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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA13 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	2	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	25	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	51	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	77	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	103	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	129	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	155	
8 Numerical Methods Locating roots, iteration	10	77	181	

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1

CHAPTER 1 • KEY SKILLS

Algebraic Methods

Specification P3: Algebraic methods • 10 questions • 69 marks • about 85 minutes

1 Factorise and cancel

Factorise the numerator and the denominator fully, then cancel common factors, never terms.

e.g. $\frac{x^2 - 9}{x^2 + 3x} = \frac{x - 3}{x}$

2 Multiplying and dividing

To divide, multiply by the reciprocal.
Factorise everything before cancelling.

3 Adding and subtracting

Use the lowest common denominator: factorise the denominators first to find it.

4 Equations with fractions

Multiply through by the common denominator. Reject any value that makes a denominator zero.

5 Improper fractions

The numerator's degree is at least the denominator's: divide first.

e.g. $\frac{3x^2 - 5x + 4}{x - 2} = 3x + 1 + \frac{6}{x - 2}$

6 Algebraic division

Long division by $(x + a)$ or by a quadratic: the remainder has lower degree than the divisor.

7 Identities

$\equiv$ means true for all x : multiply out and compare coefficients, or substitute convenient values of x .

8 “Show that”

Show every step: the common denominator, the expanded numerator and the factor that cancels.

9 Excluded values

x values that make *any* denominator zero are excluded, even if the factor later cancels.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	6	5	5	9	8	8	9	10
1 Factorise and cancel	•	•	•		•	•		•		
2 Multiplying and dividing		•						•		
3 Adding and subtracting	•		•		•				•	•
4 Equations with fractions		•	•	•	•	•		•		•
5 Improper fractions		•		•		•	•			•
6 Algebraic division		•		•		•	•			
7 Identities		•	•				•		•	•
8 “Show that”	•		•			•	•	•	•	•
9 Excluded values		•			•	•		•		•
Interleaved: factor theorem, sigma notation (P2)						•			•	
Interleaved: quadratics, surds and inequalities (P1)			•	•	•		•		•	•
Interleaved: graphs and asymptotes (P1)										•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. When asked for A , B and C , also write the whole expression out in the required form. Factorise with integer coefficients, e.g. $(3x - 5)$, not $(x - \frac{5}{3})$.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{3x^2 - 27}{2x^2 + x - 15} + \frac{4}{2x - 5} \quad x > \frac{5}{2}$$

Show that $f(x) = \frac{3x - 5}{2x - 5}$

(4)

(Total for Question 1 is 4 marks)



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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{2x^4 + 7x^3 + 2x^2 + 2x}{x^2 + 3x} \quad x > 0$$

(a) Find the values of the constants A , B , C and D such that

$$f(x) = Ax^2 + Bx + C + \frac{D}{x+3}$$

(4)

(b) Hence solve the equation $f(x) = 2x^2 + x$

(1)



Question 2 continued

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(Total for Question 2 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

3. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.****(a)** Show that

$$\frac{3}{x+2} + \frac{2}{x-1} - \frac{7x-1}{x^2+x-2}$$

can be written as $\frac{a}{x+b}$, where a and b are integers to be found.**(4)****(b)** Hence show that the equation

$$\frac{3}{x+2} + \frac{2}{x-1} - \frac{7x-1}{x^2+x-2} = x+1$$

has no real solutions.

(2)

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Question 3 continued

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(Total for Question 3 is 6 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4.

$$f(x) = \frac{6x^2 + 5x - 4}{2x + 3} \quad x \neq -\frac{3}{2}$$

(a) Write $f(x)$ in the form $Ax + B + \frac{C}{2x+3}$, where A , B and C are constants to be found.

(3)

(b) Hence find the set of values of x for which $f(x) > 3x - 2$

(2)

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Question 4 continued

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(Total for Question 4 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$\frac{3x}{x-2} - \frac{4}{x+1} = \frac{12}{x^2 - x - 2}$$

explaining clearly why one of the solutions of your working must be rejected.

(5)

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Question 5 continued

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{x^3 + 3x^2 - 4}{x^2 + x - 2} \quad x \neq 1, x \neq -2$$

(a) Use the factor theorem to show that $(x - 1)$ is a factor of $x^3 + 3x^2 - 4$

(2)

(b) Hence show that $f(x) = x + 2$

(4)

(c) Explain why the graph of $y = f(x)$ is a straight line with two points missing, and find the coordinates of the two missing points.

(3)

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Question 6 continued

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(Total for Question 6 is 9 marks)



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Solutions relying on calculator technology are not acceptable.

$$g(x) = \frac{x^4 + 2x^3 + 7x + 7}{x^2 + 1} \quad x \in \mathbb{R}$$

- (a)** Show that $g(x) \equiv x^2 + Bx + C + \frac{Dx + E}{x^2 + 1}$ and find the values of the constants B, C, D and E .

(5)

- (b)** Find the set of values of x for which

$$\frac{Dx + E}{x^2 + 1} > 1$$

giving your answer in exact form. Explain why you may multiply both sides of the inequality by $x^2 + 1$

(3)

Question 7 continued

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8.

$$h(x) = \frac{x^2 - 4}{x^2 + 3x + 2} \div \frac{x^2 - 5x + 6}{x^2 - 2x - 3}$$

- (a) Show that $h(x) = 1$ for all values of x for which it is defined. (4)
- (b) State the values of x for which $h(x)$ is not defined. (2)
- (c) Hence show that the equation $h(x) = x^2 - 3$ has no solutions. (2)

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Question 8 continued

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(Total for Question 8 is 8 marks)



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9. (a) Show that $\frac{1}{r} - \frac{1}{r+1} \equiv \frac{1}{r(r+1)}$ (2)

- (b) Hence show that

$$\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1} \quad (4)$$

- (c) Find the least value of n for which $\sum_{r=1}^n \frac{1}{r(r+1)} > 0.99$ (3)

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{2x^2 + kx - 3}{x + 2} \quad x \neq -2$$

where k is a constant. Given that $f(x) \equiv 2x + 1 + \frac{C}{x+2}$, where C is a constant,

(a) find the value of k and the value of C .

