eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2 (WST02)

Statistics 2 Practice Paper 3

Worked Solutions

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WST02/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen — the working a full-marks answer shows, with a short note on why each step is taken

M1 A1 — the mark earned on that line: M = method, A = accuracy, B = independent mark

boxed — the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	A sample size and two approximations	12	2
2	Two Poisson tests on typing errors	12	3
3	A pdf in two pieces	13	4
4	A stick broken at random	13	6
5	A binomial test with a normal approximation	12	7
6	The sampling distribution of a median	13	8

Question paper & mark scheme | Exam questions by topic | More practice papers

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

1. A factory makes electronic components, and each component is faulty with probability 0.08, independently of all other components.

(a) Find the smallest number of components that must be tested for the probability that at least one of them is faulty to be greater than 0.95

(4)

A batch contains 250 components.

(b) Using a suitable approximation, find the probability that fewer than 15 of the components in the batch are faulty. Give a reason for your choice of approximation.

(5)

A second factory makes the same components, each of which is faulty with probability 0.02

(c) Using a suitable approximation, find the probability that a batch of 250 components from the second factory contains more than 8 faulty components.

(3)

(a) $P(\text{at least one faulty in } n) = 1 - 0.92^n > 0.95$ M1

$0.92^n < 0.05$ A1

$n \ln 0.92 < \ln 0.05 \Rightarrow n > \ln 0.05 \div \ln 0.92 = 35.9$ (ln 0.92 < 0, so the sign flips) M1

Check: $0.92^{35} = 0.0540$, $0.92^{36} = 0.0497$; round UP **36** A1

(b) $np = 20$ is large and $p = 0.08$ is not small: use a normal approximation. B1

Mean $250 \times 0.08 = 20$, variance $20 \times 0.92 = 18.4$: $Y \sim N(20, 18.4)$ M1

$P(X < 15) = P(X \leqslant 14) \approx P(Y < 14.5)$ M1

$z = (14.5 - 20) \div \sqrt{18.4} = -1.282\ldots = -1.28$ M1

$P(Z < -1.28) = 1 - 0.8997$ **0.1003** A1

(c) $p = 0.02$ is small, $n = 250$ large: $W \approx Po(250 \times 0.02) = Po(5)$ B1

$P(W > 8) = 1 - P(W \leqslant 8) = 1 - 0.9319$ M1

0.0681 A1

(Total for Question 1 is 12 marks)

DO NOT WRITE IN THIS AREA

2. A typist makes errors at random, at a mean rate of 0.4 per page. A manager suspects that the typist's mean rate of errors has increased. She checks 5 pages typed by the typist.

(a) Write down suitable hypotheses to test the manager's suspicion.

(1)

(b) Find the critical region for the test at the 5% level of significance.

(3)

(c) Write down the actual significance level of the test.

(1)

The manager finds 5 errors on the 5 pages.

(d) Comment on the manager's suspicion, giving a reason for your answer.

(2)

A second typist claims that her mean rate of errors is less than 0.4 per page. The manager checks 25 pages typed by this typist and finds 4 errors.

(e) Test the second typist's claim at the 5% level of significance. State your hypotheses clearly.

(5)

(a) 5 pages: $\lambda = 5 \times 0.4 = 2$. $H_0: \lambda = 2$, $H_1: \lambda > 2$ B1

(b) Under H_0, $X \sim Po(2)$. Upper tail, at most 5%: M1

$P(X \geqslant 5) = 1 - 0.9473 = 0.0527 > 0.05$ (too big!); $P(X \geqslant 6) = 1 - 0.9834 = 0.0166$ M1

0.0527 is closer to 5% but exceeds it, so 5 is NOT in the region. $X \geqslant 6$ A1

(c) 0.0166 B1

(d) 5 is not in the critical region, so do not reject H_0. M1

There is insufficient evidence that the typist's mean rate of errors has increased. A1

(e) 25 pages: $\lambda = 25 \times 0.4 = 10$. $H_0: \lambda = 10$, $H_1: \lambda < 10$ B1

Under H_0, $Y \sim Po(10)$: $P(Y \leqslant 4)$ M1

$= 0.0293$ A1

$0.0293 < 0.05$: significant, so reject H_0. M1

There is evidence that the second typist makes fewer than 0.4 errors per page. A1

(Total for Question 2 is 12 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3.

