



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Exponentials and Logarithms

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Exponentials and Logarithms worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Exponentials and Logarithms worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Exponentials & Logarithms revision engine

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $e^{1-x} = 4$, so $1 - x = \ln 4$: $x = 1 - \ln 4$

A2 $\ln(x-3) = \frac{5}{2}$, so $x-3 = e^{\frac{5}{2}}$: $x = 3 + e^{\frac{5}{2}}$

A3 Asymptote $y = -6$; $x = 0$: $(0, -4)$; $e^x = 3$: $(\ln 3, 0)$

STRETCH

A4 Multiply by e^x : $u^2 - 2u - 8 = 0$, $u = 4$ or $u = -2$; $e^x > 0$, so reject -2 : $x = \ln 4$

A5 $\ln \frac{x^2}{x+6} = 0$, so $x^2 = x+6$: $x = 3$ or $x = -2$; $\ln(-2)$ is undefined: $x = 3$

A6 12000 ; $e^{-0.15t} = \frac{1}{2}$, $t = \frac{\ln 2}{0.15}$: $t = 4.62$

A7 $\log_{10} y = \log_{10} k + x \log_{10} b$: gradient 0.2, intercept 1.2: $k = 10^{1.2} \approx 15.8$, $b = 10^{0.2} \approx 1.58$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) 5 (b) $3x$ (c) $e^{\ln 9} = 9$ (d) -2

e^x and $\ln x$ undo each other.

B2 (a) $\ln 7$ (b) $\frac{1}{2} \ln 7$ (c) $\frac{1}{2}(\ln 7 - 1)$ (d) **no solutions**, since $e^{2x+1} > 0$

Each change adds one step to the rearrangement.

B3 (a) e^2 (b) $\frac{e^2}{3}$ (c) $\frac{e^2 + 1}{3}$ (d) $3x - 1 = 2x$: $x = 1$

When every term is a single $\ln$, combine and remove the logs.

B4 (a) $y = 2$; $(0, 3)$; **no x -intercept** (b) $y = -3$; $(0, -2)$; $(-\ln 3, 0)$
 (c) $x = 2$; $(3, 0)$; **no y -intercept** (d) $x = 0$; $\left(\frac{1}{2}e^{-3}, 0\right)$; **no y -intercept**

Exponentials have horizontal asymptotes; logarithms have vertical ones.

B5 (a) 50 (b) $50e^2 = 369.4 \dots$: 369 (c) $e^{0.4t} = 4$: $t = 3.47$

The initial value is the coefficient in front of e

B6 (a) **gradient 2, intercept $\log_{10} 3$** (b) **gradient $\log_{10} 2$, intercept $\log_{10} 3$**
 (c) $y = 10^{0.5}x^2 = 3.16x^2$ (d) $y = 10 \times (10^{-0.3})^x = 10 \times 0.501^x$

Powers of x give lines against $\log x$; powers of a constant give lines against x

STRETCH

B7 (a) $\ln 3, \ln 4$ (b) $(u+3)(u-2) = 0$: $\ln 2$ (c) $2u^2 - 7u + 3 = 0$: $\ln 3, -\ln 2$
 (d) $u^2 - 2u + 5 = 0$ has discriminant $-16 < 0$: **no solutions**

$u = e^x$ must be positive, so check each root.



B8 (a) $x^2 - 1 = 8, x > 1$: $x = 3$ (b) $x + 4 = 3x$: $x = 2$ (c) $x^2 + 3x - 4 = 0$, reject -4 : $x = 1$ (d) $\ln 2x = 1$:
 $x = \frac{e}{2}$

Every answer must keep each $\ln$ argument positive.

B9 (a) 90 (b) 20 (c) $-3.5e^{-0.05t}$ at $t = 10$: -2.12 (d) $e^{-0.05t} = \frac{3}{7}$: $t = 16.9$

A negative rate means the temperature is falling.