(4)

(b) Find the x coordinate of the point where the curve $y = f(x)$ meets the line $y = 2x - 1$

(2)

(c) Explain why, for $x > -2$, the curve $y = f(x)$ lies below the line $y = 2x + 1$

(1)

(d) Find the set of values of x for which $f(x) > 2x$

(3)



Question 10 continued

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(Total for Question 10 is 10 marks)**TOTAL FOR CHAPTER 1 IS 69 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

2

CHAPTER 2 • KEY SKILLS

Functions and Graphs

Specification P3: Functions • 10 questions • 76 marks • about 90 minutes

1 The modulus function $|x| = x$ for $x \geq 0$ and $-x$ for $x < 0$. $y = |ax + b|$ is a V shape with its vertex on the x -axis.**2 Modulus equations**Solve $+(\dots)$ and $-(\dots)$ separately, then check each answer in the original equation. A sketch shows how many solutions.**3 Domain and range**The range of f is the set of outputs for the domain given. Find *both* ends: sketch it.**4 Composite functions** $fg(x) = f(g(x))$: apply g first. Its outputs must lie in the domain of f .**5 Inverse functions**Only one-to-one functions have inverses. Swap x and y , rearrange, write $f^{-1}(x) = \dots$.
Domain of f^{-1} = range of f .**6 $|f(x)|$ and $f(|x|)$** $y = |f(x)|$: reflect the parts below the x -axis upwards. $y = f(|x|)$: reflect the part for $x \geq 0$ in the y -axis.**7 Combined transformations** $y = af(bx + c) + d$: change x in the order " $+c$ then $\div b$ ", and y in the order " $\times a$ then $+d$ ".**8 Points under transformations**Track a point: e.g. $(3, -4)$ on $y = f(x)$ maps to $(5, 1)$ on $y = f(x - 2) + 5$ **9 Parameters in modulus graphs**

Write key points in terms of the constants; substitute known intersections into the correct branch.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	5	5	5	5	10	10	10	10	10
1 The modulus function		•	•				•		•	
2 Modulus equations		•	•				•	•	•	
3 Domain and range						•	•			•
4 Composite functions					•	•				•
5 Inverse functions						•				
6 $ f(x) $ and $f(x)$	•			•				•		
7 Combined transformations				•				•		
8 Points under transformations	•			•			•			
9 Parameters in modulus graphs			•				•			
Interleaved: algebraic fractions (Chapter 1)						•				•
Interleaved: quadratics, completing the square, surds (P1)					•			•	•	•
Interleaved: inequalities and simultaneous equations (P1)		•					•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.**Must learn.** Always give the domain of an inverse function. Write $f^{-1}(x) = \dots$ in terms of x , and check every modulus solution in the original equation.

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A graph of a function $y = f(x)$ is shown on a Cartesian coordinate system. The x-axis has tick marks at -4, 1, and 5. The y-axis has tick marks at 3 and -4. The origin is labeled O . The function passes through the points $(-2, 6)$, $(0, 3)$, $(3, -4)$, and $(5, 0)$. The point $(3, -4)$ is a local minimum, and $(-2, 6)$ is a local maximum.

Figure 1

On separate diagrams, sketch the curve with equation

$$\textbf{(a)} \quad y = |\mathbf{f}(x)| \tag{3}$$

$$\textbf{(b)} \quad y = f(|x|) \tag{3}$$

On each sketch, show clearly the coordinates of any turning points and of any points where the curve meets or crosses the coordinate axes.

Question 1 continued

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Solve the equation $|3x - 5| = x + 7$

(3)

(b) Hence find the set of values of x for which $|3x - 5| < x + 7$

(2)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = |3x + a| + a \quad x \in \mathbb{R}$$

where a is a positive constant.

(a) Sketch the graph of $y = f(x)$, showing, in terms of a , the coordinates of its vertex and of the point where it meets the y -axis.

(2)

(b) Solve, in terms of a , the equation $f(x) = 5a$

(3)

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Question 3 continued

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4. The point $P(3, -4)$ lies on the curve with equation $y = f(x)$.

Find the coordinates of the point, or points, to which P is mapped on the curve with equation

(a) $y = |f(x)| - 2$

(1)

(b) $y = f(|x|) + 1$

(2)

(c) $y = 2f(x + 1)$

(1)

(d) $y = -f(2x)$

(1)

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Question 4 continued

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = x^2 - 4x \quad g(x) = 2x + 1 \quad x \in \mathbb{R}$$

(a) Find $fg(x)$ in its simplest form.

(2)

(b) Solve the equation $fg(x) = gf(x)$, giving your answers in simplest surd form.

(3)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. The functions f and g are defined by

$$f(x) = \frac{2x+3}{x+1} \quad x \geq 0 \qquad g(x) = x^2 - 1 \quad x \in \mathbb{R}$$

(a) Find the range of f .

(2)

(b) Find $f^{-1}(x)$, stating its domain.

(4)

(c) Find the exact value of $fg(-2)$.

(1)

(d) Show that the equation $gf(x) = 3$ has no solutions.

(3)

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Question 6 continued

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Question 6 continued

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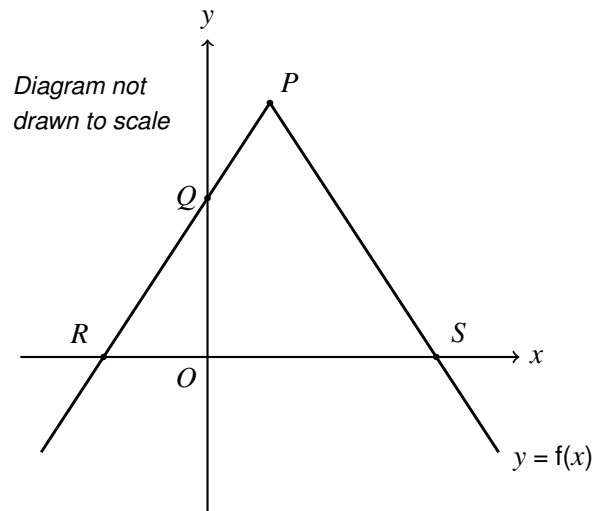


Figure 2

Figure 2 shows a sketch of the graph with equation $y = f(x)$, where

$$f(x) = a - |2x - b|$$

and a and b are positive constants with $a > b$. The graph has a maximum at P , meets the y -axis at Q and meets the x -axis at R and S .

(a) Find, in terms of a and b , the coordinates of

- (i) P
- (ii) Q
- (iii) R and S

(4)

The graph of $y = |x| - 1$ meets the graph of $y = f(x)$ at the points where $x = -2$ and $x = 4$

(b) Find the value of a and the value of b .

(5)

(c) Hence find the set of values of k for which the equation $f(x) = k$ has exactly two solutions.

