



eClassroom

A-LEVEL MATHEMATICS • STATISTICS 1

Discrete Random Variables

Worked Solutions

Pearson Edexcel International A Level (WST01) & A Level (9MA0) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Discrete Random Variables worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 1 Discrete Random Variables worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dm** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Statistics 1 exam questions

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $k(3+4+5) = 12k = 1$: $k = \frac{1}{12}$

A2 $0.4 - 0.15$: 0.25 ; $1 - 0.4$: 0.6

A3 $E(X) = 0.4 + 0.6 = 1$; $E(X^2) = 0.4 + 1.2 = 1.6$: $E(X) = 1, \text{Var}(X) = 0.6$

STRETCH

A4 $3(4) + 2$: 14 ; $3^2 \times 1.5$: 13.5

A5 $n = 10$: $E(X) = 5.5, \text{Var}(X) = \frac{99}{12} = 8.25$

A6 $a + b = 0.7$ and $a + 2b + 0.6 = 1.6$, so $a + 2b = 1$: $b = 0.3, a = 0.4$

A7 $(0, 2), (2, 0), (1, 1)$: $2(0.2)(0.3) + 0.5^2 = 0.12 + 0.25$: 0.37

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**CORE**

B1 (a) $k(4+3+2+1) = 1$: $k = 0.1$ (b) 0.1×3 : 0.3 (c) $0.4 + 0.3$: 0.7 (d) 1

Substitute each value of x , then add.

B2 (a) $0.55 - 0.25$: 0.3 (b) $1 - 0.25$: 0.75 (c) $F(3)$: 0.55 (d) $F(4) - F(1)$: 0.7

F is cumulative: subtract to get a single value or a range.

B3 (a) $-0.2 + 0.8 + 0.5$: 1.1 (b) $0.2 + 1.6 + 2.5$: 4.3 (c) $4.3 - 1.21$: 3.09 (d) 1.76

Negative values of x give positive contributions to $E(X^2)$ 

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B4 (a) $4(2.5) - 1$: **9** (b) 16×1.2 : **19.2** (c) $(-1)^2 \times 1.2$: **1.2** (d) $1.2 + 2.5^2$: **7.45**

The constant moves the mean only; the multiplier is squared for the variance.

CORE

B5 (a) $\frac{9}{2}$: **4.5** (b) $\frac{63}{12}$: **5.25** (c) $\frac{2}{8}$: **0.25**

Discrete uniform with $n = 8$

B6 (a) $X > 3$: **0.4** (b) $X \leq 3$: **0.6** (c) $\frac{0.75}{0.85}$: **0.882** (d) **3.15**

Turn each inequality into a set of values of X first.

STRETCH

B7 (a) $a + b = 0.5$, $a + 3b + 1.4 = 2.5$: **$a = 0.2$, $b = 0.3$** (b) $0.2 + 1.2 + 2.7 + 3.2$: **7.3**

(c) $7.3 - 6.25$: **1.05** (d) 4×1.05 : **4.2**

The sum of probabilities and the mean give two equations.

B8 (a) $\text{Var}(Y) = a^2 \times 4 = 36$: **$a = 3$** (b) $9 + b = 20$: **$b = 11$** (c) $4 + 9$: **13** (d) $36 + 400$: **436**

$\text{Var}(aX + b) = a^2 \text{Var}(X)$ gives a ; then the mean gives b

B9 (a) $2(0.4)(0.25) + 0.35^2$: **0.3225** (b) $0.16 + 0.1225 + 0.0625$: **0.345**

(c) $\frac{1 - 0.345}{2}$: **0.3275** (d) $2(0.35)(0.25) + 0.25^2$: **0.2375**

By symmetry, $P(X_1 > X_2) = P(X_1 < X_2)$

B10 $P(X = x) = \frac{x^2 - (x-1)^2}{16} = \frac{2x-1}{16}$

(a) **$\frac{5}{16}$** (b) $1 - \frac{1}{16}$: **$\frac{15}{16}$** (c) $\frac{1+6+15+28}{16}$: **$\frac{25}{8}$** (d) **4**

$P(X = x) = F(x) - F(x-1)$, with $F(0) = 0$



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** The multiplier is squared and the constant has no effect; a variance cannot be negative. $(-2)^2 \times 3 = 12$
- C2** $E(X^2)$ is the mean of the squares, not the square of the mean. $5.5 - 2^2 = 1.5$
- C3** $F(3.5) = P(X \leq 3.5) = P(X \leq 3)$: 0.65
- C4** The probabilities already weight the values; there is nothing to divide by. $E(X) = 2.7$

STRETCH

- C5** $\text{Var}(aX - 1) = a^2 \text{Var}(X)$: $2.5a^2 = 40$, $a^2 = 16$: $a = 4$
- C6** A draw is neither a win for the first player nor for the second:
 $P(\text{first wins}) = 1 - P(\text{second wins}) - P(\text{draw})$
- C7** The formulae need the values 1 to n . Here $X = 2Y$ with Y uniform on 1 to 5:
 $E(X) = 2(3) = 6$, $\text{Var}(X) = 4(2) = 8$
- C8** (a) $E(aX + b) = \sum (ax + b)P(X = x) = a \sum xP(X = x) + b \sum P(X = x) = aE(X) + b$
 (b) $\text{Var}(aX + b) = E[(aX + b - a\mu - b)^2] = E[a^2(X - \mu)^2] = a^2 \text{Var}(X)$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