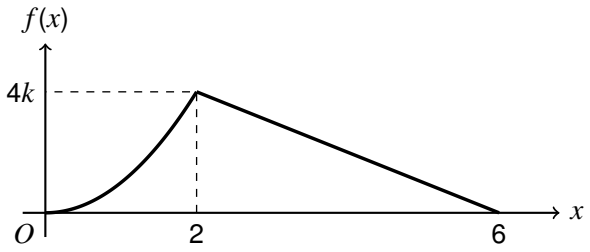

Figure 1

The continuous random variable X has probability density function

$$f(x) = \begin{cases} kx^2 & 0 \leqslant x \leqslant 2 \\ k(6-x) & 2 < x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant, as shown in Figure 1.

(a) Find the value of k

(3)

(b) Find the lower quartile of X

(2)

(c) Find F(x) for $2 < x \leqslant 6$

(3)

(d) Find the median of X

(3)

(e) Describe the skewness of the distribution, giving a reason for your answer.

(2)

(a) $\int$ from 0 to 2 of $kx^2\,dx + \int$ from 2 to 6 of $k(6-x)\,dx = 1$ (second part: a triangle) M1

$k(8/3) + k(\tfrac{1}{2} \times 4 \times 4) = 8k/3 + 8k = 32k/3 = 1$ A1

$\boxed{k = 3/32}$ A1

(b) $F(2) = (3/32) \times (8/3) = 1/4$ M1

So the lower quartile is exactly at the join: $\boxed{Q_1 = 2}$ A1

(c) $F(x) = 1/4 + \int$ from 2 to x of $(3/32)(6 - t)\,dt$ M1

$= 1/4 + (3/32)(6x - x^2/2 - 12 + 2) = 1/4 + (3/32)(6x - x^2/2 - 10)$ A1

$\boxed{F(x) = (36x - 3x^2 - 44)/64}$ (check $F(6) = 64/64 = 1$ ✓) A1

(d) $F(2) = 1/4 < 1/2$, so the median is in the second piece: $(36m - 3m^2 - 44)/64 = 1/2$ M1

$36m - 3m^2 - 44 = 32 \Rightarrow 3m^2 - 36m + 76 = 0 \Rightarrow m = 6 \pm \sqrt{32/3}$ A1

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

$6 + 3.27 = 9.27 > 6$ is outside the range: reject. median = 2.73 A1

(e) f increases to $x = 2$ then decreases, so mode = 2; median = 2.73 M1

mode < median $\Rightarrow$ positive skew A1

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 13 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4. A stick AB of length 60 cm is broken at a point P, chosen at random along its length. The random variable X is the length, in cm, of AP

(a) Find the probability that the shorter piece is less than one third of the length of the longer piece.

(3)

(b) Given that the shorter piece is less than one third of the length of the longer piece, find the probability that the shorter piece is less than 6 cm long.

(3)

The piece AP is bent to form the four sides of a square.

(c) Find the probability that the area of the square is greater than 100 cm^2

(2)

(d) Find the expected area of the square.

(3)

Ten such sticks are broken at random, independently.

(e) Find the probability that, for at least 6 of the sticks, the shorter piece is less than one third of the length of the longer piece.

(2)

(a) $X \sim U[0, 60]$. Shorter s, longer $60 - s$: $s < (60 - s)/3 \Rightarrow 4s < 60 \Rightarrow s < 15$ M1

The shorter piece can be AP or PB: $X < 15$ or $X > 45$ M1

$P = (15 + 15) \div 60$ **0.5** A1

(b) Shorter piece < 6: $X < 6$ or $X > 54$, probability $12/60 = 0.2$ M1

$P(\text{shorter} < 6 \mid \text{shorter} < 15) = 0.2 \div 0.5$ (the first event is inside the second) M1

0.4 A1

(c) Side $X/4$, area $X^2/16 > 100 \Rightarrow X^2 > 1600 \Rightarrow X > 40$ M1

$P(X > 40) = 20/60$ **1/3** A1

(d) $E(\text{area}) = E(X^2)/16$, $E(X^2) = \text{Var}(X) + (E(X))^2$ M1

$\text{Var}(X) = 60^2/12 = 300$, $E(X) = 30 \Rightarrow E(X^2) = 300 + 900 = 1200$ A1

