



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Equations and Inequalities

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Equations and Inequalities worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Equations and Inequalities worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $x = 1 + 2y$: $(1 + 2y)y = 3$, so $2y^2 + y - 3 = 0$, so $(2y + 3)(y - 1) = 0$

$y = 1$ gives $x = 3$; $y = -\frac{3}{2}$ gives $x = -2$ $x = 3, y = 1$ or $x = -2, y = -\frac{3}{2}$

A2 $3x^2 - 10x - 8 \leq 0$, so $(3x + 2)(x - 4) \leq 0$: critical values $-\frac{2}{3}$ and 4

A U-shape is below the axis *between* its roots: $-\frac{2}{3} \leq x \leq 4$

A3 $7 - x^2 = x + 1$, so $x^2 + x - 6 = 0$, so $(x + 3)(x - 2) = 0$: they meet at $x = -3$ and $x = 2$

Test $x = 0$: the curve gives 7, the line gives 1, so R is below the curve and above the line.

$y \leq 7 - x^2$ and $y \geq x + 1$

STRETCH

A4 Linear: $6 > 2x$, so $x < 3$. Quadratic: $(2x - 3)(x + 2) \geq 0$, so $x \leq -2$ or $x \geq \frac{3}{2}$

Values in both: $\{x : x \leq -2\} \cup \left\{x : \frac{3}{2} \leq x < 3\right\}$

A5 Multiply by x^2 : $x(5x - 4) > 3x^2$, so $2x^2 - 4x > 0$, so $2x(x - 2) > 0$

Critical values 0 and 2, outside region: $x < 0$ or $x > 2$ (0 is already excluded)

A6 $x^2 + kx = 4x - x^2 - 2$, so $2x^2 + (k - 4)x + 2 = 0$

Two distinct points: $(k - 4)^2 - 16 > 0$, so $k^2 - 8k > 0$, so $k(k - 8) > 0$: $k < 0$ or $k > 8$

A7 $2x^2 + 3 = 9 - x$, so $2x^2 + x - 6 = 0$, so $(2x - 3)(x + 2) = 0$: they meet at $x = \frac{3}{2}$

R is above the curve and below the line, and $x \geq 0$ is needed for the y -axis:

$x \geq 0, y \geq 2x^2 + 3, y \leq 9 - x$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

- B1** (a) $x^2 + 1 = x + 3$, so $x^2 - x - 2 = 0$, $(x - 2)(x + 1) = 0$, and $y = x + 3$: $(2, 5)$, $(-1, 2)$
 (b) $x^2 - x = 0$, $x(x - 1) = 0$, and $y = x + 3$: $(0, 3)$, $(1, 4)$
 (c) $x^2 + 2x + 1 = 0$, $(x + 1)^2 = 0$, and $y = x + 3$: $(-1, 2)$ only: the line touches the curve
 (d) $x^2 + 2 = 0$ has no real roots: **the graphs do not meet**

The line stays the same and the curve moves up: two points, two points, one touching point, none.

- B2** (a) $y = 6 - x$, $x(6 - x) = 8$, $x^2 - 6x + 8 = 0$: $x = 2$, $y = 4$ or $x = 4$, $y = 2$
 (b) $x = y + 2$, $(y + 2)^2 + y^2 = 20$, $y^2 + 2y - 8 = 0$: $x = 4$, $y = 2$ or $x = -2$, $y = -4$
 (c) $y = 1 - 2x$, $x^2 + (1 - 2x)^2 = 2$, $5x^2 - 4x - 1 = 0$: $x = 1$, $y = -1$ or $x = -\frac{1}{5}$, $y = \frac{7}{5}$
 (d) $x = 5 - 2y$, $(5 - 2y)^2 + 4y^2 = 13$, $2y^2 - 5y + 3 = 0$: $x = 3$, $y = 1$ or $x = 2$, $y = \frac{3}{2}$

Make the subject the letter with coefficient 1. Then pair each value with its own partner.

- B3** (a) $3x < 18$: $x < 6$ (b) $-2x \geq 6$: $x \leq -3$ (c) $2x - 8 > 5x + 4$, $-3x > 12$: $x < -4$
 (d) $2(x + 1) - 3(x - 2) \leq 6$, so $-x + 8 \leq 6$, $-x \leq -2$: $x \geq 2$

The sign reverses only when you multiply or divide by a negative number, as in (b), (c) and (d).

- B4** (a) $2 < x < 5$ (b) $x \leq 2$ or $x \geq 5$ (c) $(x - 2)(x - 5) < 0$: $2 < x < 5$
 (d) $x^2 - 7x + 10 < 0$, $(x - 2)(x - 5) < 0$: $2 < x < 5$

(c) and (d) are (a) in disguise: multiplying by -1 reverses the sign, and (d) must be rearranged first.

- B5** (a) $x > 3$ and $2 < x < 5$: $3 < x < 5$
 (b) $x > 3$ or $2 < x < 5$: $x > 2$
 (c) $x > 3$ and $(x < 2$ or $x > 5)$: $x > 5$

"And" keeps the values in both; "or" keeps the values in either. Use a number line.

- B6** (a) $x < y < 6 - x^2$ (b) $y > 6 - x^2$ and $y > x$ (c) $y < 6 - x^2$ and $y < x$
 (d) $y > 6 - x^2$, $y < x$, $x > 2$

In (d), "above the curve and below the line" also describes a second piece where $x < -3$, so an x restriction is needed.



STRETCH

B7 (a) $6x > 2x^2$, $2x(x-3) < 0$: $0 < x < 3$ (b) $6x > -2x^2$, $2x(x+3) > 0$: $x < -3$ or $x > 0$

(c) $x(x+6) < 2x^2$, $x(x-6) > 0$: $x < 0$ or $x > 6$

(d) Multiply by $(x-1)^2$: $6(x-1) > 2(x-1)^2$, so $2(x-1)(x-4) < 0$: $1 < x < 4$

Always multiply by the square of the denominator, which is positive. In (d) the denominator is $(x-1)$, not x .

B8 $x^2 + 2x + 5 = kx + 1$, so $x^2 + (2-k)x + 4 = 0$, with $b^2 - 4ac = (2-k)^2 - 16$

(a) $(2-k)^2 - 16 > 0$: $k < -2$ or $k > 6$ (b) $(2-k)^2 = 16$: $k = -2$ or $k = 6$

(c) $(2-k)^2 - 16 < 0$: $-2 < k < 6$

(d) $k = 6$: $x^2 - 4x + 4 = 0$, $(x-2)^2 = 0$, and $y = 6x + 1$: $(2, 13)$

One discriminant answers three questions: > 0 , $= 0$ and < 0 .

B9 (a) $x = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$: $2 - \sqrt{5} < x < 2 + \sqrt{5}$

(b) $(x-2)^2 > 0$, true except at $x = 2$: $x < 2$ or $x > 2$ (every x except 2)

(c) $(x-2)^2 + 1 > 0$ for every x : all real values of x

(d) $(x-2)^2 + 1$ is never negative: no real values of x

Only the constant changes: two surd roots, a repeated root, then no roots at all.

B10 (a) $y = 5 - x$, $x(5-x) = 6$, $x^2 - 5x + 6 = 0$: $(2, 3)$, $(3, 2)$

(b) $y = 5 - x$, $x^2 + (5-x)^2 = 13$, $x^2 - 5x + 6 = 0$: $(2, 3)$, $(3, 2)$

(c) $(x+y)(x-y) = 5$, so $x-y = 1$; with $x+y = 5$: $(3, 2)$ only

(d) $y = 5 - x$, $2x^2 + 2(5-x)^2 = 25$, $4x^2 - 20x + 25 = 0$, $(2x-5)^2 = 0$: $\left(\frac{5}{2}, \frac{5}{2}\right)$

Same line, four curves: (a) and (b) give the same points; in (c) the squared terms cancel; in (d) the line touches.



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** $x > -3$ includes $x = 0$, but $0^2 = 0$, which is not greater than 9, and Tom also misses every x below -3
 $x^2 - 9 > 0$, $(x - 3)(x + 3) > 0$, outside region: $x < -3$ or $x > 3$

- C2** Kim's statement says x is less than $-\frac{1}{2}$ and greater than 4 at the same time, which no number can be.

