eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 1 (WFM01)

Further Pure 1 Practice Paper 1

Edexcel IAL Further Pure Mathematics 1

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM01/01) • Calculator allowed

A full 75-mark paper of multi-step exam questions covering every Further Pure 1 chapter, with a separate mark scheme

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 9 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A matrix that is never singular	6	
2	Numerical solution of an equation	7	
3	Roots of a quadratic equation	7	
4	A complex equation with a conjugate	8	
5	Series and sigma notation	8	
6	Transformations using matrices	9	
7	The normal to a rectangular hyperbola	9	
8	Complex roots of a quartic	10	
9	Proof by induction	11	
	Total	**75**	

Rough grade guide (out of 75): A **63** • B **56** • C **49**

Based on recent WFM01 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

1.

$$\mathbf{A} = \begin{pmatrix} k & 3 \\ k-2 & k+1 \end{pmatrix}$$

where k is a real constant.

(a) Find $\det \mathbf{A}$ in terms of k, giving your answer in simplest form.

(1)

(b) Show that $\mathbf{A}$ is non-singular for all real values of k

(2)

(c) Find $\mathbf{A}^{-1}$ in terms of k

(3)

(Total for Question 1 is 6 marks)

DO NOT WRITE IN THIS AREA

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2.

$$f(x) = x^3 - \frac{2}{x} - 6, \qquad x > 0$$

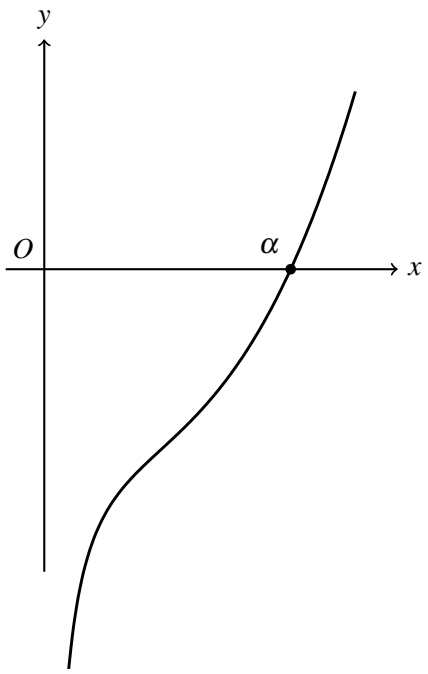

Figure 1

Figure 1 shows part of the curve with equation $y = f(x)$. The equation $f(x) = 0$ has a single root α

(a) Show that α lies in the interval $[1.8, 2]$

(2)

(b) Taking 2 as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to find a second approximation to α. Show your working.

(3)

(c) Use linear interpolation once on the interval $[1.8, 2]$ to find another approximation to α, giving your answer to 3 decimal places.

(2)

(Total for Question 2 is 7 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

3. The roots of the equation

$$2x^2 - 3x + 5 = 0$$

are α and β

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

(1)

(b) Find the value of $\alpha^2 + \beta^2$

(2)

(c) Find a quadratic equation, with integer coefficients, whose roots are

$$\alpha + \frac{1}{\beta} \quad \text{and} \quad \beta + \frac{1}{\alpha}$$

(4)

(Total for Question 3 is 7 marks)

DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The complex number z satisfies

$$z + 2iz^* = 8 + 7i$$

where z^* is the complex conjugate of z

(a) Find z, giving your answer in the form $a + bi$, where a and b are real.

(4)

(b) Find $|z|$ and $\arg z$, giving the argument in radians to 3 significant figures.

(2)

(c) Find z^2, and show z and z^2 on a single Argand diagram.

(2)

(Total for Question 4 is 8 marks)

DO NOT WRITE IN THIS AREA

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5. **(a)** Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+3) = \tfrac{1}{3}n(n+1)(n+5)$$

(4)

(b) Hence show that

$$\sum_{r=n+1}^{2n} r(r+3) = \tfrac{1}{3}n\left(an^2+bn+c\right)$$

where a, b and c are integers to be found.

(4)

(Total for Question 5 is 8 marks)

DO NOT WRITE IN THIS AREA

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6.

$$\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(a) Describe fully the single geometrical transformation represented by the matrix $\mathbf{P}$

(2)

The transformation Q is an enlargement, scale factor 3, centre $(0, 0)$

(b) Write down the matrix $\mathbf{Q}$ that represents Q

(1)

The transformation represented by $\mathbf{P}$ is followed by the transformation Q, and the combined transformation is represented by the matrix $\mathbf{R}$

(c) Find $\mathbf{R}$

(2)

(d) A triangle T has area 5 cm^2. Find the area of the image of T under the transformation represented by $\mathbf{R}$

(2)

(e) Find the coordinates of the point that is mapped by $\mathbf{R}$ onto the point $(-6, 12)$

(2)

(Total for Question 6 is 9 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7.

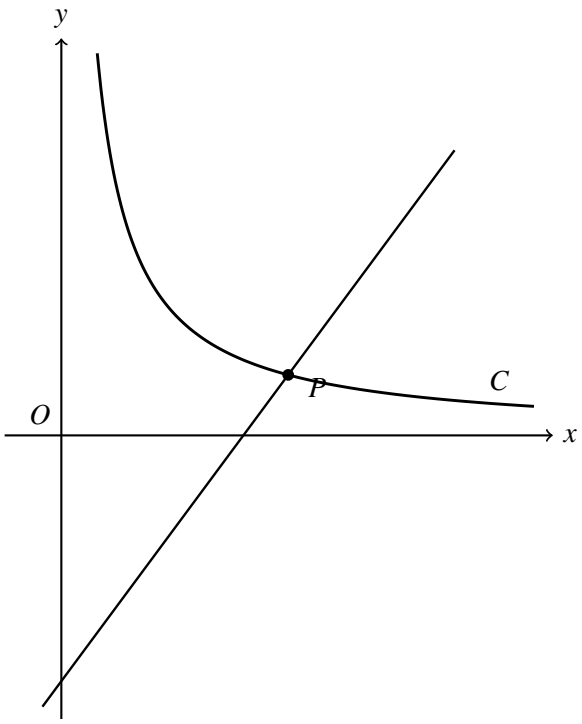

Figure 2

The rectangular hyperbola C has equation $xy = 16$. The point $P\left(4t, \frac{4}{t}\right)$, $t \neq 0$, lies on C, as shown in Figure 2.

(a) Show that an equation of the normal to C at P is

$$t^3x - ty = 4t^4 - 4$$

(4)

The normal to C at the point where $t = 2$ meets the x-axis at A and the y-axis at B

(b) Find the coordinates of A and the coordinates of B

(3)

(c) Find the area of the triangle OAB, where O is the origin.

(2)

DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 9 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(z) = z^4 - 4z^3 + 14z^2 - 4z + 13$$

Given that $2 - 3i$ is a root of the equation $f(z) = 0$

(a) write down another complex root of the equation.

(1)

(b) Hence find a quadratic factor of $f(z)$ with real coefficients.

(2)

(c) Find the other two roots of $f(z) = 0$

(5)

(d) Show all four roots of $f(z) = 0$ on a single Argand diagram.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9. **(i)** Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

(5)

(ii) Prove by induction that, for all positive integers n, $3^{2n}+7$ is divisible by 8

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk