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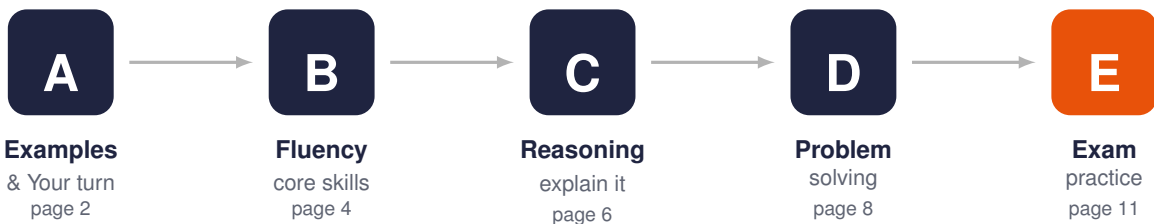
A-LEVEL MATHEMATICS • PURE 3

Algebraic Methods

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Algebraic Methods worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: simplify, multiply, divide, add and subtract algebraic fractions • write improper fractions in mixed form • *Stretch:* division by quadratics, identities and proofs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Fractions that collapse, division by quadratics, composite functions and a minimum by substitution.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Simplify fully $\frac{x^2 - 16}{2x^2 + 7x - 4}$

Factorise the top and the bottom fully, then cancel common factors.

$$\frac{(x-4)(x+4)}{(2x-1)(x+4)}$$

$$\frac{x-4}{2x-1}$$

Your turn A1

Simplify fully $\frac{3x^2 - 12}{x^2 + x - 6}$

Worked example A2

Simplify $\frac{x^2 - 1}{x^2 + 3x} \div \frac{x+1}{x^2 - 9}$

Multiply by the reciprocal, factorise everything, then cancel.

$$\frac{(x-1)(x+1)}{x(x+3)} \times \frac{(x-3)(x+3)}{x+1}$$

$$\frac{(x-1)(x-3)}{x}$$

Your turn A2

Simplify $\frac{x^2 + 5x + 6}{x^2 - 4} \div \frac{x+3}{4x-8}$

Worked example A3

Write $\frac{4}{x-3} - \frac{1}{x+2}$ as a single fraction.

Common denominator $(x-3)(x+2)$; bracket the second numerator.

$$\frac{4(x+2) - (x-3)}{(x-3)(x+2)}$$

$$\frac{3x+11}{(x-3)(x+2)}$$

Your turn A3

Write $\frac{5}{2x+1} + \frac{3}{x-4}$ as a single fraction.



STRETCH

Worked example A4: the lowest common denominator

Write $\frac{2}{x^2-4} + \frac{1}{x+2}$ as a single fraction in its simplest form.

$x^2 - 4 = (x-2)(x+2)$ already contains $(x+2)$: that is the common denominator.

$$\frac{2 + (x-2)}{(x-2)(x+2)}$$

$$\frac{x}{(x-2)(x+2)}$$

Your turn A4

Write $\frac{3}{x^2+x-2} - \frac{1}{x-1}$ as a single fraction in its simplest form.

Worked example A5: an improper fraction

Write $\frac{2x^2+3x-4}{x+2}$ in the form $Ax+B+\frac{C}{x+2}$

Divide: $(x+2)(2x-1) = 2x^2+3x-2$, so the remainder is -2

$$2x-1-\frac{2}{x+2}$$

Your turn A5

Write $\frac{3x^2-7x+5}{x-1}$ in the form $Ax+B+\frac{C}{x-1}$

Worked example A6: dividing by a quadratic

Write $\frac{x^3+2x^2-5}{x^2+1}$ in the form $Ax+B+\frac{Cx+D}{x^2+1}$

$(x^2+1)(x+2) = x^3+2x^2+x+2$; the remainder has degree less than 2

remainder = $-x-7$

$$x+2+\frac{-x-7}{x^2+1}$$

Your turn A6

Write $\frac{2x^3-x^2+3x+4}{x^2-x+1}$ in the form $Ax+B+\frac{Cx+D}{x^2-x+1}$

Worked example A7: comparing coefficients

Find A , B and C given $\frac{3x^2+4x+7}{x+1} \equiv Ax+B+\frac{C}{x+1}$

$$3x^2+4x+7 \equiv (Ax+B)(x+1)+C$$

$$x^2: A=3; \quad x: A+B=4; \quad x=-1: C=3-4+7$$

$$A=3, B=1, C=6$$

Your turn A7

Find A , B and C given $\frac{4x^2-2x+1}{2x-1} \equiv Ax+B+\frac{C}{2x-1}$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Simplify fully

(a) $\frac{x^2 - 25}{x^2 + 5x}$

(b) $\frac{2x^2 - 8}{x^2 - 4x + 4}$

(c) $\frac{x^2 - 7x + 12}{16 - x^2}$

(d) $\frac{6x^2 + x - 2}{4x^2 - 1}$

[4 marks]**B2** Simplify

(a) $\frac{x+3}{x-2} \times \frac{x^2-4}{x^2+3x}$

(b) $\frac{x^2-9}{2x} \div \frac{x-3}{4x^2}$

(c) $\frac{x^2-x-6}{x^2-9} \times \frac{x+3}{x+2}$

(d) $\frac{x^2+4x}{x^2-16} \div \frac{x^2}{x-4}$

[4 marks]**B3** Write each as a single fraction.

(a) $\frac{1}{x} + \frac{1}{x+1}$

(b) $\frac{3}{x-2} - \frac{2}{x+1}$

(c) $\frac{x}{x+3} + \frac{3}{x-3}$

(d) $2 - \frac{3}{x+1}$

[4 marks]**B4** Write each in the form (polynomial) + (proper fraction).

(a) $\frac{x^2+5x+7}{x+2}$

(b) $\frac{2x^2-x-4}{x-2}$

(c) $\frac{x^3-2x+3}{x-1}$

(d) $\frac{4x^2+1}{2x+1}$

[4 marks]

CORE continued: equations and identities**B5** Solve, stating any values of x that are excluded.

(a) $\frac{3}{x-1} = \frac{2}{x+1}$

(b) $\frac{x}{x-2} - \frac{1}{x+2} = 1$

(c) $\frac{x^2-4}{x-2} = 5$

[3 marks]

B6 Find the constants in each identity.

(a) $\frac{2x+5}{x+1} \equiv A + \frac{B}{x+1}$

(b) $\frac{x^2+3}{x-1} \equiv Ax+B + \frac{C}{x-1}$

(c) $\frac{6x^2+5x-1}{2x+1} \equiv Ax+B + \frac{C}{2x+1}$

(d) $\frac{x^3+1}{x^2+1} \equiv Ax+B + \frac{Cx+D}{x^2+1}$

[4 marks]

STRETCH**B7** Simplify fully

(a) $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x^2}}$

(b) $\frac{x - \frac{4}{x}}{x+2}$

(c) $\frac{1}{x-1} - \frac{1}{x+1} - \frac{\frac{x}{x+1}}{\frac{x^2}{x^2-1}}$

[4 marks]

B8 Find the quotient and the remainder when

(a) $x^3 + x^2 + x + 3$ is divided by $x^2 + 1$

(b) $x^4 - 1$ is divided by $x^2 + 1$

(c) $2x^3 + 3x^2 - x + 5$ is divided by $x^2 + x - 1$

(d) $x^4 + 2x^2 + 5$ is divided by $x^2 - 1$

[4 marks]

B9 Show that

(a) $\frac{x+1}{x^2-x-2} \equiv \frac{1}{x-2}$

(b) $\frac{4}{x^2-4} + \frac{1}{x+2} \equiv \frac{1}{x-2}$

(c) $\frac{2x}{x^2-1} - \frac{1}{x-1} \equiv \frac{1}{x+1}$

(d) $\frac{5}{x^2+x-6} - \frac{1}{x-2} \equiv -\frac{1}{x+3}$

[4 marks]

B10 $f(x) = \frac{x^2+4x+7}{x+2}, x \neq -2$

(a) Write $f(x)$ in the form $x+A + \frac{B}{x+2}$

(b) State the equation of the vertical asymptote of $y = f(x)$ (c) Explain why $f(x) > x+2$ for $x > -2$ (d) Use your form to find $f(0)$, and check it from the original fraction.

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

C1 Sam simplifies $\frac{x^2+9}{x+3}$ to $x+3$

Explain Sam's mistake.

[2 marks]

C2 Lily writes $\frac{3}{x-2} - \frac{x+1}{x-2} = \frac{4-x}{x-2}$

Explain Lily's mistake, and simplify the expression correctly.

[2 marks]

C3 Kai simplifies $\frac{2x^2-18}{x^2-x-6}$ as $\frac{(x-3)(x+3)}{(x-3)(x+2)} = \frac{x+3}{x+2}$

Explain Kai's mistake, and give the correct simplified fraction.

[3 marks]

C4 Jo uses her calculator to find the roots of $3x^2+x-2=0$ and writes $3x^2+x-2 = (x+1)\left(x-\frac{2}{3}\right)$

Explain Jo's mistake, and simplify $\frac{3x^2+x-2}{3x^2-2x}$

[3 marks]



STRETCH

- C5** Asked to write $f(x) = \frac{2x^2 - x - 11}{x - 3}$ in the form $Ax + B + \frac{C}{x - 3}$, Max finds $A = 2$, $B = 5$ and $C = 4$ and stops.
Explain why Max would lose a mark, and check his constants. [3 marks]
- C6** Ali writes $\frac{1}{x^2 - 4} + \frac{1}{x - 2}$ over the common denominator $(x^2 - 4)(x - 2)$
Explain why this is not the best choice, and write the sum as a single fraction in its simplest form. [3 marks]
- C7** Ali says that, because $\frac{x^2 - 1}{x - 1} = x + 1$, the graph of $y = \frac{x^2 - 1}{x - 1}$ is exactly the straight line $y = x + 1$
Explain why Ali is not quite right. [3 marks]
- C8** (a) Prove that $\frac{1}{n} - \frac{1}{n+1} \equiv \frac{1}{n(n+1)}$ (1)
- (b) Prove that, for positive a and b , $\frac{a}{b} + \frac{b}{a} - 2 \equiv \frac{(a-b)^2}{ab}$ (2)
- (c) Hence prove that $\frac{x}{x+1} + \frac{x+1}{x} > 2$ for all $x > 0$ (2)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

D1 The function f is defined by

$$f(x) = \frac{4x}{x^2 - 4} - \frac{2}{x + 2} \quad x > 2$$

Show that $f(x) = \frac{2}{x - 2}$, making each step of your working clear. You should factorise the first denominator, state the lowest common denominator that you use, and show the factor that cancels at the end.

[3 marks]

D2 An improper algebraic fraction is to be written as a quadratic plus a proper fraction. Given that

$$\frac{x^4 + 3x^2 + 2x + 5}{x^2 + 2} \equiv Ax^2 + Bx + C + \frac{Dx + E}{x^2 + 2}$$

find the values of the constants A , B , C , D and E , showing clearly the method that you use. You may use long division or compare coefficients after multiplying both sides by $x^2 + 2$

[4 marks]

D3 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve the equation

$$\frac{2}{x - 1} + \frac{3}{x + 2} = 1$$

giving your answers in exact form. State the values of x for which the left-hand side is not defined, and explain why neither of your answers needs to be rejected.

[3 marks]



D4 The function g is defined for all real values of x greater than 1 by

$$g(x) = \frac{3x^2 - 4x + 5}{x - 1} \quad x > 1$$

(a) Write $g(x)$ in the form $Ax + B + \frac{C}{x-1}$, where A , B and C are constants to be found.

(2)

(b) Hence explain why $g(x) > 3x - 1$ for all $x > 1$, referring to the sign of each part of your answer to (a).

(1)

[3 marks]

STRETCH

D5 The function f is defined by $f(x) = \frac{2x+1}{x-3}$, $x \neq 3$. The composite function ff is defined wherever $f(x) \neq 3$

(a) Show that $ff(x) = \frac{5x-1}{10-x}$, where it is defined, writing each stage as a single fraction and simplifying the numerator and the denominator separately.

(3)

(b) Hence find the exact values of x for which $ff(x) = x$, giving your answers in simplest surd form.

