



eClassroom

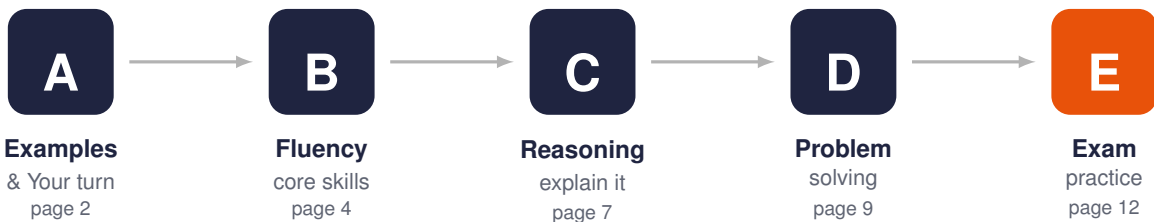
A-LEVEL MATHEMATICS • PURE 1

Equations and Inequalities

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Equations and Inequalities worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: solve a linear and a quadratic equation simultaneously, pairing each x with its y • solve linear and quadratic inequalities and write the answer in correct notation • use inequalities to define a region on a graph • *Stretch:* use the discriminant to decide where graphs meet, solve inequalities with x in the denominator, combine inequalities in set notation and prove results.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Intersections that depend on a constant, inequalities with x in the denominator, regions that need an x restriction, and proof.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Solve the simultaneous equations

$$2x + y = 5 \quad x^2 + xy = 6$$

Make y the subject of the linear equation: $y = 5 - 2x$

$$x^2 + x(5 - 2x) = 6, \text{ so } x^2 - 5x + 6 = 0$$

$(x - 2)(x - 3) = 0$. Put each x into $y = 5 - 2x$ to find its own y .

$$x = 2, y = 1 \quad \text{or} \quad x = 3, y = -1$$

Your turn A1

Solve the simultaneous equations

$$x - 2y = 1 \quad xy = 3$$

Worked example A2

Find the set of values of x for which $2x^2 + 5x > 3$

Rearrange to 0 first: $2x^2 + 5x - 3 > 0$

$$(2x - 1)(x + 3) > 0: \text{critical values } \frac{1}{2} \text{ and } -3$$

A U-shape is above the axis outside its roots. Write "or", never $-3 > x > \frac{1}{2}$

$$x < -3 \text{ or } x > \frac{1}{2}$$

Your turn A2

Find the set of values of x for which $3x^2 \leq 10x + 8$

Worked example A3

The finite region R is bounded by the curve $y = x^2 - 2x - 3$ and the line $y = x + 1$. Use inequalities to define R , including its boundaries.

They meet where $x^2 - 3x - 4 = 0$, so $x = -1$ or $x = 4$

Test $x = 0$: the line gives 1, the curve gives -3 , so R is above the curve and below the line.

$$y \geq x^2 - 2x - 3 \text{ and } y \leq x + 1$$

Your turn A3

The finite region R is bounded by the curve $y = 7 - x^2$ and the line $y = x + 1$. Use inequalities to define R , including its boundaries.



STRETCH

Worked example A4: two inequalities at once

Using set notation, find the values of x for which both $2(x-3) < x+1$ and $x^2 - 4x - 12 > 0$

Linear: $x < 7$. Quadratic: $x < -2$ or $x > 6$

On a number line, keep only the values in both.

$$\{x : x < -2\} \cup \{x : 6 < x < 7\}$$

Your turn A4

Using set notation, find the values of x for which both $7 - 2x > 1$ and $2x^2 + x - 6 \geq 0$

Worked example A5: x in the denominator

Solve $\frac{x+6}{x} \geq 4$, $x \neq 0$

x may be negative, so multiply by x^2 , which is positive.

$x(x+6) \geq 4x^2$, so $3x^2 - 6x \leq 0$, so $0 \leq x \leq 2$, but $x \neq 0$

$$0 < x \leq 2$$

Your turn A5

Solve $\frac{5x-4}{x} > 3$, $x \neq 0$

Worked example A6: when do two curves meet?

Find the set of values of k for which the curves $y = 2x^2 + kx + 1$ and $y = x^2 - 2$ do not meet.

Equate: $x^2 + kx + 3 = 0$. Do not meet: no real roots, so $b^2 - 4ac < 0$

$k^2 - 12 < 0$: critical values $\pm\sqrt{12} = \pm 2\sqrt{3}$

$$-2\sqrt{3} < k < 2\sqrt{3}$$

Your turn A6

Find the set of values of k for which the curves $y = x^2 + kx$ and $y = 4x - x^2 - 2$ meet at two distinct points.

Worked example A7: an x restriction

The region R lies in the first quadrant and is bounded by the curve $y = 10 - x^2$, the line $y = x + 4$ and the y -axis. Use inequalities to define R , including its boundaries.

They meet where $x^2 + x - 6 = 0$, so $x = 2$ ($x = -3$ is not in the first quadrant).

R is below the curve and above the line. Without $x \geq 0$ the region would carry on to $x = -3$

$$x \geq 0, y \geq x+4, y \leq 10-x^2$$

Your turn A7

The region R lies in the first quadrant and is bounded by the curve $y = 2x^2 + 3$, the line $y = 9 - x$ and the y -axis. Use inequalities to define R , including its boundaries.



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 Solve each pair of simultaneous equations. If the graphs do not meet, say so.

(a) $y = x + 3, y = x^2 + 1$

(b) $y = x + 3, y = x^2 + 3$

(c) $y = x + 3, y = x^2 + 3x + 4$

(d) $y = x + 3, y = x^2 + x + 5$

[4 marks]

B2 Solve the simultaneous equations.

(a) $x + y = 6, xy = 8$

(b) $x - y = 2, x^2 + y^2 = 20$

(c) $2x + y = 1, x^2 + y^2 = 2$

(d) $x + 2y = 5, x^2 + 4y^2 = 13$

[4 marks]

B3 Solve

(a) $3x - 5 < 13$

(b) $5 - 2x \geq 11$

(c) $2(x - 4) > 5x + 4$

(d) $\frac{x+1}{3} - \frac{x-2}{2} \leq 1$

[4 marks]

B4 Find the set of values of x for which

(a) $(x - 2)(x - 5) < 0$

(b) $(x - 2)(x - 5) \geq 0$

(c) $(2 - x)(x - 5) > 0$

(d) $x^2 < 7x - 10$

[4 marks]



CORE

continued: combining inequalities and regions

B5 Find the set of values of x for which

(a) $2x - 1 > 5$ and $x^2 - 7x + 10 < 0$

(b) $2x - 1 > 5$ or $x^2 - 7x + 10 < 0$

(c) $2x - 1 > 5$ and $x^2 - 7x + 10 > 0$

[3 marks]

B6 Figure 1 shows the curve with equation $y = 6 - x^2$ and the line with equation $y = x$. They meet at $(-3, -3)$ and $(2, 2)$

The boundaries of the regions are not included.

Use inequalities to define each region.

(a) R_1 (b) R_2 (c) R_3 (d) R_4

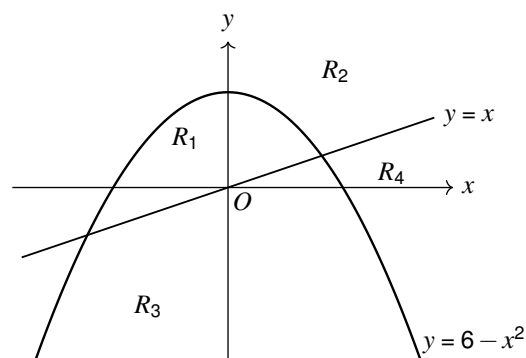


Figure 1

[4 marks]

STRETCH

B7 Find the set of values of x for which

(a) $\frac{6}{x} > 2$

(b) $\frac{6}{x} > -2$

(c) $\frac{x+6}{x} < 2$

(d) $\frac{6}{x-1} > 2$

[4 marks]

B8 The line $y = kx + 1$ and the curve $y = x^2 + 2x + 5$ are drawn on the same axes, where k is a constant. Find the values of k for which the line

(a) meets the curve at two distinct points

(b) touches the curve

(c) does not meet the curve.

