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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 5: Matrices

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	separate file	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	separate file	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	separate file	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	separate file	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	2	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	separate file	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	separate file	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 5 • KEY SKILLS

5 Matrices

Specification FP1: Matrix algebra • 10 questions • 80 marks • about 96 minutes

1 Order

An $m \times n$ matrix times an $n \times p$ matrix gives $m \times p$. The inner numbers must match, or the product is undefined.

2 Multiplication

Row by column. In general $\mathbf{AB} \neq \mathbf{BA}$, so keep the order exactly as written.

3 Identity

$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ with $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$. Write $5\mathbf{I}$, not 5, inside matrix equations.

4 Determinant

$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$, with care over minus signs. Singular means determinant 0.

5 Inverse

$\mathbf{A}^{-1} = \dfrac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$: swap the leading diagonal, change the signs of the other diagonal, divide by the determinant.

6 Matrix equations

$\mathbf{AX} = \mathbf{B}$ gives $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$; $\mathbf{XA} = \mathbf{B}$ gives $\mathbf{X} = \mathbf{BA}^{-1}$. The inverse goes on the same side as $\mathbf{A}$.

7 Products

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ and $\det(\mathbf{AB}) = \det\mathbf{A}\det\mathbf{B}$

8 Simultaneous equations

Write as $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$. If $\mathbf{A}$ is singular there are no solutions or infinitely many.

9 Unknown entries

Multiply out with the unknowns, then equate corresponding entries and check every equation.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Order	•									
2 Multiplication	•		•		•	•	•	•	•	
3 Identity							•		•	
4 Determinant	•	•		•		•	•			•
5 Inverse		•	•	•		•	•	•	•	•
6 Matrix equations			•			•		•		
7 Products	•					•	•			
8 Simultaneous equations				•			•			•
9 Unknown entries		•			•	•		•	•	•
Interleaved: quadratics and simultaneous equations (P1)		•		•	•	•				•
Interleaved: algebraic proof and generalising patterns (P1, P2)							•	•	•	
Interleaved: fractions and surds (P1)		•		•					•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the determinant out with its brackets before simplifying. When solving a matrix equation, decide which side the inverse goes on before multiplying.

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1.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 4 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{pmatrix}$$

(a) Find $\mathbf{AB}$.

(3)

(b) State the order of the matrix $\mathbf{BA}$.

(1)

(c) Explain why $\mathbf{A}^2$ is not defined.

(1)

(d) Find $\det(\mathbf{AB})$.

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. A 2×2 matrix $\mathbf{M}$ has k and $k-5$ on its leading diagonal, 3 in the top right and 2 in the bottom left, where k is a real number.

(a) Show that $\det \mathbf{M} = (k-6)(k+1)$, and state the values of k for which $\mathbf{M}$ has no inverse.

(3)

(b) Find the inverse of $\mathbf{M}$ when $k = 2$

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$.

(2)

(b) The matrix $\mathbf{X}$ satisfies $\mathbf{XA} = \mathbf{B}$. Find $\mathbf{X}$.

(3)

(c) Explain why $\mathbf{A}^{-1}\mathbf{B}$ is not the answer to part (b).

(1)

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Question 3 continued

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4.

$$3x - 2y = 7, \qquad 5x + 4y = 19$$

(a) Write these equations as a single matrix equation.

(1)

(b) Find the inverse of the matrix of coefficients.

(2)

(c) Use your answer to part (b) to solve the equations.

(3)

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Question 4 continued

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5.

$$\mathbf{A} = \begin{pmatrix} a & 2 \\ 1 & b \end{pmatrix}, \qquad \mathbf{A}^2 = \begin{pmatrix} 11 & 8 \\ 4 & 3 \end{pmatrix}$$

(a) Find the value of a and the value of b.

(4)

(b) With $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, show that $\mathbf{AB} \neq \mathbf{BA}$

(2)

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Question 5 continued

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6.

$$\mathbf{A} = \begin{pmatrix} p & 5 \\ 2 & p+3 \end{pmatrix}, \quad \text{where } p \text{ is a constant.}$$

(a) Find the values of p for which $\mathbf{A}$ is singular.

(3)

Given that $p = 3$,

(b) find $\mathbf{A}^{-1}$,

(3)

(c) find the matrix $\mathbf{B}$ such that $\mathbf{AB} = \begin{pmatrix} -2 & 11 \\ -4 & 10 \end{pmatrix}$

(3)

(d) verify that $\det(\mathbf{AB}) = \det\mathbf{A} \times \det\mathbf{B}$.

(1)

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Question 6 continued

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Question 6 continued

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7.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

(a) Show that, for any non-singular 2×2 matrices $\mathbf{A}$ and $\mathbf{B}$, $(\mathbf{AB})\left(\mathbf{B}^{-1}\mathbf{A}^{-1}\right) = \mathbf{I}$

(2)

(b) Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$.

(2)

(c) Hence find $(\mathbf{AB})^{-1}$, and check your answer by finding $\mathbf{AB}$.

(3)

(d) Solve $\mathbf{AB}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(3)

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Question 7 continued

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Question 7 continued

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8.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

(a) Find $\mathbf{M}^2$ and $\mathbf{M}^3$, and suggest a form for $\mathbf{M}^n$, where n is a positive integer.

(3)

(b) Find $\mathbf{M}^{-1}$, and show that it fits your form with $n = -1$

(3)

(c) Find the matrix $\mathbf{X}$ such that $\mathbf{MX} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

(4)

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Question 8 continued

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9.

$$\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$$

(a) Show that $\mathbf{A}^2 - 5\mathbf{A} + 6\mathbf{I} = \mathbf{0}$

(3)

(b) Hence show that $\mathbf{A}^{-1} = \frac{1}{6}(5\mathbf{I} - \mathbf{A})$

(3)

(c) Find $\mathbf{A}^{-1}$, and check your answer using the formula for the inverse of a 2×2 matrix.

(2)

(d) Express $\mathbf{A}^3$ in the form $p\mathbf{A} + q\mathbf{I}$, where p and q are integers.

(2)

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Question 9 continued

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10. The simultaneous equations

$$kx + 2y = 4, \qquad 8x + ky = 8$$

where k is a constant, are written as $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

(a) Find the values of k for which $\mathbf{M}$ is singular.

(2)

(b) For $k = 2$, use $\mathbf{M}^{-1}$ to solve the equations.

(4)

(c) For each of the values of k in part (a), determine whether the equations have no solutions or infinitely many solutions, giving a reason.

(4)

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Question 10 continued

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TOTAL FOR CHAPTER 5 IS 80 MARKS

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