(1)

[4 marks]

D6 Figure 1 shows part of the curve C with equation

$$y = \frac{x^2 + 3x + 6}{x + 1} \quad x > -1$$

and the line l with equation $y = 2x + 2$. For $x > -1$, the curve and the line meet at exactly one point, P .

(a) Write the equation of C in the form

$$y = Ax + B + \frac{D}{x+1}$$

(2)

(b) Show that the x coordinate of P satisfies $x^2 + x - 4 = 0$, and find it exactly, explaining why only one root of the quadratic is the x coordinate of P

(2)

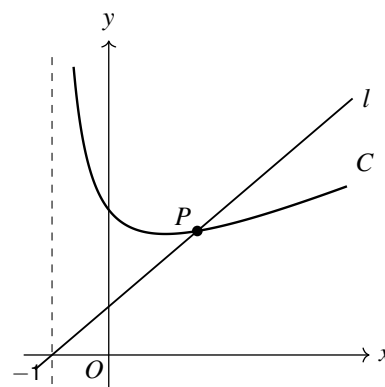


Figure 1

[4 marks]



D7 In this question you should factorise every numerator and denominator before cancelling.

(a) Show that $\frac{x^2+2x-3}{x^2-9} \div \frac{x^2-x}{x^2-6x+9} = \frac{x-3}{x}$ for all values of x where both sides are defined.

(3)

(b) Hence solve $\frac{x^2+2x-3}{x^2-9} \div \frac{x^2-x}{x^2-6x+9} = \frac{1}{2}$, checking that your answer is not an excluded value.

(1)

[4 marks]

D8 The function f is defined by

$$f(x) = \frac{3}{x+1} - \frac{2}{x-2} + \frac{x+10}{x^2-x-2} \quad x \neq -1, x \neq 2$$

(a) Show that $f(x) = \frac{2}{x-2}$

(3)

(b) Explain why $f(-1)$ is not defined, even though the simplified expression $\frac{2}{x-2}$ has a value when $x = -1$

(1)

(c) Solve $f(x) = x$, giving your answers in exact form and checking that neither is an excluded value.

(1)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The function f is defined by

$$f(x) = \frac{x^2 + 7x + 10}{x^2 + x - 2} - \frac{3}{x - 1} \quad x > 1$$

- (a) Show that $f(x) = \frac{x+2}{x-1}$, showing each step of your working.

(3)

- (b) Hence solve the equation $f(x) = 4$, explaining why your answer is valid.

(2)

(Total for Question E1 is 5 marks)



E2 Given that

$$\frac{2x^4 - 3x^3 + 6x^2 - x + 5}{x^2 - x + 2} \equiv Ax^2 + Bx + C + \frac{Dx + E}{x^2 - x + 2}$$

(a) find the values of the constants A , B and C , showing clearly your method (long division, or comparing coefficients),

.....
(3)

(b) find the values of the constants D and E

.....
(2)

(c) Hence write down the remainder when $2x^4 - 3x^3 + 6x^2 - x + 5$ is divided by $x^2 - x + 2$

.....
(1)

(Total for Question E2 is 6 marks)



STRETCH

- E3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 2 shows part of the curve C with equation

$$y = \frac{2x^2 + x + 3}{x + 1} \quad x > -1$$

The curve has a minimum point P , and the line $x = -1$ is a vertical asymptote of C , as shown in Figure 2.

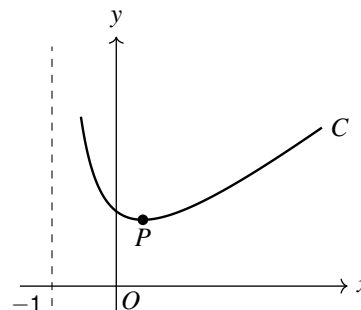


Figure 2

- (a) Write y in the form $Ax + B + \frac{D}{x+1}$, where A , B and D are constants to be found.

.....
(3)

- (b) By writing $u = x + 1$, show that $y = 2u - 3 + \frac{4}{u}$

(1)

- (c) Hence, by differentiating with respect to u , find the exact coordinates of P

.....
(3)

- (d) State the set of values of k for which the equation $\frac{2x^2 + x + 3}{x + 1} = k$ has two solutions with $x > -1$

.....
(1)

(Total for Question E3 is 8 marks)

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