(d) For the positive value of k in part (b), find the coordinates of the point where the line touches the curve.

[4 marks]



STRETCH*continued: inequalities and simultaneous equations*

B9 Find the set of values of x for which each inequality is true. Use set notation where it helps.

(a) $x^2 - 4x - 1 < 0$

(b) $x^2 - 4x + 4 > 0$

(c) $x^2 - 4x + 5 > 0$

(d) $x^2 - 4x + 5 < 0$

[4 marks]

B10 Solve each pair of simultaneous equations.

(a) $x + y = 5, xy = 6$

(b) $x + y = 5, x^2 + y^2 = 13$

(c) $x + y = 5, x^2 - y^2 = 5$

(d) $x + y = 5, 2x^2 + 2y^2 = 25$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Tom solves the inequality $x^2 > 9$ and writes $x > \pm 3$
Explain why Tom's answer is wrong, and give the correct set of values of x . **[2 marks]**
- C2** Kim solves the inequality $2x^2 - 7x - 4 > 0$. She finds the critical values correctly and writes $-\frac{1}{2} > x > 4$
Explain why Kim's answer would lose the final mark, and write her answer correctly. **[2 marks]**
- C3** Show that the line with equation $y = 4x - 7$ is a tangent to the curve with equation $y = x^2 - 2x + 2$, and find the coordinates of the point where the line touches the curve. **[3 marks]**
- C4** The region R lies in the first quadrant and is bounded by the curve $y = 9 - x^2$ and the coordinate axes, including its boundaries.
Ali defines the region as $0 \leq y \leq 9 - x^2$
Give the coordinates of a point that satisfies Ali's inequalities but is not in R , and write down a correct definition of R . **[3 marks]**



STRETCH

- C5** Priya solves $\frac{3}{x} < 1$, $x \neq 0$, by multiplying both sides by x . She writes $3 < x$, so $x > 3$

Show that $x = -1$ satisfies the inequality. Explain Priya's mistake, and find the full set of values of x .

[3 marks]

- C6** Prove that, for every real value of the constant k , the line with equation $y = kx + 1$ meets the curve with equation $y = x^2 + 2x - 3$ at two distinct points.

[3 marks]

- C7** Jo says: " $x^2 + kx + 9 > 0$ for all real values of x whenever $k < 6$ "

Show that Jo is wrong by giving a value of k and a value of x for which the statement fails. Find the correct set of values of k for which $x^2 + kx + 9 > 0$ for all real x .

[3 marks]

- C8** The real numbers x and y satisfy the simultaneous equations

$$x + y = p \quad x^2 + y^2 = 8$$

where p is a constant.

- (a) Show that $2x^2 - 2px + p^2 - 8 = 0$

(1)

- (b) Hence prove that these equations have real solutions only when $-4 \leq p \leq 4$

(2)

- (c) Deduce the greatest possible value of $x + y$ when $x^2 + y^2 = 8$, and the values of x and y that give it.

(2)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

D1 Figure 2 shows the plan of a metal plate.

The plate is made by cutting a square of side 2 cm from one corner of a rectangle that measures x cm by y cm, where $x > y$

The perimeter of the plate is 26 cm and the area of the plate is 36 cm^2

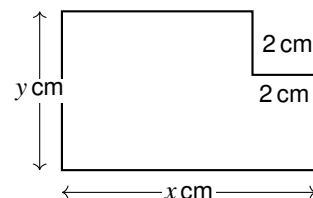


Diagram NOT accurately drawn

Figure 2

(a) Show that $x + y = 13$ and $xy = 40$

(1)

(b) Hence find the value of x and the value of y .

(2)

[3 marks]

D2 The curve C_1 has equation $y = 3x^2 - 2x + k$, where k is a constant.

