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A-LEVEL MATHEMATICS • DECISION 1

Route Inspection

Worksheet

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Free Route Inspection worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 11

Tap a section to jump straight to it

By the end of this worksheet you can: recognise Eulerian and semi-Eulerian graphs • pair odd vertices and find route lengths • *Stretch:* choose start and finish points, and handle new roads and unknown lengths.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Inspecting every arc of a network by the shortest route: odd vertices, pairings, and where to start and finish.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: Eulerian or not

A graph has degrees 2, 4, 4, 2, 3, 3. Is it Eulerian, semi-Eulerian or neither?

Exactly two odd vertices

semi-Eulerian

Your turn A1

A graph has degrees 2, 4, 6, 2, 4. Is it Eulerian, semi-Eulerian or neither?

Worked example A2: two odd vertices

A network has total weight 50 and odd vertices B and E, with shortest path B to E of 9. Find the closed inspection route length.

Repeat the shortest path B to E

59

Your turn A2

A network has total weight 64 and odd vertices C and F, with shortest path C to F of 11. Find the closed inspection route length.

Worked example A3: pairings

Odd vertices A, B, C, D have shortest distances AB 5, AC 8, AD 7, BC 6, BD 9, CD 4. Find the least pairing.

$AB + CD = 9$, $AC + BD = 17$, $AD + BC = 13$

$AB + CD = 9$

Your turn A3

Odd vertices P, Q, R, S have shortest distances PQ 7, PR 3, PS 9, QR 8, QS 5, RS 6. Find the least pairing.

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STRETCH

Worked example A4: the route length

The network in A3 (example) has total weight 120. Find the closed route length.

$120 + 9 = 129$

Your turn A4

The network in A3 (your turn) has total weight 95. Find the closed route length.

Worked example A5: any start and finish

For the network in A3 (example), where should a route start and finish to be as short as possible?

Repeat only the shortest single path, CD 4; start and finish at the other two

A and B, 124

Your turn A5

For the network in A3 (your turn), where should a route start and finish to be as short as possible?

Worked example A6: a given start

For the network in A3 (example), the route must start at A. Where should it finish?

Finish B: repeat CD 4; C: BD 9; D: BC 6

finish at B

Your turn A6

For the network in A3 (your turn), the route must start at Q. Where should it finish?

Worked example A7: passes through a vertex

In a closed route of an Eulerian network, how many times is a vertex of degree 6 passed through (not counting the start)?

3

Your turn A7

In a closed route of an Eulerian network, how many times is a vertex of degree 4 passed through (not counting the start)?

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A network has arcs AB 8, AC 5, BC 4, BD 12, CD 7, CE 6, DE 3.

(a) State the degree of each vertex.

(b) State the odd vertices.

(c) State whether it is Eulerian, semi-Eulerian or neither.

(d) Find its total weight.

[4 marks]

B2 For the network in B1, a closed inspection route is needed.

(a) Find the shortest path between the odd vertices.

(b) State the arcs repeated.

(c) Find the route length.

(d) Explain why BD is not repeated.

[4 marks]

B3 A network has arcs PQ 6, PR 9, PS 4, QR 5, QT 8, RS 7, RU 3, ST 10, TU 2.

(a) State the odd vertices.

(b) Find the shortest distances between them.

(c) List the three pairings with totals.

(d) Find the closed route length.

[4 marks]

B4 A network has arcs AB 5, BC 7, CD 4, DA 6, AC 9.

(a) State the odd vertices.

(b) State where a route using each arc once must start and finish.

(c) Find its length.

(d) Find the closed route length.

[4 marks]

B5 For the network in B3, the route may start and finish anywhere.

(a) List the six choices of start and finish with the distance repeated.

(b) State the best start and finish.

(c) Find the route length.

[3 marks]

B6 For the network in B3, the route must start at P.

(a) List the possible finishes.

(b) Give the repeated distance for each.

(c) State the best finish.

(d) Find the route length.

[4 marks]

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STRETCH

B7 A network has arcs AB 4, BC 6, CA 5, CD 7, DE 3, EC 8.

(a) Show that it is Eulerian.

(b) Write down a closed route from A.

(c) State how many times it passes through C

(d) State the route length.

[4 marks]

B8 A new road BD of length 10 is added to the network in B1.

(a) State the new odd vertices.

(b) State whether the network is now Eulerian.

(c) Find the new closed route length.

(d) Explain why it is shorter, despite the extra road.

[4 marks]

B9 In the network in B4, the arc AC has length x

(a) State the total weight in terms of x

(b) Find the shortest path from A to C not using AC

(c) Find the values of x for which AC is repeated.

(d) Find the closed route length when $x = 7$

[4 marks]

B10 For the network in B3, Dijkstra's algorithm from P gives final values Q 6, R 9, S 4, T 14, U 12

(a) State three of the pairing distances needed.

(b) Find the other three directly.

(c) Explain why Dijkstra from P alone is not enough.

(d) Confirm the least pairing.

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 For the network in B3, Amir uses the first pairing he finds, PQ + ST. Explain his error. **[2 marks]**

C2 For the network in B1, Bea gives the closed route length as 11. Explain her error. **[2 marks]**

C3 For the network in B1, Cal repeats the arc BD because B and D are the odd vertices. Explain his error and give the correct route length. **[3 marks]**

C4 For the network in B3, Dev says the shortest open route should start at R because R has the highest degree. Explain his error. **[3 marks]**

STRETCH

C5 For B6, Eve writes “repeat QT, length 62” as her full answer. Explain what is missing. **[3 marks]**

C6 For the closed route in B7, Fay says the route passes through C four times because C has degree 4. Explain her error. **[3 marks]**

C7 Gus says a network can have exactly three odd vertices. Explain why this is impossible. **[3 marks]**

C8 **(a)** Explain why a closed route that uses every arc exactly once is only possible if every vertex has even degree.

(3)

(b) Explain why the shortest inspection route has length equal to the total weight plus the least pairing total.

(2)

[5 marks]

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D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 Figure 1 shows a network of footpaths in a country park. The number on each arc is the length of the path in metres. A ranger must walk along every path at least once, starting and finishing at the visitor centre at A.

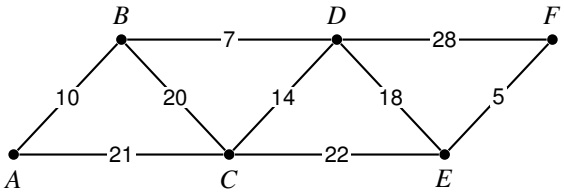


Figure 1

[The total weight of the network is 145]

Determine the length of the ranger's shortest route, stating which paths must be walked twice. You must make your method clear. **[3 marks]**

D2 Figure 2 shows a network of roads, with lengths in km. Every road must be checked, starting and finishing at the same point.

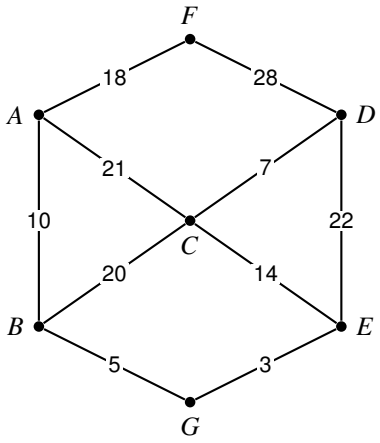


Figure 2

[The total weight of the network is 148]

(a) By considering the pairings of all relevant vertices, find the roads that must be travelled twice. You must show the total for every pairing.

(3)

(b) State the length of the shortest route.

(1)

[4 marks]

D3 Figure 3 shows a network of corridors in a museum, with lengths in metres. A guard must walk every corridor, starting and finishing at A.

(a) Explain why the guard need not walk any corridor twice, and state the length of the route.

(1)

(b) Write down a possible route.

(1)

(c) State how many times the route passes through E

(1)

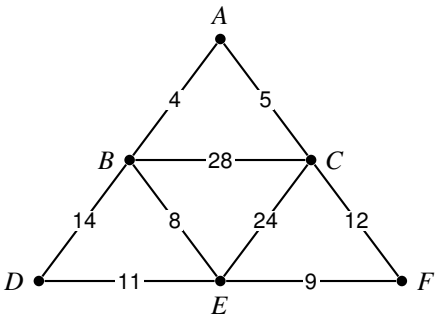


Figure 3

[The total weight of the network is 115]

[3 marks]

D4 Figure 4 shows a network of streets, with lengths in metres. A street sweeper must clean every street, and can start and finish anywhere.

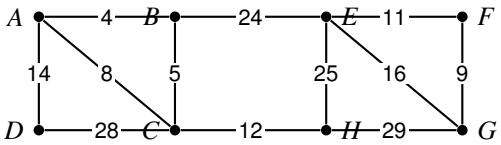


Figure 4

[The total weight of the network is 185]

Determine where the sweeper should start and finish to minimise the length of the route, and find this length. You must give a reason for your choice. **[3 marks]**

STRETCH

D5 Figure 5 shows a network of cycle paths, with lengths in km. An inspector must ride along every path.

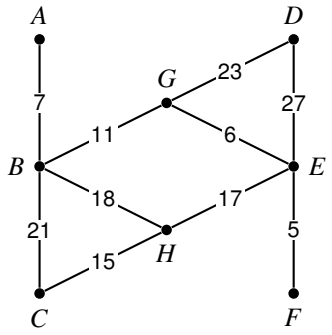


Figure 5

[The total weight of the network is 150]

(a) The route must start at A and finish at F. Determine its length, stating which paths are repeated.

(2)

(b) The inspector now starts at G and finishes at H. Determine the change in length, explaining your answer.

(2)

[4 marks]

D6 Figure 6 shows a network of pipes, with lengths in metres, that must be inspected in a closed route.

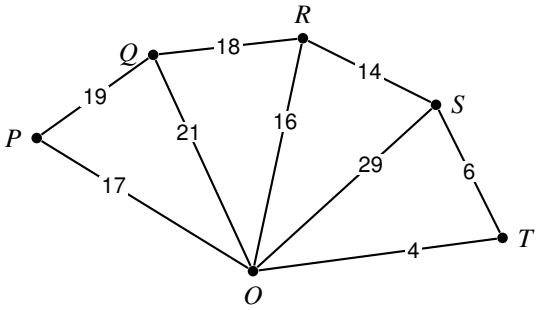


Figure 6

[The total weight of the network is 144]

(a) Find the length of the shortest closed inspection route.

(2)

(b) A new pipe QS, of length 13 m, is added and must also be inspected. Determine the change in the length of the shortest closed route, giving a reason.

(2)

[4 marks]

D7 Figure 7 shows a network of tracks in a forest, with lengths in hundreds of metres. A route must use every track, starting and finishing at A.

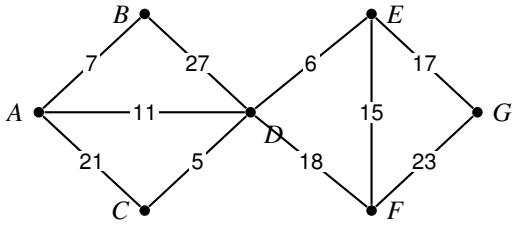


Figure 7

[The total weight of the network is 150]

(a) Find the length of the shortest such route.

(2)

(b) The route must still start at A but may finish elsewhere. Determine where it should finish, giving a reason, and the saving made.

(2)

[4 marks]

D8 Figure 8 shows a network of roads, with lengths in km. The length of road AF is x km, where x is a positive integer. A closed route must use every road.

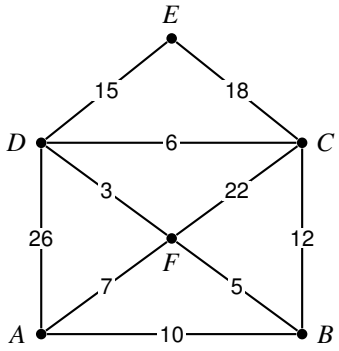


Figure 8

[The total weight of the network is 124]

(a) State the odd vertices, and determine the values of x for which road AF is travelled twice.

(2)

(b) Find, in terms of x, the length of the shortest route in each case.

(2)

(c) Determine the least possible length of the route.

(1)

[5 marks]

E Exam practice

Exam-style questions in the style of WDM11. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 Figure 9 represents a network of 12 roads in a town. The number on each arc is the time taken, in minutes, to drive along the corresponding road. A council officer needs to check that the roads are in good repair. She must travel along each road at least once, starting and finishing at the depot at A, and must minimise the total time taken for her inspection route. She has asked for a clear explanation of which roads she will drive along twice, so that she can plan where to stop and make notes.

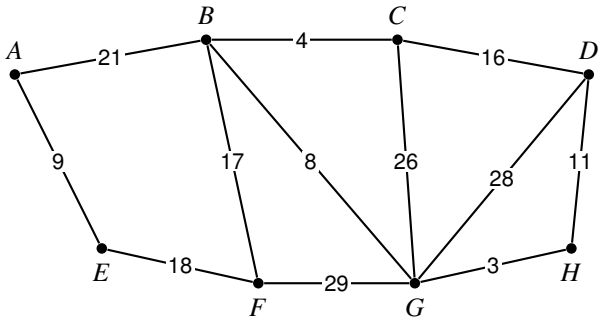


Figure 9

[The total weight of the network is 190]

(a) By considering the pairings of all relevant vertices, find the roads that need to be travelled twice, and the time taken for the route. You must show your working.

(4)

(b) Write down a possible route for the officer, starting and finishing at A, that achieves this minimum time.

(2)

(c) On another day the officer decides to start her route at C, because she lives near C, and she may finish at any other vertex. Determine where she should finish, giving a reason, and the time taken. You must make clear which roads are now travelled twice.

(3)

(d) For the route in part (b), state the number of times the officer passes through vertex B, not counting the start or the finish.

(1)

(Total for Question E1 is 10 marks)

Answer book: Question E1

Work in the space on this page, as you would in the WDM11 answer book.

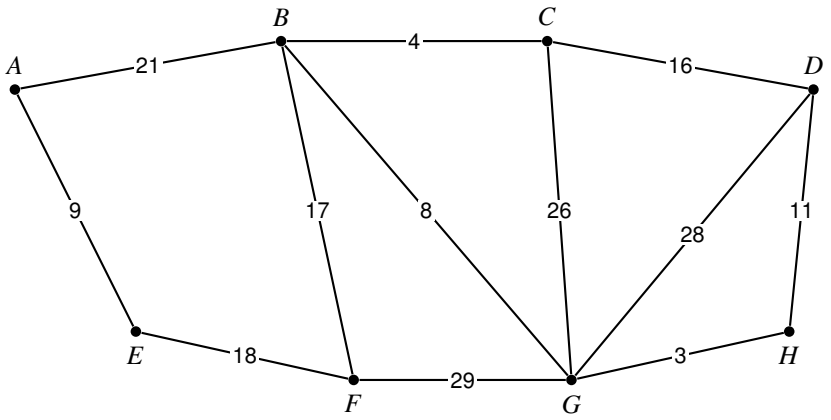


Figure 9 (printed again for you to mark up)

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E2 Figure 10 represents the paths between nine attractions in a theme park. The number on each arc is the length of the path in metres. A maintenance worker is based at attraction B. He needs to know the shortest distances from B to the other attractions, and later in the week he must inspect every path in the park at least once, walking as short a distance as possible.

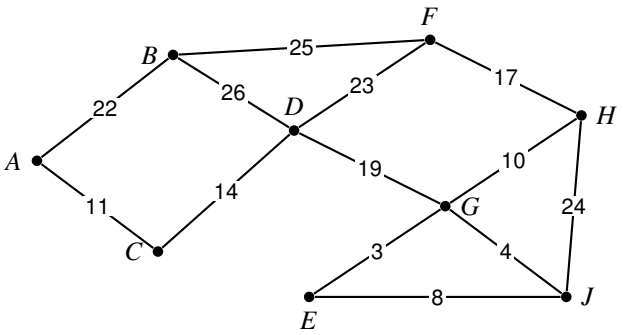


Figure 10

[The total weight of the network is 206]

(a) Use Dijkstra's algorithm to find the shortest path from B to J, and state its length. Complete the boxes on the answer-book page.

(5)

(b) The worker must travel along every path at least once, starting and finishing at B. Using your answer to part (a) where possible, consider the pairings of all relevant vertices to find the length of the shortest route.

(4)

(c) The worker now starts at B but may finish at a different attraction, where a colleague will collect him. Determine where the route should finish, giving a reason for your choice, and the length of the route.

(3)

(Total for Question E2 is 12 marks)

Answer book: Question E2

Work in the space on this page, as you would in the WDM11 answer book.

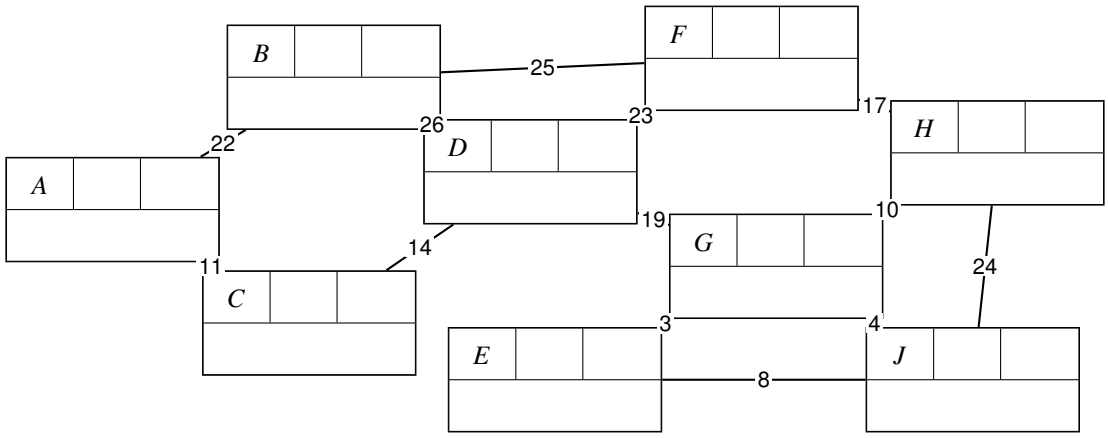


Diagram 1

Key:	Vertex	Order of labelling	Final value
	Working values		

Shortest path from B to J _____

Length _____

STRETCH

E3 Figure 11 represents a network of roads between eight villages. The number on each arc is the length of the road in km. A highways engineer must drive along every road at least once, starting and finishing at village A, and wants the route to be as short as possible. The council is also considering two changes to the network, and the engineer wants to know how each change would affect the length of the inspection route.

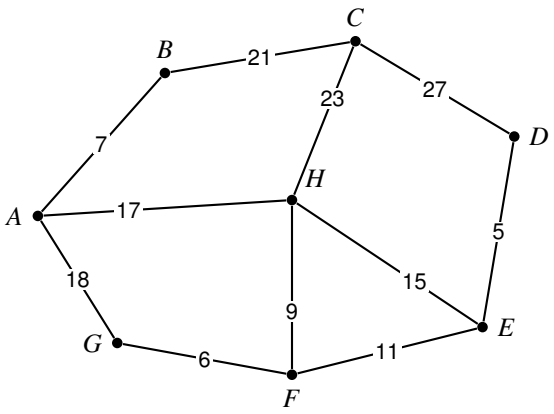


Figure 11

[The total weight of the network is 159]

(a) Use Dijkstra's algorithm to find the shortest route from A to E, and its length. Complete the boxes on the answer-book page.

(5)

(b) By considering the pairings of all relevant vertices, determine the length of the engineer's shortest route, stating which roads are travelled twice.

(4)

(c) The road EF is to be rebuilt, and its new length will be x km. Determine the values of x for which EF would still be travelled twice in the shortest route, making your reasoning clear.

(3)

(d) Instead of rebuilding EF, the council builds a new road CF of length 9 km, which must also be inspected. Determine the length of the engineer's shortest route on the new network, giving a reason for your answer.

(2)

(Total for Question E3 is 14 marks)

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Answer book: Question E3

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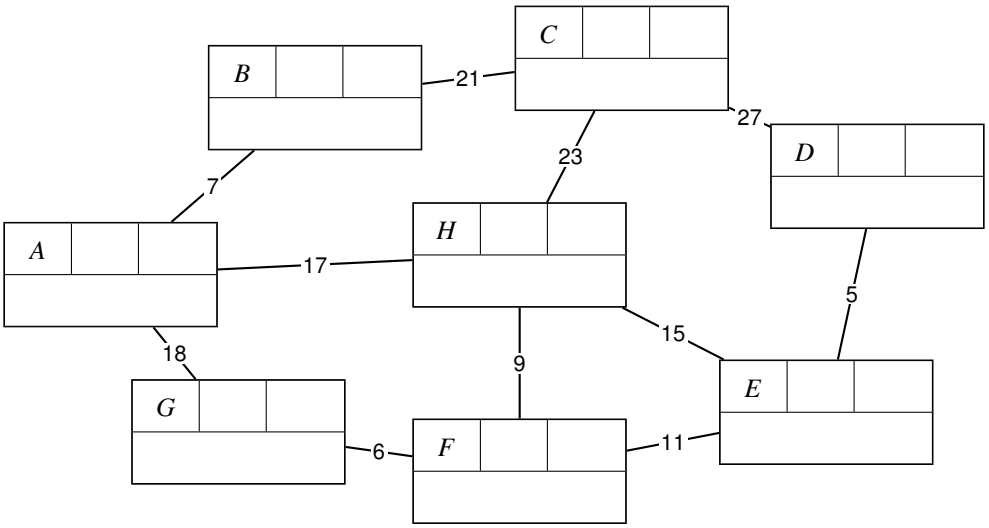


Diagram 2

Key:

Vertex	Order of labelling	Final value
Working values		

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