The two parts must be joined by "or": $x < -\frac{1}{2}$ or $x > 4$

- C3** $x^2 - 2x + 2 = 4x - 7$, so $x^2 - 6x + 9 = 0$
 $b^2 - 4ac = 36 - 36 = 0$, so there is exactly one (repeated) root: the line is a tangent.
 $(x - 3)^2 = 0$, so $x = 3$, and $y = 4 \times 3 - 7 = 5$: $(3, 5)$

- C4** For example $(-1, 4)$: $0 \leq 4 \leq 8$, so it satisfies Ali's inequalities, but $x < 0$, so it is not in the first quadrant.
 Ali has no boundary for the y -axis: $x \geq 0$, $y \geq 0$, $y \leq 9 - x^2$

STRETCH

- C5** $\frac{3}{-1} = -3$, which is less than 1, so $x = -1$ satisfies the inequality.
 When $x < 0$, multiplying by x reverses the sign, so Priya's method only works for positive x .
 Multiply by x^2 : $3x < x^2$, so $x(x - 3) > 0$: $x < 0$ or $x > 3$

- C6** $x^2 + 2x - 3 = kx + 1$, so $x^2 + (2 - k)x - 4 = 0$
 $b^2 - 4ac = (2 - k)^2 + 16$
 $(2 - k)^2 \geq 0$, so $b^2 - 4ac \geq 16 > 0$ for every real k : always two distinct points. **as required**

- C7** For example $k = -10$ and $x = 5$: $25 - 50 + 9 = -16$, which is not greater than 0, so Jo is wrong.
 For all x , the U-shape must stay above the axis: no real roots, so $k^2 - 36 < 0$
 $-6 < k < 6$ (Jo has forgotten the lower limit.)



- C8** (a) $y = p - x$, so $x^2 + (p - x)^2 = 8$, so $2x^2 - 2px + p^2 - 8 = 0$ **as required**
- (b) Real x needs $b^2 - 4ac \geq 0$: $4p^2 - 8(p^2 - 8) \geq 0$, so $64 - 4p^2 \geq 0$, so $p^2 \leq 16$
- $-4 \leq p \leq 4$
- (c) The greatest value of $x + y$ is the greatest p : **4**
- $p = 4$: $2x^2 - 8x + 8 = 0$, so $(x - 2)^2 = 0$: **$x = 2, y = 2$**

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

- D1** (a) Cutting the corner square does not change the perimeter: $2x + 2y = 26$, so $x + y = 13$
- Area: $xy - 4 = 36$, so $xy = 40$ **as required**
- (b) $y = 13 - x$, $x(13 - x) = 40$, so $x^2 - 13x + 40 = 0$, $(x - 5)(x - 8) = 0$
- $x = 8, y = 5$ or $x = 5, y = 8$. Only one pair has $x > y$: **$x = 8, y = 5$**

- D2** (a) $3x^2 - 2x + k = x^2 + 6x - 1$, so $2x^2 - 8x + k + 1 = 0$ **as required**
- (b) Exactly one point: $b^2 - 4ac = 0$, so $64 - 8(k + 1) = 0$: **$k = 7$**
- (c) $2x^2 - 8x + 8 = 0$, so $(x - 2)^2 = 0$, and $y = x^2 + 6x - 1$: **$(2, 15)$**

- D3** (a) $x^2 - x - 6 = x + 9$, so $x^2 - 2x - 15 = 0$, so $(x - 5)(x + 3) = 0$: **$x = -3, x = 5$**
- (b) C below l : $-3 < x < 5$. C above the x -axis: $(x - 3)(x + 2) > 0$, so $x < -2$ or $x > 3$
- Both: **$-3 < x < -2$ or $3 < x < 5$**

- D4** (a) $40x - 2x^2 - 150 \geq 42$, so $2x^2 - 40x + 192 \leq 0$, so $x^2 - 20x + 96 \leq 0$ **as required**
- (b) $(x - 8)(x - 12) \leq 0$, so $8 \leq x \leq 12$: the price is from **£8** to **£12** inclusive

STRETCH

- D5** (a) $kx^2 + 3 = 4x - x^2$, so $(k + 1)x^2 - 4x + 3 = 0$ **as required**
- (b) If $k \neq -1$ the equation is a quadratic: exactly one root needs $16 - 12(k + 1) = 0$, so **$k = \frac{1}{3}$**
- If $k = -1$ the x^2 term disappears: $-4x + 3 = 0$ is linear, with exactly one root, so **$k = -1$**