The curve C_2 has equation $y = x^2 + 6x - 1$

Given that C_1 and C_2 meet at exactly one point P ,

(a) show that the x coordinate of P satisfies the equation $2x^2 - 8x + k + 1 = 0$

(1)

(b) find the value of k ,

(2)

(c) find the coordinates of P .

(1)

[4 marks]

D3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 3 shows the curve C with equation

$y = x^2 - x - 6$ and the line l with equation $y = x + 9$

The line l meets C at the points A and B , as shown in Figure 3.

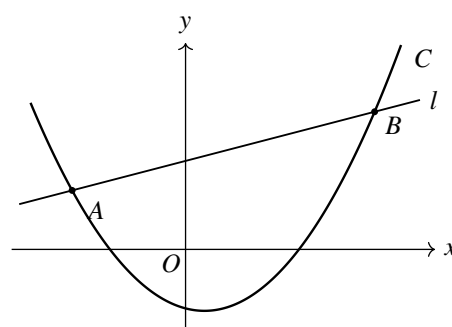


Figure 3

(a) Use algebra to find the x coordinates of A and B .

(2)

(b) Hence, using Figure 3, find the set of values of x for which C lies below l and above the x -axis.

(1)

[3 marks]



- D4** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

A company makes and sells garden chairs. Its weekly profit, £ P hundred, when each chair is sold for £ x , is modelled by the equation

$$P = 40x - 2x^2 - 150$$

The company needs a weekly profit of at least £4200

- (a) Show that the selling price must satisfy $x^2 - 20x + 96 \leq 0$ (1)

- (b) Hence find, according to the model, the range of selling prices that give the company the profit it needs. (2)

[3 marks]

STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = kx^2 + 3$, where k is a constant, and the curve C_2 has equation $y = 4x - x^2$

- (a) Show that the x coordinates of any points where C_1 and C_2 meet satisfy the equation $(k+1)x^2 - 4x + 3 = 0$ (1)

Given that C_1 and C_2 meet at exactly one point,

- (b) find the two possible values of k , explaining how each value arises. (3)

[4 marks]

- D6** Figure 4 shows the curve C with equation $y = x^2 - 3x + 5$ and the straight line l with equation $y = x + c$, where c is a constant.

The line l is a tangent to C at the point P .

- (a) Find the value of c . (2)

The region R , shown shaded in Figure 4, is bounded by C , l and the y -axis.

- (b) Use inequalities to define R . (2)

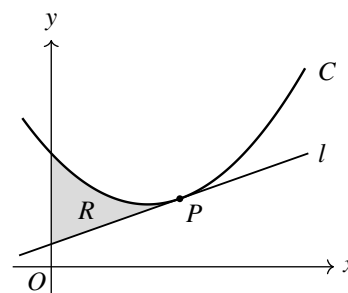


Figure 4

[4 marks]



D7 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 5 shows a sketch of the curve C with equation $y = (x - 1)(x - a)$, where a is a constant and $a > 1$

The line l passes through the point $A(1, 0)$ and has gradient 2. The line l meets C again at the point P .

(a) Show that P has coordinates $(a + 2, 2a + 2)$

(2)

Given that the length of AP is $4\sqrt{5}$,

(b) find the value of a .

(2)

[4 marks]

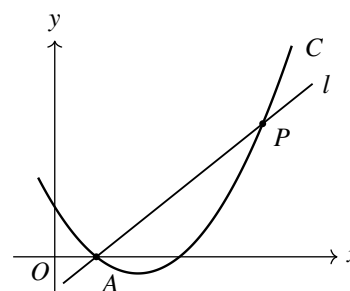


Diagram NOT accurately drawn

Figure 5

D8 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 6 shows the curve C with equation $y = x^2 - 2x + 6$ and the line l with equation $y = mx$, where m is a constant.

The line l meets C at the two distinct points A and B , as shown in Figure 6.

(a) Show that $m^2 + 4m - 20 > 0$

(2)

(b) Hence find the set of possible values of m , giving your answer in simplest surd form.

(2)

Given that the midpoint of AB has x coordinate 3,

(c) find the value of m .