B10 (a) $f(x) > -1$ (b) $\frac{1}{2} \ln(x+1), x > -1$ (c) $e^x - 3, x \in \mathbb{R}$ (d) $e^{2x} = 4$: $x = \ln 2$

The domain of an inverse is the range of the function.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $\ln x + \ln 3 = \ln 3x$, not $\ln(x+3)$. $x+3 = e^2$: $x = e^2 - 3$

C2 The temperature is falling, so the rate is negative: $\frac{dT}{dt} = -2.1$

C3 Rounding early carries an error into later answers, and 0.03 is only 1 significant figure: $k = 0.0288$

C4 0.69 is a rounded decimal, not exact: $x = \ln 2$

STRETCH

C5 $x = -1$ makes $\ln x$ and $\ln(x-2)$ undefined: reject $x = -1$; $x = 3$

C6 The base is 3, not 10: $N = 3^{2+0.5t} = 9 \times (3^{0.5})^t$: $N = 9 \times (\sqrt{3})^t$

C7 N is in thousands, so the rate is 4.73 thousand items per month: 4730 items per month



C8 (a) $\ln(e^x(e^x - 1)) = \ln e^x + \ln(e^x - 1) = x + \ln(e^x - 1)$ as required

(b) $e^x - 1 = 3$: $x = \ln 4$ (c) $e^x - 1 > 0$ only when $x > 0$, so the logarithm is defined only then

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) gradient $\frac{1.1 - 2.3}{2} = -0.6$: $\log_{10} I = 2.9 - 0.6 \log_{10} d$

(b) $\log_{10} I = \log_{10} a + n \log_{10} d$: $n = -0.6$, $a = 10^{2.9} \approx 794$

D2 (a) 15 mg per litre (b) $6 = 15e^{-4k}$, $e^{4k} = \frac{5}{2}$: $k = \frac{1}{4} \ln \frac{5}{2}$ as required

(c) $15e^{-kt} = 1$, $t = \frac{\ln 15}{k} = 11.82$ hours: 11 hours 49 minutes

D3 (a) $A(0, 3)$; as x decreases, $e^{\frac{x}{2}} \rightarrow 0$, so $y \rightarrow 5$

(b) $e^{\frac{x}{2}} = \frac{5}{2}$: $x = 2 \ln \frac{5}{2}$

D4 (a) $V = e^{9.2} (e^{-0.12})^t$: $a = e^{9.2}$, $b = 0.887$; the value is multiplied by 0.887 each year, a fall of about 11% a year

(b) $e^{8.6} = 5431.7$: £5430

STRETCH

D5 (a) $3e^x = 2 + 8e^{-x}$; multiply by e^x : $3u^2 = 2u + 8$, so $3u^2 - 2u - 8 = 0$ as required

(b) $(3u + 4)(u - 2) = 0$; $u = -\frac{4}{3}$ is rejected as $e^x > 0$; $u = 2$: $P(\ln 2, 6)$

D6 (a) $\ln(2x + 1) + \ln x = 2$, so $x(2x + 1) = e^2$: $2x^2 + x - e^2 = 0$ as required

(b) $x > 0$: $x = \frac{-1 + \sqrt{1 + 8e^2}}{4} = 1.69$



- D7 (a) $40 = 80e^{-6k}$, so $e^{6k} = 2$: $k = \frac{1}{6} \ln 2$ as required
- (b) $\frac{dm}{dt} = -80ke^{-kt} = -2.91$: decreasing at 2.91 grams per day

- D8 (a) $G = 70$; $G \rightarrow 40$
- (b) $\frac{dG}{dt} = -6e^{-0.1t} + 12e^{-0.4t} = 0$, so $e^{0.3t} = 2$, $t = \frac{10}{3} \ln 2$; the derivative is positive before and negative after as required
- (c) $G = 75.7$

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE*Exam questions for everyone*

- E1 (a) $P = 10^{2.1} \times (10^{0.04})^t$ M1 M1
- $P = 125.9 \times 1.096^t$ A1
- (b) $2.1 + 0.04t > 3$, so $t > 22.5$ M1
- 23 weeks A1
- E2 (a) 5 grams B1
- (b) $M = 25$: the mass approaches 25 grams but never reaches it B1
- (c) $e^{-0.08t} = \frac{1}{4}$, $t = \frac{\ln 4}{0.08}$ M1
- $t = 17.33$ A1
- (d) $\frac{dM}{dt} = 1.6e^{-0.08t} = 0.08(25 - M)$ M1
- $\frac{dM}{dt} = 2 - 0.08M$ A1



STRETCH

- E3** (a) $(u - 1)(u - 3) = 0$; $u = 3$ at B M1
- $x = \ln 3$ A1
- (b) $y = (e^x - 2)^2 - 1$ M1 A1
- $P(\ln 2, -1)$ A1
- (c) $f(x) \geq -1$ B1
- (d) the curve falls from the asymptote $y = 3$ to -1 and then rises M1
- $-1 < k < 3$ A1