- D1** (a) $F(2.5) = F(2)$: 0.55
 (b) $3X > 7$, so $X \geq 3$: $0.25 + 0.2$: 0.45



- D2** (a) $E(X^2) = 0.2 + 4.5 + 10.8 = 15.5$; $15.5 - 3.5^2$: **3.25**
 (b) $3.25a^2 = 29.25$, so $a = 3$; $3(3.5) - b = 9$: **$a = 3, b = 1.5$**

- D3** (a) $a + 1 + 3(0.5 - a)$: **$E(X) = 2.5 - 2a$**
 (b) $0 \leq a \leq 0.5$: **$1.5 \leq E(X) \leq 2.5$**

- D4** $8k = 1$: **$k = 0.125$** ; F: 0.2, 0.25, 0.625, 1
 $P(X = x)$: 0.2, 0.05, 0.375, 0.375

STRETCH

- D5** (a) $2a + 2b + 0.1 = 1$ and $a + 2a + 3b + 4b + 0.5 = 3.05$: $a + b = 0.45$, $3a + 7b = 2.55$
 $3a + 7(0.45 - a) = 2.55$, so $4a = 0.6$: **$a = 0.15$** (and $b = 0.3$)
 (b) $0.15 + 0.6 + 2.7 + 4.8 + 2.5$: **10.75**

- D6** (a) $(1, 2), (1, 4), (2, 2), (3, 2)$: **$\frac{4}{9}$**
 (b) $A = 2$: $B = 2, 4$; $A = 3$: $B = 2, 4, 6$: **$\frac{5}{9}$**

- D7** $X_1 = 1$: $2 > X_2 + 2$ never; $X_1 = 2$: needs $X_2 = 1$; $X_1 = 4$: $8 > X_2 + 2$ always
 $0.3 \times 0.5 + 0.2 \times 1$: **0.35**

- D8** (a) $0.8 + 1 + 0.3$: **2.1** (b) $E\left(\frac{16}{X^2}\right) = 3.2 + 2 + 0.3 = 5.5$; $5.5 - 2.1^2$: **1.09**
 (c) $\frac{4}{X} < 3$ means $X = 2$ or 4 : $\frac{0.5}{0.8}$: **0.625**

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.



CORE

Exam questions for everyone

- E1** (a) $1 - 0.7$: $p = 0.3$ (b) $F(2) = 0.1 + 0.25 + 0.3$: 0.65 B1 B1
- (c) $0 + 0.25 + 0.6 + 0.6 + 0.6$ M1
- 2.05 A1
- (d) $E(X^2) = 0.25 + 1.2 + 1.8 + 2.4 = 5.65$ M1 A1
- $5.65 - 2.05^2 = 1.4475$ A1*
- (e) 9×1.4475 : 13.0275 B1
- (f) $(3, 4)$ or $(4, 3)$: $2(0.2)(0.15)$ M1
- 0.06 A1

- E2** (a) **discrete uniform** B1
- (b) $E(R) = 3, \text{Var}(R) = \frac{25 - 1}{12} = 2$ B1 B1
- (c) B is 0, 1, 2 each with probability $\frac{1}{3}$: $E(B) = 1; E(B^2) = \frac{5}{3}$ M1 A1
- $\frac{5}{3} - 1$: $\text{Var}(B) = \frac{2}{3}$ A1
- (d) $(4, 2), (5, 1), (5, 2)$: $3 \times \frac{1}{5} \times \frac{1}{3}$ M1
- $\frac{1}{5}$ A1
- (e) $B = 0$: all 5 values; $B = 1$: $R = 3, 4, 5$; $B = 2$: $R = 5$ M1 M1
- $\frac{5 + 3 + 1}{15}$: $\frac{3}{5}$ A1

STRETCH

- E3** (a) $F(5) = \frac{25 + 5k}{40} = 1$: $k = 3$ B1
- (b) F : 0.1, 0.25, 0.45, 0.7, 1 M1
- $P(X = x)$: **0.1, 0.15, 0.2, 0.25, 0.3** A1 A1
- (c) $0.1 + 0.3 + 0.6 + 1 + 1.5$: **3.5** B1
- (d) $E(X^2) = 0.1 + 0.6 + 1.8 + 4 + 7.5 = 14$ M1
- $14 - 3.5^2$: **1.75** A1
- (e) $(4, 5), (5, 4)$: $2(0.25)(0.3)$ M1
- 0.15** A1
- (f) $\text{sum } 7: 2(0.15)(0.3) + 2(0.2)(0.25) = 0.19; X_1 = 2: 0.15 \times 0.3 = 0.045$ M1 M1
- $\frac{0.045}{0.19}$: $\frac{9}{38} \approx 0.237$ A1

