = 3$$

giving your answers to 1 decimal place. You should find every value of $2x + 10^\circ$ in the transformed interval before finding x , and you must show clearly how you change the interval for the transformed angle.

[3 marks]



- D4** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve, for $0 \leq x < 360^\circ$, the equation $\sqrt{3} \sin x = \cos x$, giving your answers as exact angles in degrees.

You must explain why it is valid to divide both sides of the equation by $\cos x$ without losing any solutions, and you should check each of your answers in the original equation.

[3 marks]

STRETCH

- D5** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve, for $0 \leq x < \pi$, where x is measured in radians, the equation

$$2 \cos \left(2x - \frac{\pi}{6} \right) = -\sqrt{3}$$

giving your answers as exact multiples of π . You should state the interval for $2x - \frac{\pi}{6}$ before you solve, and check that each answer lies in the interval $0 \leq x < \pi$

[4 marks]

- D6** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve, for $0 \leq x < 360^\circ$, the equation $\tan x = 4 \sin x$, writing $\tan x$ in terms of $\sin x$ and $\cos x$ first, and giving non-exact answers to 1 decimal place.

A student divides both sides of this equation by $\sin x$ and finds only two solutions. You must show clearly how you avoid this mistake, and give every solution.

[4 marks]



D7 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

A harbour master uses a model to decide when ships can enter a harbour. The depth of water, D metres, in the harbour t hours after midnight is modelled by

$$D = 5 + 3 \sin(30t)^\circ \quad 0 \leq t < 24$$

(a) Find the greatest depth of water according to the model, and the first time of day, after midnight, at which it occurs.

(2)

(b) A large ship can only enter the harbour safely when the depth is at least 6.5 metres. Find all the times of day at which the depth of water is exactly 6.5 metres.

(2)

[4 marks]

D8 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

You may use the factorisation $a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$ without proof.

(a) Prove that $\sin^3 x + \cos^3 x \equiv (\sin x + \cos x)(1 - \sin x \cos x)$

(2)

(b) Hence solve $\sin^3 x + \cos^3 x = 0$ for $0 \leq x < 360^\circ$, giving each answer as an exact angle in degrees and explaining why one of the factors cannot be zero.

(3)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

A student is asked to solve an equation that contains both $\sin^2 x$ and $\cos x$, so it must first be written in terms of one function only.

- (a) Using a suitable trigonometric identity, show that the equation

$$3 \sin^2 x + 7 \cos x = 5$$

can be written in the form

$$3 \cos^2 x - 7 \cos x + 2 = 0$$

(2)

- (b) Hence solve, for $0 \leq x < 360^\circ$, the equation $3 \sin^2 x + 7 \cos x = 5$, giving your answers to one decimal place. You must factorise the quadratic, and you must explain clearly why one of the factors gives no solutions in this interval.

(3)

(Total for Question E1 is 5 marks)



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E2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Show that the equation

$$4 \tan \theta = 3 \cos \theta$$

can be written as

$$3 \sin^2 \theta + 4 \sin \theta - 3 = 0$$

(2)

(b) Hence solve, for $0 \leq x < 360^\circ$, the equation

$$4 \tan (x + 20^\circ) = 3 \cos (x + 20^\circ)$$

giving your answers to one decimal place. You must show clearly how you deal with the interval for $x + 20^\circ$, and explain why one root of your quadratic in $\sin$ gives no solutions.

.....

(4)

(Total for Question E2 is 6 marks)



STRETCH

- E3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 1 shows the curve C with equation

$$y = 4 \sin \left(2\theta - \frac{\pi}{6} \right) + 1 \quad 0 \leq \theta \leq \pi$$

The point P is the maximum point of C and the point Q is the minimum point of C , as shown in Figure 1.

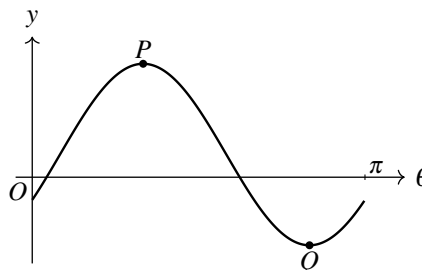


Figure 1

- (a) State the maximum and the minimum values of y

.....
(1)

- (b) Find the exact coordinates of P

.....
(2)

- (c) Find the coordinates of the point where C crosses the y -axis.

.....
(1)

- (d) Solve, for $0 \leq \theta \leq \pi$, the equation $4 \sin \left(2\theta - \frac{\pi}{6} \right) + 1 = 3$, giving exact answers.

.....
(3)

- (e) The line $y = k$ meets C at exactly one point. State the possible values of k

.....
(1)

(Total for Question E3 is 8 marks)

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