(1)

[5 marks]

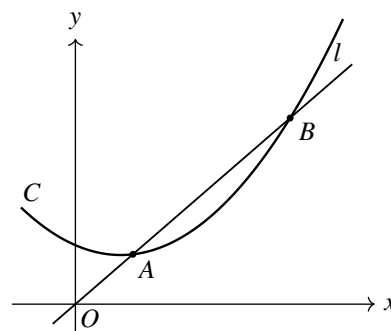


Figure 6



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 7 shows the curve C with equation

$$y = 2x^2 - 3x - 2$$

and the line l with equation $y = x + 4$

The line l meets C at the points P and Q , as shown in Figure 7.

- (a) Use algebra to find the coordinates of P and the coordinates of Q .

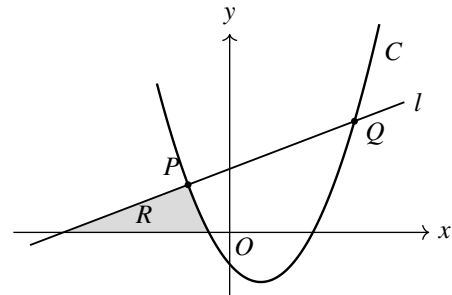


Figure 7

(3)

The region R , shown shaded in Figure 7, is bounded by C , l and the x -axis.

- (b) Use inequalities to define the region R .

(2)

(Total for Question E1 is 5 marks)



E2 Figure 8 shows the lines with equations

$$y = 4 \quad y = kx \quad y = 12 - x$$

where k is a positive constant.

The region R_1 , shown shaded in Figure 8, is defined by the inequalities

$$0 \leq y \leq 4 \quad y \leq kx \quad y \leq 12 - x$$

(a) Find, in terms of k , the x coordinate of the point where the line $y = kx$ meets the line $y = 4$

(1)

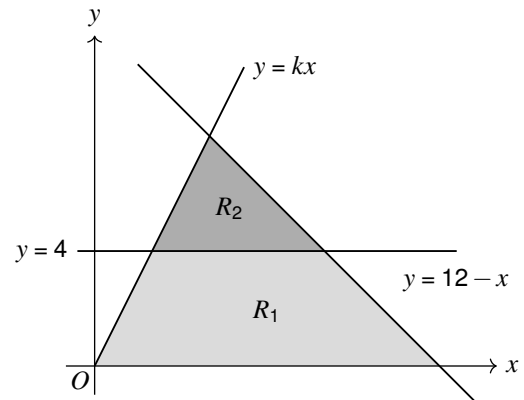


Diagram NOT accurately drawn

Figure 8

(b) Show that the area of R_1 is $40 - \frac{8}{k}$

(2)

Given that the area of R_1 is 36 square units,

(c) find the value of k .

.....

(1)

The region R_2 , also shown shaded in Figure 8, is bounded by the three lines.

(d) Use inequalities to define the region R_2 .

.....

(2)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 2 + \frac{4}{x} \quad x \neq 0$$

The line l has equation $y = k - x$, where k is a constant.

(a) Show that the x coordinates of any points where l meets C satisfy the equation

$$x^2 + (2 - k)x + 4 = 0$$

(2)

(b) Find the set of values of k for which l does not meet C .

(3)

Figure 9 shows part of the curve C and the line l in the case $k = 7$

The region R , shown shaded in Figure 9, is bounded by C and l .

(c) Use inequalities to define the region R .

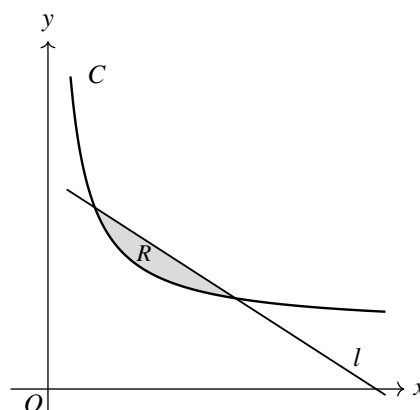


Figure 9

(3)

(Total for Question E3 is 8 marks)

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