$E(\text{area}) = 1200 \div 16$ **75 cm^2** (not $30^2/16 = 56.25$) A1

(e) $T \sim B(10, 0.5)$: $P(T \geqslant 6) = 1 - P(T \leqslant 5) = 1 - 0.6230$ M1

0.3770 A1

(Total for Question 4 is 13 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

5. In the past, 35% of the customers of a bank used its mobile app. After the app is redesigned, the bank believes that the proportion has fallen. A random sample of 200 customers is taken, and X is the number who use the app.

(a) Write down suitable hypotheses for a test of the bank's belief.

(1)

(b) Explain why a normal approximation is suitable for the distribution of X under the null hypothesis.

(1)

(c) Using a normal approximation, find the critical region for the test at the 5% level of significance.

(5)

(d) Using a normal approximation, find the actual significance level of the test.

(3)

In the sample, 59 customers use the app.

(e) State, with a reason, the conclusion of the test.

(2)

(a) $H_0: p = 0.35$, $H_1: p < 0.35$ B1

(b) $n = 200$ is large and $p = 0.35$ is near 0.5 ($np = 70$ and $n(1-p) = 130$ are both large). B1

(c) $X \sim B(200, 0.35)$: mean 70, variance $200 \times 0.35 \times 0.65 = 45.5$: $Y \sim N(70, 45.5)$ B1

Want the largest c with $P(X \leqslant c) < 0.05$: $P(X \leqslant c) \approx P(Y < c + 0.5)$ M1

$(c + 0.5 - 70) \div \sqrt{45.5} < -1.6449$ M1

$c + 0.5 < 70 - 1.6449 \times 6.745 = 58.905 \Rightarrow c < 58.4$ A1

$X \leqslant 58$ A1

(d) Actual level $= P(X \leqslant 58) \approx P(Y < 58.5)$ M1

$z = (58.5 - 70) \div \sqrt{45.5} = -1.704\ldots = -1.70$ M1

$P(Z < -1.70) = 1 - 0.9554$ 0.0446 A1

(e) 59 is not in the critical region (just outside it), so do not reject H_0. M1

There is insufficient evidence that the proportion using the app has fallen. A1

(Total for Question 5 is 12 marks)

DO NOT WRITE IN THIS AREA

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6. A large bag contains coins. Of the coins, 60% are 10c coins, 30% are 20c coins and 10% are 50c coins. A random sample of three coins is taken from the bag, and M is the median value, in cents, of the three coins.

(a) Explain why M is a statistic.

(1)

(b) Find the sampling distribution of M

(7)

(c) Find E(M)

(2)

(d) Find the probability that the median value of the three coins is equal to their mean value.

(3)

(a) M is found from the sample values only, with no unknown parameters. B1

(b) The median of three coins is 10, 20 or 50. B1

M = 10 needs at least two 10c: (10,10,10) or two 10c and one other, in 3 orders M1

$P(M = 10) = 0.6^3 + 3 \times 0.6^2 \times 0.4 = 0.216 + 0.432 = 0.648$ A1

M = 50 needs at least two 50c: $0.1^3 + 3 \times 0.1^2 \times 0.9$ M1

$P(M = 50) = 0.001 + 0.027 = 0.028$ A1

$P(M = 20) = 1 - 0.648 - 0.028$; or $0.3^3 + 3(0.3^2)(0.7) + 6(0.6)(0.3)(0.1)$ M1

M: 10, 20, 50 with P = 0.648, 0.324, 0.028 A1

(c) $E(M) = 10 \times 0.648 + 20 \times 0.324 + 50 \times 0.028 = 6.48 + 6.48 + 1.4$ M1

14.36 A1

(d) 10, 20, 50 are not equally spaced, so median = mean only if all three are equal. M1

$P = 0.6^3 + 0.3^3 + 0.1^3 = 0.216 + 0.027 + 0.001$ M1

0.244 A1

(Total for Question 6 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END

DO NOT WRITE IN THIS AREA

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk