



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Numerical Methods

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Numerical Methods worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Numerical Methods worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Pure 3 exam questions

More worksheets



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Page 1 of 7

A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $f(1) = -0.2817 < 0$, $f(2) = 1.3891 > 0$: **a sign change, and f is continuous, so there is a root**

A2 $e^x = 3x$; take $\ln$ of both sides: **$x = \ln 3x$**

A3 $x_2 = \ln 4.5$: **$x_2 = 1.5041$** , **$x_3 = 1.5068$**

STRETCH

A4 $f(1.5115) = -0.0010 < 0$, $f(1.5125) = 0.0006 > 0$; f is continuous: **$\alpha = 1.512$ to 3 d.p.**

A5 $\tan 1 = 1.557$, $\tan 2 = -2.185$, but $\tan x$ has an asymptote at $x = \frac{\pi}{2}$, so it is not continuous:
no root is shown

A6 $x_2 = \sqrt{5}$: **$x_2 = 2.2361$** , **$x_3 = 2.2596$**

A7 $\ln x = 4 - x^2$ is not needed: $x^2 = 4 - \ln x$, and $x > 0$: **$x = \sqrt{4 - \ln x}$**



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

- B1** (a) $-3, 5$ (b) $1, -0.4597$ (c) $-0.3069, 1.0986$ (d) $-1, 4$

each: a sign change, and the function is continuous, so there is a root

Every answer needs the values, the sign change, continuity and a conclusion.

- B2** (a) $x^3 = 5 - x$ (b) $x(x^2 + 1) = 5$ (c) $x^2 + 1 = \frac{5}{x}$ **as required**

(d) $x_2 = 1.5183$, $x_3 = 1.5156$

One equation can be rearranged in many ways; not all of them converge.

- B3** (a) 3.0986 (b) 3.1310 (c) 3.1413 (d) 3.146

Keep iterating until the value is stable to the accuracy you need.

- B4** (a) -0.0038 (b) 0.0041

(c) a sign change and f is continuous, so $1.5155 < \alpha < 1.5165$: $\alpha = 1.516$

(d) **every number in this interval rounds to 1.516**

The interval is the rounding interval of the claimed answer.

- B5** (a) sign change, but an asymptote at $x = 3$: **no root**

(b) no sign change, but the graph touches the axis: **a root at $x = 1$**

(c) sign change, but an asymptote at $x = \frac{\pi}{2}$: **no root**

The sign-change test needs a continuous function, and it misses repeated roots.

- B6** (a) 0.1065 , -0.0512 : **a sign change, and f is continuous, so there is a root** (b) $x_2 = 0.6065$,

$x_3 = 0.5452$ (c) $x_4 = 0.5797$ (d) 0.567

Here the iterates jump above and below the root.



STRETCH

B7 (a) $f(x) = \ln x + x^2 - 3$: $f(1) = -2$, $f(2) = 1.6931$: **a sign change, and f is continuous, so there is a root**

(b) $x^2 = 3 - \ln x$ **as required**

(c) $x_2 = 1.6108$, $x_3 = 1.5885$ (d) 1.592

Intersections become roots of a new function, $f(x) - g(x)$

B8 (a) 0.1477 , -0.1814 : **a sign change, and f is continuous, so there is a root** (b) $x_2 = 1.8186$,

$x_3 = 1.9389$

(c) $x_4 = 1.8660$; the iterates are alternately below and above the root (d) 1.895

Radian mode is essential.

B9 (a) $\frac{dy}{dx} = e^x - 3x^2 = 0$, so $x^2 = \frac{e^x}{3}$, $x > 0$ **as required**

(b) $x_2 = 0.9519$ (c) $x_3 = 0.9293$, $x_4 = 0.9188$ (d) 0.910

A stationary point is a root of $\frac{dy}{dx} = 0$

B10 (a) $f(1) = 4$, $f(2) = -7$: **a sign change, and f is continuous, so there is a root** (b) $6h^2 = h^3 + 9$, $h > 0$

as required

(c) $h_2 = 1.4361$, $h_3 = 1.4120$ (d) $h = 1.3985 \dots$: **1.40 m**

Give the answer in the units and accuracy asked for.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 **Sam must say f is continuous on $[1, 2]$ and state that there is a sign change.**

C2 f has an asymptote at $x = 1$, so it is not continuous on $[0.5, 1.5]$; the sign change comes from the asymptote.

The only root is $x = 0$, outside the interval.

C3 The formula uses radians. In radians: $x_2 = \frac{1}{2} + \cos 1 =$ **1.0403**



- C4** Iterates must be given to 4 d.p. and not rounded between steps; rounding early changes later iterates.

$$x_2 = 2.0801$$

STRETCH

- C5** A sign change on $[2.094, 2.096]$ allows a root such as 2.0942, which rounds to 2.094.

$$\text{Use } [2.0945, 2.0955]$$

- C6** The iterates move away from the root: the formula diverges, because $|g'(x)| > 1$ near 2.09.

$$\text{Use } x_{n+1} = \sqrt[3]{2x_n + 5}$$

- C7** $x = 2$ is a repeated root: the graph touches the axis without crossing it, so there is no sign change.

$$\text{there is a root at } x = 2$$

- C8** (a) $\alpha^3 = 2\alpha + 5$, so $\alpha = \sqrt[3]{2\alpha + 5}$ **as required** (b) $g'(x) = \frac{2}{3}(2x + 5)^{-\frac{2}{3}} \leq \frac{2}{3} \times 9^{-\frac{2}{3}} = 0.154$:

$$0 < g'(x) < 0.16$$

- (c) **each step multiplies the error by less than 0.16, so the error shrinks quickly**

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

- D1** (a) $f(1.9) = -0.1752 < 0$, $f(2) = 0.3628 > 0$; a sign change and f is continuous: $1.9 < \alpha < 2$

(b) $x^2 = 4 \sin x$, and $x > 0$: $x = 2\sqrt{\sin x}$

- D2** (a) $h(x) = e^{\frac{x}{2}} + x^2 - 4$: $h(1.4) = -0.0262$, $h(1.5) = 0.3670$, a sign change, and h is continuous:

$$P \text{ lies between } 1.4 \text{ and } 1.5$$

(b) $x_2 = 1.4093$, $x_3 = 1.4060$

- D3** (a) $70e^{-0.04t} = 20 + 0.5t$, so $-0.04t = \ln \frac{20 + 0.5t}{70}$ **as required**

(b) $t_2 = 21.182$; $\beta = 20.8 \text{ minutes}$



D4 $f(1.4055) = -0.0070 < 0$, $f(1.4065) = 0.0019 > 0$: a sign change and f is continuous, so $1.4055 < \alpha < 1.4065$

$$\alpha = 1.406 \text{ to 3 d.p.}$$

STRETCH

D5 (a) $f(1.5) = -4.4375$, $f(2) = 5$: **a sign change, and f is continuous, so there is a root**

(b) $x^4 = 3x + 5$: $x_2 = 1.7556$, $x_3 = 1.7900$ (c) $\alpha = 1.7952$

D6 (a) $\frac{dy}{dx} = -e^{-x} + \cos x = 0$, so $\cos x = e^{-x}$ **as required**

(b) $x_2 = 1.1941$, $x_3 = 1.2630$; $x = 1.293$

D7 (a) $f(1.5) = 7.5$, $f(2.5) = -3.5$: **opposite signs**

(b) f has an asymptote at $x = 2$: **not continuous on the interval**

(c) $3 + x(2 - x) = 0$, $x^2 - 2x - 3 = 0$: $x = 3$, $x = -1$, both outside $[1.5, 2.5]$

D8 (a) $f(0) = 3$, $f(1) = -3$; $f(2) = -3$, $f(3) = 9$; f is continuous: **a root in each interval**

(b) $x_2 = 0.4464$, $x_3 = 0.4413$ (c) 2.6607, 3.1195, 4.7651: **diverges**; the second formula gives 2.398

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $f(1.1) = 0.0244 > 0$, $f(1.2) = -0.1106 < 0$

sign change, f continuous: $1.1 < \alpha < 1.2$

(b) $4 \cos x = 3 - x^2$, so $\cos x = \frac{3 - x^2}{4}$ **as required**

(c) $x_2 = 1.1068$

$\alpha = 1.118$

M1

A1

B1

M1

A1



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

- E2** (a) $4xe^{-x} = 1$, so $e^x = 4x$, $x = \ln 4x$ as required B1
- (b) $4(2)e^{-2} - 1 = 0.0827$, $4(2.5)e^{-2.5} - 1 = -0.1792$ M1
- sign change, continuous: $2 < \beta < 2.5$ A1
- (c) $x_2 = 2.0794$, $x_3 = 2.1184$ M1 A1
- (d) $x_1 = 0.3$: $\ln 1.2 = 0.1823$, then $\ln 0.729 = -0.316$, then $\ln$ of a negative:
the iterates move away from A and fail B1

STRETCH

- E3** (a) $h = 0$: $0.4e^{0.03x} = 1.2$, $e^{0.03x} = 3$ M1
- $d = \frac{100}{3} \ln 3 = 36.62$ A1
- (b) $\frac{dh}{dx} = 1.2 - 0.4e^{0.03x} - 0.012xe^{0.03x}$ M1 A1
- $= 0$: $e^{0.03x}(1 + 0.03x) = 3$, so $0.03x = \ln \frac{3}{1 + 0.03x}$ as required A1*
- (c) $x_2 = 20.954$; by repeated iteration $x = 20.59$ M1 A1
- (d) 9.43 metres B1

