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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	3	
Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA14.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.

Marking a proof. A proof by contradiction earns its final mark only when it has all three parts: a clear assumption at the start (“Assume there is ...”), correct deductions in the right order, and a statement of the contradiction with a conclusion. A sketch or calculator values are not a proof.



5

Chapter 5: Differentiation

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	Differentiates implicitly: $xy \rightarrow y + x \frac{dy}{dx}$ or $y^3 \rightarrow 3y^2 \frac{dy}{dx}$	M1
	$4x + y + x \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$	A1
	Substitutes (2, 1): $8 + 1 + 2 \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$	M1
	$\frac{dy}{dx} = 9$	A1
	Normal gradient $-\frac{1}{9}$: $y - 1 = -\frac{1}{9}(x - 2)$	M1
	$x + 9y - 11 = 0$	A1
	The “= 0” must be seen, and any integer multiple is accepted. Finding the tangent ($y = 9x - 17$) scores the first four marks only.	
		(6)
Total for Question 1: 6 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = 8 \sin t \cos t$ and $\frac{dy}{dt} = 3 \sec^2 t$	B1
	Divides $\frac{dy}{dt}$ by $\frac{dx}{dt}$	M1
	$\frac{dy}{dx} = \frac{3}{8 \sin t \cos^3 t}$	A1
		(3)
(b)	$t = \frac{\pi}{6}$: $x = 1, y = \sqrt{3}$	B1
	Gradient $\frac{3}{8 \times \frac{1}{2} \times \frac{3\sqrt{3}}{8}} = \frac{2}{\sqrt{3}}$, and $y - \sqrt{3} = \frac{2}{\sqrt{3}}(x - 1)$	M1
	$y = \frac{2\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$	A1
		(3)
Total for Question 2: 6 marks		



Question 3

5 marks

Part	Scheme	Marks
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dS}{dr} = 8\pi r$	B1
	$\frac{dr}{dt} = \frac{50}{4\pi r^2} = \frac{1}{2\pi}$ when $r = 5$	M1
	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = 40\pi \times \frac{1}{2\pi}$	M1
	$20 \text{ cm}^2 \text{ s}^{-1}$	A1
	Alternatively $S = (36\pi V^2)^{\frac{1}{3}}$ and $\frac{dS}{dt} = \frac{dS}{dV} \times 50$ with $V = \frac{500\pi}{3}$.	
		(5)
Total for Question 3: 5 marks		