D6 (a) $x^2 - 3x + 5 = x + c$, so $x^2 - 4x + 5 - c = 0$

Tangent: $b^2 - 4ac = 0$, so $16 - 4(5 - c) = 0$: $c = 1$

(b) $x^2 - 4x + 4 = 0$, so $(x - 2)^2 = 0$: P is at $x = 2$. The curve is above the tangent on both sides of P , so R needs $x \leq 2$ as well as $x \geq 0$:

$$0 \leq x \leq 2, \quad x + 1 \leq y \leq x^2 - 3x + 5$$

D7 (a) $l: y = 2(x - 1)$. $(x - 1)(x - a) = 2(x - 1)$, so $(x - 1)(x - a - 2) = 0$

$x = 1$ is A , so at P , $x = a + 2$ and $y = 2(a + 2) - 2 = 2a + 2$ as required

(b) $AP^2 = (a + 1)^2 + (2a + 2)^2 = 5(a + 1)^2$, so $AP = \sqrt{5}(a + 1)$

$\sqrt{5}(a + 1) = 4\sqrt{5}$, so $a + 1 = 4$: $a = 3$

D8 (a) $x^2 - 2x + 6 = mx$, so $x^2 - (2 + m)x + 6 = 0$

Two distinct points: $(2 + m)^2 - 24 > 0$, so $m^2 + 4m - 20 > 0$ as required

(b) $m = \frac{-4 \pm \sqrt{96}}{2} = -2 \pm 2\sqrt{6}$, outside region:

$$m < -2 - 2\sqrt{6} \text{ or } m > -2 + 2\sqrt{6}$$

(c) The roots add to $2 + m$, so the midpoint has $x = \frac{2 + m}{2} = 3$: $m = 4$ (in the range, as $2\sqrt{6} < 5$)



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $2x^2 - 3x - 2 = x + 4$

M1: equates and collects to a 3-term quadratic

$2x^2 - 4x - 6 = 0$, so $x^2 - 2x - 3 = 0$, so $(x - 3)(x + 1) = 0$

dM1: solves with working shown

$y = x + 4$: $P(-1, 3)$, $Q(3, 7)$

A1: both points

(b) $y \geq 0$, $y \leq x + 4$, $y \leq 2x^2 - 3x - 2$, $x \leq 0$

M1: two correct; A1: all four

Any x restriction that removes the second piece (for $x \geq 2$) scores; strict inequalities used consistently also score.

E2 (a) $kx = 4$: $x = \frac{4}{k}$

B1

(b) $y = 12 - x$ meets $y = 4$ at $x = 8$. R_1 is a trapezium with parallel sides 12 and $8 - \frac{4}{k}$, and height 4

M1

Area = $\frac{1}{2} \left(12 + 8 - \frac{4}{k} \right) \times 4 = 40 - \frac{8}{k}$ as required

A1*

(c) $40 - \frac{8}{k} = 36$, so $\frac{8}{k} = 4$: $k = 2$

B1

(d) $y \geq 4$, $y \leq 2x$, $y \leq 12 - x$

M1: two correct; A1: all three, with $k = 2$

STRETCH

E3 (a) $2 + \frac{4}{x} = k - x$

M1: equates

Multiply by x : $2x + 4 = kx - x^2$, so $x^2 + (2 - k)x + 4 = 0$

as required

A1*: cso

(b) Does not meet: $(2 - k)^2 - 16 < 0$

M1: discriminant < 0

$(2 - k)^2 = 16$ gives critical values -2 and 6

dM1

$-2 < k < 6$

A1: inside region

 $x = 0$ is never a root, as the constant term is 4, so no value of k is lost by multiplying by x .

(c) $k = 7$: $x^2 - 5x + 4 = 0$, so $(x - 1)(x - 4) = 0$

M1: solves for the ends of R

$2 + \frac{4}{x} \leq y \leq 7 - x$, $x > 0$

M1: both curve inequalities; A1: with $x > 0$ Without $x > 0$, the same two inequalities also hold for every negative x below the line. $1 \leq x \leq 4$ is an equally good restriction.