(1)



Question 7 continued

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Solutions relying on calculator technology are not acceptable.

$$f(x) = x^2 - 6x + 5 \quad x \in \mathbb{R}$$

- (3)**

- (2)

- (1)

- (2)

- (2)

Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



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9. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

- (a) On the same axes, sketch the graphs of $y = |2x + 3|$ and $y = x^2$, showing the coordinates of any points where the graphs meet the coordinate axes. (3)
- (b) Use algebra to find the coordinates of the points where the two graphs meet. (3)
- (c) Hence solve the inequality $x^2 < |2x + 3|$ (2)
- (d) Solve the equation $|2x + 3| = 3x - 1$, explaining why one of the solutions of your working must be rejected. (2)

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Question 9 continued

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(Total for Question 9 is 10 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{4}{x+2} \quad x \geq 0 \qquad g(x) = x^2 - 2x \quad x \in \mathbb{R}$$

- (a) Find the range of f . (2)
- (b) Show that $gf(x) = \frac{-8x}{(x+2)^2}$ (2)
- (c) Find the range of gf . (3)
- (d) Solve the equation $gf(x) = -\frac{3}{4}$ (3)

Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 2 IS 76 MARKS

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CHAPTER 3 • KEY SKILLS

3

Trigonometric Functions

Specification P3: Trigonometry • 10 questions • 72 marks • about 85 minutes

1 Reciprocal functions

$$\sec x = \frac{1}{\cos x}, \quad \operatorname{cosec} x = \frac{1}{\sin x},$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

2 Graphs

$\sec x$ and $\operatorname{cosec} x$: range ≤ -1 or ≥ 1 ;
asymptotes where $\cos x$ or $\sin x$ is 0.
 $\cot x$: period π .

3 Pythagorean identities

$$1 + \tan^2 x \equiv \sec^2 x$$

$$1 + \cot^2 x \equiv \operatorname{cosec}^2 x$$

(from $\sin^2 x + \cos^2 x \equiv 1$)

4 Proving identities

Start from the more complicated side;
write in $\sin$ and $\cos$; show every step
to the other side.

5 Quadratic trig equations

Use an identity so only one function
remains, factorise, then reject
impossible values (e.g. $|\sin x| > 1$).

6 Intervals and arguments

For $\tan(2\theta - 30^\circ)$ with $\theta > 0$, the
argument starts at -30° . "Second
smallest" needs all solutions in order.

7 Inverse trig functions

$$\arcsin: [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]; \arccos:$$

$$[-1, 1] \rightarrow [0, \pi]; \arctan: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

8 Triangles for inverses

If $\theta = \arccos x$, draw a right-angled
triangle with $\cos \theta = x$: then
 $\sin \theta = \sqrt{1 - x^2}$

9 Using a result

"Hence" means use the identity or
answer just found; do not start again.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	5	5	5	9	9	9	9	10
1 Reciprocal functions	•	•	•	•		•	•			•
2 Graphs			•							•
3 Pythagorean identities	•	•				•	•			
4 Proving identities				•		•			•	
5 Quadratic trig equations		•					•			
6 Intervals and arguments		•				•	•			•
7 Inverse trig functions					•			•	•	
8 Triangles for inverses					•				•	
9 Using a result				•		•	•		•	
Interleaved: functions: range, inverse, modulus (Chapter 2)			•					•		•
Interleaved: algebraic fractions (Chapter 1)				•		•				
Interleaved: quadratics and surds (P1)	•	•					•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Check degrees or radians before you start. Give every solution in the interval and no others; with a transformed argument, change the interval first.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that $\sec \theta = -3$ and that θ is obtuse, find the exact value of

(a) $\tan \theta$

(3)

(b) $\operatorname{cosec} \theta$, giving your answer in the form $a\sqrt{2}$, where a is a rational number.

(2)

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Question 1 continued

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2. Solve, for $0 < x < 2\pi$,

$$3 \operatorname{cosec}^2 x - 4 \cot x = 4$$

giving your answers to 3 significant figures.

(6)

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Question 2 continued

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3. (a) Sketch the graph of $y = \sec x$ for $0 \leq x \leq 2\pi$. Show the equations of the asymptotes and the coordinates of the turning points.

(3)

The function f is defined by $f(x) = 3 - 2\sec x$, $0 \leq x \leq 2\pi$, $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

- (b) Find the range of f .

(2)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Prove that $\sec x - \cos x \equiv \sin x \tan x$

(3)

(b) Hence, or otherwise, show that $(\operatorname{cosec} x - \sin x)(\sec x - \cos x) \equiv \sin x \cos x$

(2)

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Sketch the graph of $y = \arccos x$, showing the coordinates of its end points and of the point where it crosses the y -axis.

(2)

- (b) Solve $\arccos(2x - 1) = \frac{\pi}{3}$

(1)

- (c) Show that $\sin(\arccos x) = \sqrt{1 - x^2}$ for $-1 \leq x \leq 1$

(2)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Prove that $\frac{\tan^2 \theta}{\sec \theta - 1} \equiv \sec \theta + 1, \sec \theta \neq 1$

(3)

(b) Hence solve, for $0 < x < \pi$,

$$\frac{\tan^2 2x}{\sec 2x - 1} = 3$$

giving your answers as exact multiples of π .

(4)

(c) Explain why the equation $\frac{\tan^2 \theta}{\sec \theta - 1} = 2$ has no solutions.

(2)

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Question 6 continued

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7. (a) Show that the equation $3\sec^2 x - 5\tan x - 5 = 0$ can be written as $3\tan^2 x - 5\tan x - 2 = 0$ (2)

(b) Hence solve, for $0 \leq x < 360^\circ$, $3\sec^2 x - 5\tan x - 5 = 0$, giving your answers to one decimal place. (4)

(c) Hence find the second smallest positive solution of

$$3\sec^2(2\theta - 30^\circ) - 5\tan(2\theta - 30^\circ) - 5 = 0 \quad (3)$$

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 9 marks)



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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Sketch the graph of $y = \arctan x$, stating the equations of its asymptotes.

(2)

The function h is defined by

$$h(x) = \frac{\pi}{4} + \arctan(2x) \quad x \in \mathbb{R}$$

(b) State the range of h .

(2)

(c) Find $h^{-1}(x)$, stating its domain.

(3)

(d) Solve the equation $h(x) = \frac{\pi}{2}$

(2)



Question 8 continued

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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Given that $y = \arcsin x$, where $-1 \leq x \leq 1$, show that $\arccos x = \frac{\pi}{2} - y$ and hence that

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

(3)

- (b) Hence solve the equation $\arcsin x = 2 \arccos x$, giving your answer in exact form.

(3)

- (c) Find the exact value of $\tan\left(\arccos \frac{1}{3}\right)$

(3)

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Question 9 continued

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10. The function f is defined by

$$f(x) = 2 + \operatorname{cosec} x \quad 0 < x < 2\pi, x \neq \pi$$

- (a) Sketch the graph of $y = f(x)$ and state the range of f . (3)
- (b) Solve the equation $f(x) = 5$, giving your answers to 3 significant figures. (3)
- (c) Find the set of values of k for which the equation $f(x) = k$ has no solutions. (2)
- (d) Find the number of solutions of the equation $|f(x)| = 2$, giving a reason. (2)

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 72 MARKS

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CHAPTER 4 • KEY SKILLS

4

Trigonometric Addition Formulae

Specification P3: Trigonometry • 10 questions • 75 marks • about 90 minutes

1 Addition formulae

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

(all in the formula booklet)

2 Exact values

Split an angle into known ones:
 $75^\circ = 45^\circ + 30^\circ$. Use right-angled triangles when $\tan A$ is given.

3 Double angles

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$$
4 Proving identities

Choose the form of $\cos 2A$ that cancels. Work from one side to the other, showing each step.

5 Equations

Replace $\cos 2x$ so that only $\sin x$ (or only $\cos x$) remains, then factorise.

6 R forms

$a \sin x + b \cos x \equiv R \sin(x + \alpha)$ with
 $R = \sqrt{a^2 + b^2}$, $\tan \alpha = \frac{b}{a}$: expand and compare coefficients.

7 Maxima and minima

$R \cos(x + \alpha)$ lies between $-R$ and R .
 For $\frac{k}{c + R \cos(\dots)}$, the maximum uses the *minimum* of the denominator.

8 Solving with R forms

Solve $R \cos(x + \alpha) = c$: adjust the interval for $x + \alpha$ first, then subtract α .

9 Modelling

Interpret R as the amplitude; convert the angle back to time and answer in context (e.g. a clock time).

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	10	10	10	10	10
1 Addition formulae	•				•		•			
2 Exact values	•			•			•		•	•
3 Double angles		•		•			•		•	
4 Proving identities				•			•		•	
5 Equations		•			•	•	•		•	•
6 R forms			•			•		•		•
7 Maxima and minima			•			•		•		
8 Solving with R forms						•		•		•
9 Modelling								•		
Interleaved: sec, cosec, cot and their identities (Chapter 3)									•	
Interleaved: range and number of solutions (Chapter 2)						•				•
Interleaved: quadratics, cubics and surds (P1, P2)		•			•		•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Give R exactly (a surd if needed) and α to the accuracy asked. Check the sign in $\tan \alpha$ by expanding your form, and use “hence” results rather than starting again.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

(2)

The angles A and B are acute, with $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$

(b) Find the exact value of $\sin(A - B)$.

(3)

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2. Solve, for $0 \leq x < 360^\circ$,

$$2 \cos 2x + 5 \sin x = 3$$

giving your answers to one decimal place where appropriate.

(5)

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Question 2 continued

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3. (a) Express $5\sin x - 12\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. Give the value of α to 2 decimal places.

(3)

- (b) Write down the maximum value of $5\sin x - 12\cos x$ and find the value of x , for $0 \leq x < 360^\circ$, at which it occurs.

(2)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$

(3)

(b) Hence find the exact value of $\tan 15^\circ$

(2)

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5. Solve, for $0 \leq x < \pi$,

$$\tan\left(x - \frac{\pi}{4}\right) = 6 \tan x$$

giving your answers to 3 significant figures.

(5)

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Question 5 continued

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$$f(x) = 2 \cos x - 3 \sin x$$

(3)

$$g(x) = \frac{12}{10 + f(x)}$$

(2)

(3)

Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Using $\cos 3x = \cos(2x + x)$, prove that $\cos 3x \equiv 4\cos^3 x - 3\cos x$ (4)

(b) Hence solve, for $0 \leq x \leq \pi$, $\cos 3x + 2\cos x = 0$, giving your answers as exact multiples of π . (4)

(c) Given that $\cos \alpha = \frac{1}{3}$, find the exact value of $\cos 3\alpha$. (2)

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Question 7 continued

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- (3)**

$$H = 10 + 3 \sin\left(\frac{\pi t}{6}\right) - 4 \cos\left(\frac{\pi t}{6}\right) \quad 0 \leq t \leq 12$$

(3)

- (4)

Question 8 continued

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Question 8 continued

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Solutions relying on calculator technology are not acceptable.

- (4)

- $$\operatorname{cosec} 4\theta + \cot 4\theta = \sqrt{3}$$

(3)

- (3)**

Question 9 continued

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Question 9 continued

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Express $\sqrt{3}\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact values of R and α . (3)
- (b) Hence solve, for $0 \leq x \leq \pi$, $\sqrt{3}\cos 2x + \sin 2x = -1$, giving your answers as exact multiples of π . (4)
- (c) Find the set of values of k for which the equation $\sqrt{3}\cos x + \sin x = k$ has exactly two solutions for $0 \leq x < 2\pi$. Give a reason for your answer. (3)

Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)**TOTAL FOR CHAPTER 4 IS 75 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

CHAPTER 5 • KEY SKILLS

5

Exponentials and Logarithms

Specification P3: Exponentials and logarithms • 10 questions • 78 marks • about 95 minutes

1 Graphs

$y = e^x$: asymptote $y = 0$, through $(0, 1)$.
 $y = \ln x$: asymptote $x = 0$, through $(1, 0)$. Transform asymptotes too.

2 Inverse functions

$\ln(e^x) = x$ and $e^{\ln x} = x$. The domain of $\ln$ is $x > 0$.

3 Laws of logarithms

$\ln a + \ln b = \ln ab$, $\ln a - \ln b = \ln \frac{a}{b}$,
 $k \ln a = \ln a^k$

4 Exact solutions

Take $\ln$ of both sides:
 $3e^{2x-1} = 12 \Rightarrow 2x - 1 = \ln 4$. Leave answers in terms of $\ln$ or e .

5 Hidden quadratics

Multiply through by e^x and let $u = e^x$.
 Reject $u \leq 0$, since $e^x > 0$

6 Exponential models

Initial value: $t = 0$. Use a known point to find k exactly, e.g. $k = \frac{1}{5} \ln 2$

7 Rates of change

$\frac{d}{dt}(Ae^{kt}) = kAe^{kt}$. A negative rate means decreasing: say so in context.

8 Long-term behaviour

As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$ (for $k > 0$): the model tends to a limit. State limitations in context.

9 Logarithmic graphs

$y = ax^n$: $\log y = \log a + n \log x$. $y = kb^x$: $\log y = \log k + x \log b$. Match the base used on the axis.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	6	10	10	10	10	12
1 Graphs			•						•	
2 Inverse functions									•	
3 Laws of logarithms	•			•	•			•	•	•
4 Exact solutions	•	•			•	•	•		•	•
5 Hidden quadratics		•								•
6 Exponential models						•	•	•		•
7 Rates of change						•				•
8 Long-term behaviour			•				•	•		
9 Logarithmic graphs				•	•			•		•
Interleaved: functions, inverse and composite (Chapter 2)			•						•	
Interleaved: algebraic fractions (Chapter 1)	•						•			
Interleaved: quadratics, straight lines and logarithms (P1, P2)		•		•	•			•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Give exact answers when asked (in terms of $\ln$ or e). Find rates by differentiating, not by averaging, and finish log-graph questions by writing the equation in the form asked.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Find the exact solutions of

(a) $3e^{2x-1} = 12$, giving your answer in the form $a + \ln b$, where a and b are rational numbers, (2)

(b) $\ln(2x+3) - \ln x = 1$, $x > 0$ (3)

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Question 1 continued

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$2e^x + 3e^{-x} = 7$$

giving your answers as exact values.

(5)

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Question 2 continued

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 3 - 2e^{-x} \quad x \in \mathbb{R}$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of the points where it crosses the coordinate axes and the equation of its asymptote.

(3)

(b) Find the range of f when the domain is restricted to $x \geq 0$

(2)

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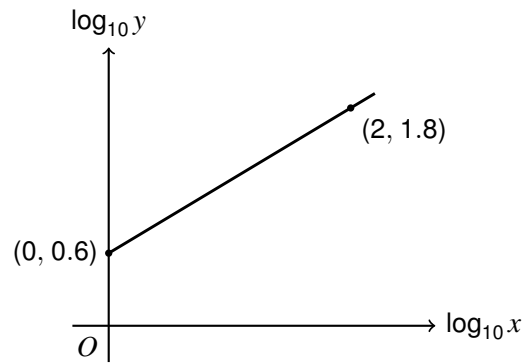
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4.

**Figure 1**

The straight line in Figure 1 shows the relationship between $\log_{10} y$ and $\log_{10} x$. The line passes through the points $(0, 0.6)$ and $(2, 1.8)$.

(a) Write down an equation for the line.

(2)

(b) Hence express y in the form ax^n , giving the value of a to 3 significant figures.

(3)

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Question 4 continued

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) The variables x and y satisfy $y = \frac{10^4}{x^2}$, $x > 0$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$, showing the coordinates of the points where it meets the coordinate axes.

(3)

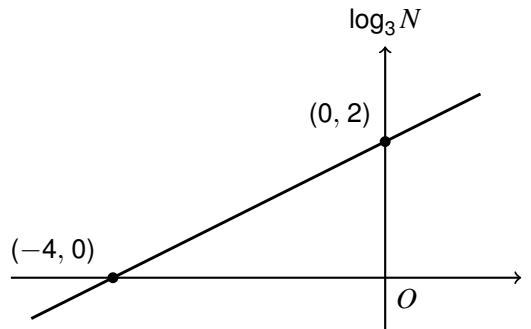


Figure 2

Figure 2 shows the linear relationship between $\log_3 N$ and t . The line passes through the points $(-4, 0)$ and $(0, 2)$.

- (b) Show that $N = ab^t$, where a and b are exact constants to be found.

(3)



Question 5 continued

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- $$\theta = 20 + Ae^{-kt}$$

(a) Find the value of A .

(1)

- (b)** Show that $k = \frac{1}{5} \ln 2$

(3)

- (c)** Find, to the nearest tenth of a minute, the time at which the temperature of the coffee is 30°C .

(3)

- (d)** Find the rate at which the temperature is decreasing when $t = 5$, giving your answer in $^{\circ}\text{C}$ per minute to 3 significant figures.

(3)

Question 6 continued

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Question 6 continued

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7. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**The number of birds, P , on an island t years after the start of a study is modelled by

$$P = \frac{500e^{kt}}{4 + e^{kt}}$$

where k is a positive constant.

- (a) Find the number of birds at the start of the study.

(1)

Given that there are 250 birds on the island 10 years after the start of the study,

- (b) show that $k = \frac{1}{5} \ln 2$

(3)

- (c) State the number of birds that the model predicts in the long term, giving a reason.

(2)

- (d) Find, according to the model, the exact number of years after the start of the study at which there are 400 birds.

(3)

- (e) Give one reason why the model may not be appropriate in the long term.

(1)

Question 7 continued

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Question 7 continued

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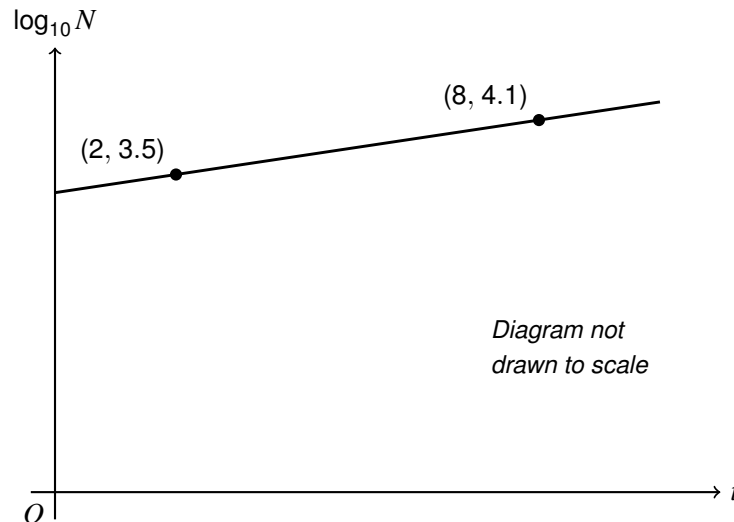
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8.

**Figure 3**

The number of subscribers, N , to an online service t years after it was launched is modelled by

$$N = ab^t$$

where a and b are constants. The line in Figure 3 shows the linear relationship between $\log_{10} N$ and t . The line passes through the points $(2, 3.5)$ and $(8, 4.1)$.

- (a) Find an equation for the line. (2)
- (b) Find the value of a and the value of b , giving the value of b to 3 significant figures. (3)
- (c) Give a practical interpretation of the constant b in this model. (1)
- (d) Find the number of subscribers predicted by the model 10 years after launch, to the nearest thousand. (1)
- (e) Use the model to find the value of t at which the number of subscribers first exceeds 50 000 (2)
- (f) State one reason why the model may not be reliable for large values of t . (1)



Question 8 continued

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Solutions relying on calculator technology are not acceptable.

$$f(x) = \ln(2x - 4) + 1 \quad x > 2 \qquad g(x) = e^{2x} \quad x \in \mathbb{R}$$

- (a) Sketch the graph of $y = f(x)$, showing the equation of its asymptote and the exact coordinates of the point where it crosses the x -axis. (3)
- (b) Find $f^{-1}(x)$ and state its domain and range. (3)
- (c) Solve the equation $\lg(x) = 1 + \ln 4$, giving your answer in exact form. (4)

Question 9 continued

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Question 9 continued

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10. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

The numbers of two species of insect, A and B , in a garden t days after the start of a study are modelled by

$$A = 40e^{0.1t} \qquad B = 200 - 80e^{-0.1t} \qquad t \geq 0$$

- (a) Find the numbers of each species at the start of the study. (1)
- (b) Show that, when $A = B$, the value $u = e^{0.1t}$ satisfies $u^2 - 5u + 2 = 0$ (3)
- (c) Hence find the exact value of t when $A = B$, giving your answer in the form $10 \ln \left(\frac{p + \sqrt{q}}{2} \right)$.
Explain why only one value of t is possible. (4)
- (d) Find the rate at which the number of species A is increasing at this time, to 3 significant figures. (2)
- (e) Show that the graph of $\ln A$ against t is a straight line, stating its gradient and the exact value of its intercept on the $\ln A$ axis. (2)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 12 marks)**TOTAL FOR CHAPTER 5 IS 78 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

6

CHAPTER 6 • KEY SKILLS

Differentiation

Specification P3: Differentiation • 10 questions • 76 marks • about 90 minutes

1 Standard derivatives

$e^{kx} \rightarrow ke^{kx}$, $\ln x \rightarrow \frac{1}{x}$,
 $\sin kx \rightarrow k \cos kx$, $\cos kx \rightarrow -k \sin kx$,
 $\tan kx \rightarrow k \sec^2 kx$

2 The chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, e.g. $\ln(2x+1) \rightarrow \frac{2}{2x+1}$

3 The product rule

$y = uv$: $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$. Factorise the answer (e.g. take out e^{kx}).

4 The quotient rule

$y = \frac{u}{v}$: $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (the order matters)

5 x as a function of y

$\frac{dy}{dx} = \frac{1}{dx/dy}$. Use an identity to write the answer in terms of x .

6 Tangents and normals

Gradient of the normal = $-\frac{1}{m}$. Keep exact values, e.g. $y = 4e^{-2}(2-x)$

7 Stationary points

Solve $\frac{dy}{dx} = 0$; $e^{kx} \neq 0$, so only the other factors give solutions. Give exact coordinates.

8 Increasing and decreasing

Find the sign of $\frac{dy}{dx}$ and finish with a statement: “so g is increasing”.

9 Using the derivative

Rearrange $f'(x) = 0$ into a given form, or use the maximum to compare values.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	10	10	10	10	11
1 Standard derivatives	•		•	•	•	•	•	•	•	•
2 The chain rule	•		•	•	•	•	•	•		•
3 The product rule	•			•		•	•			•
4 The quotient rule	•	•			•			•	•	
5 x as a function of y			•							•
6 Tangents and normals		•				•	•	•		•
7 Stationary points				•		•		•	•	•
8 Increasing and decreasing					•				•	
9 Using the derivative						•	•		•	
Interleaved: transformations and number of solutions (Chapter 2)						•				
Interleaved: trigonometric identities (Chapters 3, 4)			•					•		
Interleaved: e and $\ln$, exact values (Chapter 5)				•	•	•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. “Use calculus” means differentiate and set equal to zero. In “show that” questions, write the unsimplified product or quotient rule line first, then simplify.



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1. Differentiate with respect to x

(a) $y = e^{3x} \ln(2x), x > 0$

(2)

(b) $y = \frac{2x+1}{\cos x}, -\frac{\pi}{2} < x < \frac{\pi}{2}$, simplifying your answer.

(3)

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(Total for Question 1 is 5 marks)



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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = \frac{3x-1}{x+2}$, $x \neq -2$

(a) Show that $\frac{dy}{dx} = \frac{a}{(x+2)^2}$, where a is an integer to be found.

(2)

(b) Find an equation of the tangent to C at the point where $x = -1$

(3)

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3. The curve C has equation $x = 2\tan 3y$, $-\frac{\pi}{6} < y < \frac{\pi}{6}$

(a) Find $\frac{dy}{dx}$ in terms of y .

(2)

(b) Hence show that $\frac{dy}{dx} = \frac{2}{3(x^2 + 4)}$

(3)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = xe^{-2x}$

(a) Use calculus to find the exact coordinates of the stationary point of C .

(4)

(b) Hence write down the maximum value of xe^{-2x}

(1)

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Question 4 continued

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5.

$$g(x) = \ln(2x+1) + \frac{4}{2x+1} \quad x > -\frac{1}{2}$$

Find the set of values of x for which g is increasing.

(5)

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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

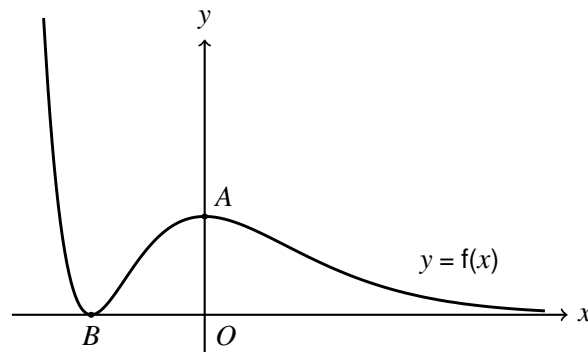


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x+1)^2 e^{-2x} \quad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at B .

- (a) Show that $f'(x) = -2x(x+1)e^{-2x}$ (3)
- (b) Use calculus to find the coordinates of A and B . (2)
- (c) State the y coordinate of the minimum point on the curve with equation $y = 3f(x) - 2$ (1)
- (d) Find the set of values of k for which the equation $f(x) = k$ has exactly three solutions. (2)
- (e) Find, in terms of e , an equation of the tangent to the curve at the point where $x = 1$ (2)



Question 6 continued

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$$f(x) = 2x \cos\left(\frac{x}{2}\right) \quad 0 < x < 2\pi$$

- (b)** Show that the equation $f'(x) = 0$ can be written as $x = 2 \cot\left(\frac{x}{2}\right)$

(c) Find an equation of the normal at P . **(3)**

- (d)** Find the exact area of the triangle OPQ , where O is the origin. **(3)**

Question 7 continued

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Question 7 continued

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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{1 + \sin 2x}{\cos 2x} \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

- (a) Show that $\frac{dy}{dx} = \frac{2}{1 - \sin 2x}$ (5)
- (b) Find an equation of the tangent to C at the point where $x = \frac{\pi}{12}$, giving your answer in the form $y = mx + c$, where m and c are exact. (3)
- (c) Explain why C has no stationary points. (2)



Question 8 continued

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Question 8 continued

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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{\ln x}{x} \quad x > 0$$

- (a) Find $f'(x)$. (2)
- (b) Find the exact coordinates of the stationary point of $y = f(x)$ and show that it is a maximum. (3)
- (c) Hence explain why $\frac{\ln \pi}{\pi} < \frac{1}{e}$ (2)
- (d) Hence show that $\pi^e < e^\pi$ (3)

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)



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Solutions relying on calculator technology are not acceptable.

(a) Find $\frac{dy}{dx}$ in terms of y .

(3)

(b) Find the exact x coordinate of P and an equation of the tangent to C at P .

(4)

(c) The tangent at P crosses the x -axis at the point Q . Find the exact x coordinate of Q .

(2)

(d) Find the exact coordinates of the point on C at which the tangent is parallel to the y -axis.

(2)

Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 6 IS 76 MARKS

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7

CHAPTER 7 • KEY SKILLS

Integration

Specification P3: Integration • 10 questions • 77 marks • about 90 minutes

1 Standard integrals

$$\int e^x dx = e^x + c, \quad \int \frac{1}{x} dx = \ln|x| + c,$$

$$\int \cos x dx = \sin x + c,$$

$$\int \sec^2 x dx = \tan x + c$$

2 Functions of $(ax + b)$ Integrate as usual, then divide by a :

$$\int \frac{1}{2x-3} dx = \frac{1}{2} \ln|2x-3| + c$$

3 Reverse chain rule

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$
 check that the numerator is (a multiple of) the derivative of the denominator.

4 Trig identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x),$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x),$$

$$\tan^2 x = \sec^2 x - 1, \quad \sin x \cos x = \frac{1}{2} \sin 2x$$

5 Improper fractions

Divide first (Chapter 1), then integrate

$$\text{each term: } \frac{C}{x+a} \text{ gives } C \ln|x+a|$$

6 Exact definite integrals

Substitute both limits; use the laws of logarithms to write one ln, e.g.

$$2 \ln 5 - 2 \ln 3 = 2 \ln \frac{5}{3}$$

7 Areas

$$\text{Area} = \int_a^b y dx;$$
 below the x -axis the integral is negative, so take its size.

8 Unknown limits

Evaluate in terms of the unknown and solve; reject values outside the domain.

9 Using “hence”

A derivative found earlier gives an integral: if $\frac{d}{dx} F(x) = f(x)$ then

$$\int f(x) dx = F(x) + c$$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	6	10	10	10	10	11
1 Standard integrals	•	•	•				•	•	•	•
2 Functions of $(ax + b)$	•	•			•	•	•		•	•
3 Reverse chain rule				•				•		
4 Trig identities			•				•			
5 Improper fractions					•	•				
6 Exact definite integrals		•		•	•	•	•	•	•	•
7 Areas						•		•		•
8 Unknown limits									•	
9 Using “hence”							•	•		
Interleaved: algebraic division (Chapter 1)					•	•				
Interleaved: addition formulae and identities (Chapters 3, 4)			•				•	•		
Interleaved: e, ln and differentiation (Chapters 5, 6)						•		•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Include “+c” for indefinite integrals. When integrating $\frac{1}{ax+b}$ do not lose the factor $\frac{1}{a}$. Give exact answers as a single logarithm where asked.



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1. Find

(a) $\int \left(3e^{2x} - \frac{4}{x} \right) dx$ (2)

(b) $\int \frac{5}{2x-3} dx$ (2)

(c) $\int \sec^2(4x) dx$ (1)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Find the exact value of

(a) $\int_1^e \left(2x + \frac{3}{x}\right) dx$ (3)

(b) $\int_0^{\ln 2} e^{2x} dx$ (2)

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Question 2 continued

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3. Using appropriate trigonometric identities, find

(a) $\int (\sin x + \cos x)^2 dx$

(3)

(b) $\int \tan^2 x dx$

(2)

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Question 3 continued

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Find $\int \frac{6x}{3x^2 + 1} dx$

(2)

(b) Find the exact value of $\int_0^{\frac{\pi}{3}} \frac{\sin x}{2 + \cos x} dx$, giving your answer as a single logarithm.

(3)

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Question 4 continued

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Write $\frac{2x^2+5x-1}{x+3}$ in the form $Ax+B+\frac{C}{x+3}$, where A , B and C are constants.

(2)

(b) Hence find the exact value of $\int_0^2 \frac{2x^2+5x-1}{x+3} dx$, giving your answer in the form $p+q\ln r$, where p , q and r are rational.

(4)

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Question 5 continued

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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

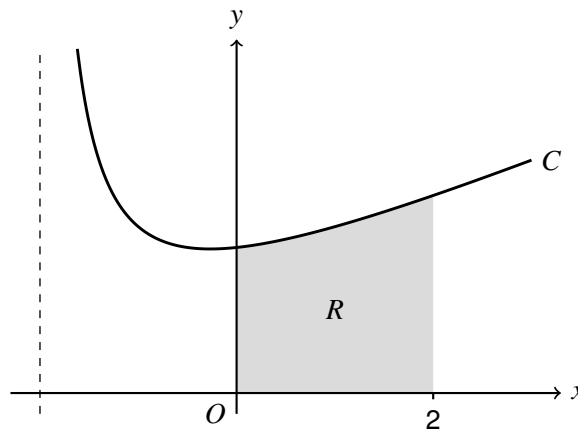


Figure 1

Figure 1 shows part of the curve C with equation

$$y = \frac{x^2 + 4x + 7}{x + 2} \quad x > -2$$

The region R , shown shaded, is bounded by C , the coordinate axes and the line $x = 2$

- (a) Show that $y = x + 2 + \frac{3}{x + 2}$ (2)
- (b) Find the exact area of R , giving your answer in the form $a + b \ln 2$ (4)
- (c) Use calculus to find the exact coordinates of the minimum point of C . (4)



Question 6 continued

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Question 6 continued

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Solutions relying on calculator technology are not acceptable.

- (4)**

- (4)

- (2)**

Question 7 continued

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Question 7 continued

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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that $\frac{d}{dx}(\ln(\sec x + \tan x)) = \sec x$, $0 < x < \frac{\pi}{2}$ (4)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{4}} \sec x dx$ (2)

(c) Show that $\int_0^{\frac{\pi}{3}} \tan x dx = \ln 2$ (2)

(d) Find the exact area of the region bounded by the curves $y = \sec x$ and $y = \tan x$, the y -axis and the line $x = \frac{\pi}{4}$, giving your answer as a single logarithm. (2)



Question 8 continued

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Question 8 continued

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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Given that $\int_0^k \frac{4}{2x+1} dx = \ln 25$, where $k > 0$, find the value of k .

(3)

(b) Given that $\int_0^{\ln p} (e^{2x} + e^x) dx = 10$, where $p > 1$, show that $p^2 + 2p - 23 = 0$ and hence find the exact value of p .

(5)

(c) Show that $\int_2^3 \frac{3}{x-1} dx = \ln 8$

(2)

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Question 9 continued

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

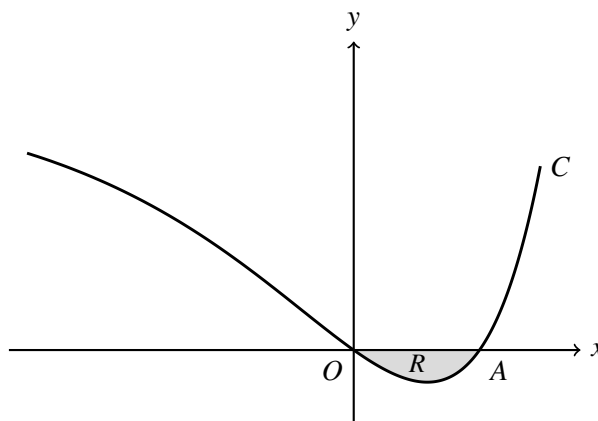


Figure 2

Figure 2 shows part of the curve C with equation

$$y = e^{2x} - 3e^x + 2 \quad x \in \mathbb{R}$$

The curve passes through the origin O and crosses the x -axis again at the point A . The region R , shown shaded, is bounded by C and the x -axis.

- (a) Show that the x coordinate of A is $\ln 2$ (3)
- (b) Write down the equation of the asymptote to C as $x \rightarrow -\infty$ (1)
- (c) Find the exact area of R , giving your answer in the form $p - \ln q$, where p and q are rational. (5)
- (d) Find an equation of the tangent to C at A . (2)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 7 IS 77 MARKS

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CHAPTER 8 • KEY SKILLS

8

Numerical Methods

Specification P3: Numerical methods • 10 questions • 77 marks • about 90 minutes

1 Locating roots

If f is continuous on $[a, b]$ and $f(a)$, $f(b)$ have different signs, there is a root between a and b . Write all three parts.

2 When it fails

A sign change across an asymptote (e.g. $\frac{1}{x-2}$ or $\tan x$ at $\frac{\pi}{2}$) does not show a root: f is not continuous there.

3 Rearranging

Show $f(x) = 0$ can be written as $x = g(x)$ step by step, e.g.
 $x^3 - 3x - 5 = 0 \Rightarrow x = \sqrt[3]{3x+5}$

4 Iteration

$x_{n+1} = g(x_n)$: use the calculator's ANS key; give each iterate to the accuracy asked.

5 Root to a given accuracy

For $\alpha = 1.749$ (3 d.p.) use the interval $[1.7485, 1.7495]$: find the sign at both ends and conclude.

6 Convergence

Iteration converges when $|g'(x)| < 1$ near the root. A different rearrangement may be needed for another root.

7 Roots from calculus

A stationary point satisfies $f'(x) = 0$; a tangent through the origin satisfies $f(\alpha) = \alpha f'(\alpha)$

8 Intersections

Curves meet where $f(x) - g(x) = 0$: locate the roots of this new function.

9 Context

Answer with units and the accuracy asked, e.g. a time in hours to 3 significant figures.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	11	10	10	10	11
1 Locating roots	•	•			•	•			•	•
2 When it fails			•						•	
3 Rearranging	•			•		•	•	•	•	•
4 Iteration	•			•	•	•	•	•	•	•
5 Root to a given accuracy		•				•	•		•	
6 Convergence				•						•
7 Roots from calculus						•	•	•		
8 Intersections					•					•
9 Context								•		
Interleaved: differentiation (Chapter 6)						•	•	•		
Interleaved: trigonometric functions (Chapters 3, 4)							•		•	
Interleaved: e, ln and inverse functions (Chapters 2, 5)		•			•	•		•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. For a sign change, give both values (or their signs), state that f is continuous on the interval, and conclude "so there is a root". Work in radians for trigonometric functions.



1.

$$f(x) = x^3 - 3x - 5$$

(a) Show that the equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$ (2)

(b) Show that $f(x) = 0$ can be written as $x = \sqrt[3]{3x+5}$ (1)

(c) Using $x_{n+1} = \sqrt[3]{3x_n+5}$ with $x_1 = 2$, find the values of x_2 , x_3 and x_4 , giving your answers to 4 decimal places. (2)

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(Total for Question 1 is 5 marks)



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2.

$$f(x) = e^x - x - 4$$

- (a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$

(2)

- (b) By choosing a suitable interval, show that $\alpha = 1.749$ correct to 3 decimal places.

(3)

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Question 2 continued

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3.

$$f(x) = \frac{1}{x-2} + 3 \quad x \neq 2$$

(a) Show that $f(1.9)$ and $f(2.1)$ have opposite signs. (1)

(b) Explain why the equation $f(x) = 0$ does not have a root between 1.9 and 2.1 (2)

(c) Find the exact root of $f(x) = 0$ (2)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. The equation $x^2 - 5x + 3 = 0$ has roots α and β , where $0 < \alpha < 1$ and $4 < \beta < 5$. The iteration formula

$$x_{n+1} = \frac{x_n^2 + 3}{5}$$

is used to try to find the roots.

- (a) Using $x_1 = 1$, find x_2 , x_3 and x_4 (2)
- (b) Using $x_1 = 4.5$, find x_2 and x_3 to 4 decimal places, and describe what happens. (2)
- (c) Suggest a rearrangement of the equation that could be used to find β by iteration. (1)



Question 4 continued

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5. The curves with equations $y = \ln x$ and $y = 3 - x$ meet at the point with x coordinate α .

(a) Show that α lies between 2 and 3

(3)

(b) Using $x_{n+1} = 3 - \ln x_n$ with $x_1 = 2$, find x_2 and x_3 to 4 decimal places.

(2)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. The curve C has equation $y = e^x + x^2$

The tangent to C at the point P with x coordinate α passes through the origin.

(a) Find $\frac{dy}{dx}$

(1)

(b) Show that $x = \alpha$ is a solution of the equation $e^x(1 - x) - x^2 = 0$

(3)

(c) Show that α lies between 0.5 and 1

(2)

(d) Show that the equation in part (b) can be written as $x = 1 - x^2e^{-x}$

(1)

(e) Using $x_{n+1} = 1 - x_n^2e^{-x_n}$ with $x_1 = 0.7$, find x_2 and x_3 to 4 decimal places.

(2)

(f) Show that $\alpha = 0.739$ correct to 3 decimal places.

(2)

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Question 6 continued

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Question 6 continued

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$$f(x) = x^2 \cos x \quad 0 < x < \frac{\pi}{2}$$

(a) Find $f'(x)$.

(2)

(b) Show that β satisfies the equation $x = \arctan\left(\frac{2}{x}\right)$

(2)

(c) Using $x_{n+1} = \arctan\left(\frac{2}{x_n}\right)$ with $x_1 = 1$, find x_2 , x_3 and x_4 to 4 decimal places.

(3)

(d) By considering the sign of $f'(x)$ on a suitable interval, show that $\beta = 1.077$ correct to 3 decimal places.

(3)

Question 7 continued

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Question 7 continued

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- (d)** Hence find, to the nearest minute, the length of time for which the drug is effective. **(3)**

Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



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9.

$$f(x) = \tan x - 3x \quad 0 < x < \pi, x \neq \frac{\pi}{2}$$

- (a) Show that the equation $f(x) = 0$ has a root between 1.2 and 1.4 (2)
- (b) Show that $f(1.5)$ and $f(1.6)$ have opposite signs, and explain why this does not show that there is a root between 1.5 and 1.6 (2)
- (c) Using $x_{n+1} = \arctan(3x_n)$ with $x_1 = 1.3$, find x_2 and x_3 to 4 decimal places. (3)
- (d) Show that the root in part (a) is 1.324 correct to 3 decimal places. (3)



Question 9 continued

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Question 9 continued

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$$f(x) = e^x - 2 \quad x \in \mathbb{R}$$

(3)

(3)

(2)

(3)

Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 8 IS 77 MARKS

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