Question 4

7 marks

Part	Scheme	Marks
(a)	$-3xy \rightarrow -3y - 3x \frac{dy}{dx}$ (product rule)	M1
	$y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$3x^2 - 3y - 3x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$	A1
	$\frac{dy}{dx} = \frac{3x^2 - 3y}{3x - 2y}$	A1
		(4)
(b)	Numerator = 0: $y = x^2$; with $y = 4$, $x = \pm 2$	M1
	$p = 2$ (as $p > 0$)	A1
	$k = 8 - 24 + 16$: $k = 0$	A1
		(3)
Total for Question 4: 7 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$\frac{d}{dx}(2^x) = 2^x \ln 2$ $xy \rightarrow y + x \frac{dy}{dx}$ $y^2 \rightarrow 2y \frac{dy}{dx}$ and collects the $\frac{dy}{dx}$ terms $2^x \ln 2 + y = (2y - x) \frac{dy}{dx}$, so $\frac{dy}{dx} = \frac{2^x \ln 2 + y}{2y - x}$ * $2^x \rightarrow x2^{x-1}$ or $2^x \rightarrow 2^x$ is B0 (Oct 2024 report: the major error with 8^x).	B1 M1 M1 A1* (4)
(b)	Gradient $\frac{1+\ln 2}{2}$; tangent $y = 1 + \frac{1+\ln 2}{2}x$, then $y = 0$ $x = -\frac{2}{1+\ln 2}$	M1 A1 (2)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = \frac{2}{2t}$ $\frac{1}{t}$	M1 A1 (2)
(b)	$P(4, 4)$ Normal gradient $-t = -2$: $y - 4 = -2(x - 4)$ $y = 12 - 2x$ *	B1 M1 A1* (3)
(c)	Substitutes $x = t^2$, $y = 2t$: $2t = 12 - 2t^2$ $t^2 + t - 6 = 0$, $(t + 3)(t - 2) = 0$; $t = 2$ is P , so $t = -3$ $Q(9, -6)$	M1 M1 A1 (3)
(d)	e.g. $\frac{1}{2} x_{PYQ} - x_{QYP} = \frac{1}{2} 4(-6) - 9(4) $ 30 Or: the normal meets the x -axis at $(6, 0)$; area = $\frac{1}{2} \times 6 \times 4 + \frac{1}{2} \times 6 \times 6 = 30$.	M1 A1 (2)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$2y \frac{dy}{dx} - 6 \frac{dy}{dx}$ $= 6x + 12$ $\frac{dy}{dx} = \frac{3(x+2)}{y-3}$	M1 A1 A1 (3)
(b)	The ends of the interval are where the tangent is vertical: $y - 3 = 0$ $y = 3: 9 - 18 = 3x^2 + 12x$ $3x^2 + 12x + 9 = 0, 3(x+1)(x+3) = 0$ $p = -3, q = -1$ Otherwise: as a quadratic in y, $y^2 - 6y - (3x^2 + 12x) = 0$ needs $36 + 12x^2 + 48x \geq 0$, i.e. $(x+1)(x+3) \geq 0$.	M1 M1 M1 A1 (4)
(c)	Assume C has a stationary point: $\frac{dy}{dx} = 0$ there, so $x = -2$ Substitutes $x = -2$: $y^2 - 6y = 12 - 24$, i.e. $y^2 - 6y + 12 = 0$ Discriminant $36 - 48 < 0$: no real y, a contradiction. So C has no stationary points * $x = -2$ lies in $-3 < x < -1$, where the curve is not defined: an equally valid reason.	B1 M1 A1* (3)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	Similar triangles: $\frac{r}{h} = \frac{2.5}{5}$, so $r = \frac{h}{2}$ $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$ * <i>Oct 2024 report: substituting $r = \frac{2}{5}h$ without showing where the link comes from cannot score.</i>	M1 A1* (2)
(b)	$\frac{dV}{dh} = \frac{\pi h^2}{4}$ $\frac{dV}{dt} = 2\pi - \frac{1}{2}\pi h$ $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ $= \frac{2\pi - \frac{1}{2}\pi h}{\frac{1}{4}\pi h^2} = \frac{8 - 2h}{h^2}$ *	B1 B1 M1 A1* (4)
(c)	1 m per minute	B1 (1)
(d)	$\frac{dh}{dt} = 0$ when $8 - 2h = 0$ $h = 4$ m ($\frac{dh}{dt} > 0$ for $h < 4$, so the depth rises towards 4 m)	M1 A1 (2)
(e)	The depth approaches 4 m, which is less than the 5 m height, so it does not overflow	B1 (1)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = 1 - \cos t, \frac{dy}{dt} = \sin t$ and divides $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$ *	M1 A1* (2)
(b)	$\sin t = 2 \sin \frac{t}{2} \cos \frac{t}{2}$ and $1 - \cos t = 2 \sin^2 \frac{t}{2}$ $\frac{2 \sin \frac{t}{2} \cos \frac{t}{2}}{2 \sin^2 \frac{t}{2}} = \cot \frac{t}{2}$ *	M1 A1* (2)
(c)	$P\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}, \frac{3}{2}\right)$ Gradient $\cot \frac{\pi}{3} = \frac{\sqrt{3}}{3}$: $y - \frac{3}{2} = \frac{\sqrt{3}}{3} \left(x - \frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right)$ $y = \frac{\sqrt{3}}{3}x + 2 - \frac{2\sqrt{3}\pi}{9}$	B1 M1 A1 (3)
(d)	Normal gradient $-\frac{1 - \cos t}{\sin t}$ through $(t - \sin t, 1 - \cos t)$ Sets $y = 0$: $-(1 - \cos t) = -\frac{1 - \cos t}{\sin t}(x - t + \sin t)$ $\sin t = x - t + \sin t$, so $x = t$: the normal passes through $(t, 0)$ * <i>Dividing by $1 - \cos t$ is valid because $1 - \cos t \neq 0$ for $0 < t < 2\pi$</i>	M1 M1 A1* (3)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$\ln y = \cos x \ln x$	M1
	$\frac{1}{y} \frac{dy}{dx}$ on the left	M1
	Product rule on the right: $-\sin x \ln x + \frac{\cos x}{x}$	M1
	$\frac{dy}{dx} = x^{\cos x} \left(\frac{\cos x}{x} - \sin x \ln x \right)$ *	A1*
		(4)
(b)	$x^{\cos x} > 0$, so $\frac{\cos x}{x} = \sin x \ln x$	M1
	Multiplies by $\frac{x}{\cos x}$ ($\cos x \neq 0$): $x \tan x \ln x = 1$ *	A1*
(c)		(2)
	$g(x) = x \tan x \ln x - 1$: $g(1.2) = -0.437$, $g(1.3) = 0.229$	M1
	Sign change and g is continuous on $[1.2, 1.3]$, so $1.2 < \alpha < 1.3$	A1
(d)		(2)
	$g(1.2725) = -0.0027$ and $g(1.2735) = 0.0049$	M1
	Sign change, g continuous, so $\alpha = 1.273$ (3 d.p.)	A1
(e)		(2)
	$1.2729^{\cos 1.2729} = 1.07$	B1
(f)		(1)
Total for Question 10: 11 marks		

